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Coupled Fixed Point Theorems in S-metric Spaces

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ABSTRACT *: The aim of this paper is to establish some coupled fixed point results in S-metric space which are the generlaizations of some fixed point theorems in metric spaces* [6].

MSC: *47H10, 54H25*

KEYWORDS *- Coupled fixed point, mixed monotone property, S-metric space, ICS property.*

I. INTRODUCTION

In 1922, the Polish mathematicians, Banach, proved a theorem which ensures, under approiate conditions, the existence and uniqueness of fixed point. This result is called Banach's fixed point theorem or Banach contraction principle. Many authors extend this principle in different ways and imporoved it. Bahaskar and Lakshikanthamin [1] introduced the concept of coupled fixed point of a mapping and proved some coupled fixed point results in partially ordered metric spaces. After that many result are obtained by several authors. Mustafa and Sims [4] introduced the concept of generalized metric space called G-metric space. Now, recently Sedghi et al. [3] have introduced the notion of S-metric spaces as the generalization of G-metric and D^*-metric spaces. After this they proved some fixed point theorems in S-metric spaces. After that several authors obtained many result on S-metric space.

II. PRELIMINARIES

Definition 1 ([3]). Let X be a nonempty set. An S-metric on X is a function $S : X^3 \to [0,\infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$.

(i) $S(x, y, z) \geq 0$

(ii) $S(x, y, z) = 0$ if and only if $x = y = z$

(iii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$

Then the pair (X, S) is called an S-metric space.

Definition 2 ([1]). Let $(X, \leq)$ be a partially ordered set and $F : X \times X \to X$ be a mapping. F is said to have the mixed monotone property if $F(x, y)$ is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any $x, y \in X$,

$x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$, for $x_1, x_2 \in X$ and

$y_1 \leq y_2 \Rightarrow F(x, y_2) \leq F(x, y_1)$, for $y_1, y_2 \in X$.

Definition 3 ([1]). An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

For further definition see [7-10].

Definition 4. Let (X, d) be a metric space. A mapping $T : X \to X$ is said to be ICS if T is injective, continuous and has the property: for every sequence $\{x_n\}$ in X, if $\{Tx_n\}$ is convergent then $\{x_n\}$ is also convergent.

Let Φ be the set of all functions $\phi : [0,1) \to [0,1)$ such that

(i) ϕ is non-decreasing,

(ii) $\phi(t) < t$ for all $t > 0$,

(iii) $\lim_{r \to t^+} \phi(r) < t$ for all $t > 0$.

Definition 5 ([1]). Let $(X, \leq)$ be a partially ordered set equipped with a metric S such that (X, S) is a metric space. Further, equip the product space $X \times X$ with the following partial ordering: for $(x, y), (u, v) \in X \times X$, define $(u, v) \leq (x, y) \Leftrightarrow x \geq u, y \leq v$.

Theorem 1. Let $(X, \leq)$ be a partially ordered set and suppose there is a metric S on X such that (X, S) is complete S-metric space. Suppose $T : X \to X$ is an ICS mapping and $F : X \times X \to X$ is such that F has the mixed monotone property. Assume that there exists $\phi \in \Phi$ such that

$$S(TF(x, y), TF(x, y), TF(u, v)) \leq \phi(\max\{S(Tx, Tx, Tu), S(Ty, Ty, Tv)\}) \tag{1}$$

for any $x, y, u, v \in X$ for which $x \leq u$, $v \leq y$.

Suppose either

(a) F is continuous or

(b) X has the following property:

(i) if non-decreasing sequence $x_n \to x$, then $x_n \leq x$ for all n.

(ii) if non-increasing sequence $y_n \to y$, then $y_n \geq y$ for all n.

If there exist $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$, $y_0 \geq F(y_0, x_0)$, then there exist $x, y \in X$ such that $F(x, y) = x$, $F(y, x) = y$

that is F has a coupled fixed point.

Proof. Let $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$. Define

$$x_1 = F(x_0, y_0),\ y_1 = F(y_0, x_0) \tag{2}$$

Like this, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$x_{n+1} = F(x_n, y_n), \text{ and } y_{n+1} = F(y_n, x_n) \tag{3}$$

Since F has the mixed monotone property, so it is easy that

$$x_n \leq x_{n+1},\ \ y_{n+1} \leq y_n \text{ for } n = 0,1,2,...$$

Let for some $n \in N$, we take

$$x_n = x_{n+1} \text{ or } y_n = y_{n+1}$$

then form (3), F has the coupled fixed point.

Let if possible, for any $n \in N$,

$$x_n \neq x_{n+1} \text{ or } y_n \neq y_{n+1}. \tag{4}$$

Since T is injective, then for any $n \in N$, by (4)

$$0 < \max\{S(Tx_n, Tx_n, Tx_{n+1}), S(Ty_n, Ty_n, Ty_{n+1})\} \tag{5}$$

Using (1) and (3), we get

$$\begin{aligned} S(Tx_n, Tx_n, Tx_{n+1}) &= S(TF(x_{n-1}, y_{n-1}), TF(x_{n-1}, y_{n-1}), TF(x_n, y_n)\}. \\ &\leq \phi(\max\{S(Tx_{n-1}, Tx_{n-1}, Tx_n), S(Ty_{n-1}, Ty_{n-1}, Ty_n)\}) \end{aligned} \tag{6}$$

Also

$$\begin{aligned} S(Ty_n, Ty_n, Ty_{n+1}) &= S(Ty_{n+1}, Ty_{n+1}, Ty_n) \\ &= S(TF(y_n, x_n), TF(y_n, x_n), TF(y_{n-1}, x_{n-1})\}. \\ &\leq \phi \max\{S(Ty_n, Ty_n, Ty_{n-1}), S(Tx_n, Tx_n, Tx_{n-1})\} \end{aligned}$$

Also, we are given that $\phi(t) < t$ for all $t > 0$, so from (6) and (7), we set

$$\begin{aligned} 0 &< (\max\{S(Tx_n, Tx_n, Tx_{n+1}), S(Ty_n, Ty_n, Ty_{n+1})\}) \\ &\leq \phi(\max\{S(Tx_{n-1}, Tx_{n-1}, Tx_n), S(Ty_{n-1}, Ty_{n-1}, Ty_n)\}) \\ &< (\max\{S(Tx_{n-1}, Tx_{n-1}, Tx_n), S(Ty_{n-1}, Ty_{n-1}, Ty_n)\}) \end{aligned} \tag{8}$$

This shows that

$$\max\{S(Tx_n,Tx_n,Tx_{n+1}),S(Ty_n,Ty_n,Ty_{n+1})\}<\max\{S(Tx_{n-1},Tx_{n-1},Tx_n),S(Ty_{n-1},Ty_{n-1},Ty_n)\}$$

Thus $(\max\{S(Tx_n,Tx_n,Tx_{n+1}),S(Ty_n,y_n,y_{n+1})\})$ is a positive decreasing sequence, so there exist $r\geq 0$ such that

$$\lim_{n\to\infty}\max\{S(Tx_n,Tx_n,Tx_{n+1}),S(Ty_n,y_n,y_{n+1})\}=r$$

Assume that $r>0$. Letting $n\to\infty$ in (8), we get

$$\begin{aligned}0<r&\leq\lim_{n\to\infty}\phi(\max\{S(Tx_{n-1},Tx_{n-1},Tx_n),S(Ty_{n-1},Ty_{n-1},Ty_n)\})\\&\leq\lim_{t\to r^+}\phi(t)<r\end{aligned}\tag{9}$$

which is a contradiction, so we deduce that

$$\lim_{n\to\infty}\max\{S(Tx_n,Tx_n,Tx_{n+1}),S(Ty_n,Ty_n,Ty_{n+1})\}=0\tag{10}$$

Now, we shall prove that $\{Tx_n\}$ and $\{Ty_n\}$ are Cauchy sequences. Suppose, on the contrary, $\{Tx_n\}$ or $\{Ty_n\}$ is not a Cauchy sequences, that is,

$$\lim_{n,m\to\infty}S(Tx_m,Tx_m,Tx_n)\neq 0$$

or

$$\lim_{n,m\to\infty}S(Ty_m,Ty_m,Ty_n)\neq 0$$

This means that there exist $\varepsilon>0$ for which we can find subsequences of integers (m_k) and (n_k) with $n_k>m_k>k$ such that

$$\max\{S(Tx_{m_k},Tx_{m_k},Tx_{n_k}),S(Ty_{m_k},Ty_{m_k},Ty_{n_k})\}\geq\varepsilon\tag{11}$$

Again correspoidng to m_k we can find n_k in such a way that it is the smallest integers with $n_k>m_k$ and satisfying (11). Then

$$\max\{S(Tx_{m_k},Tx_{m_k},Tx_{n_k-1}),S(Ty_{m_k},Ty_{m_k},Ty_{n_k-1})\}<\varepsilon\tag{12}$$

with the help of triangular inequality and (12), we get

$$\begin{aligned}S(Tx_{m_k},Tx_{m_k},Tx_{n_k})&\leq S(Tx_{m_k},Tx_{m_k},Tx_{m_k-1})+S(Tx_{m_k},Tx_{m_k},Tx_{m_k-1})+S(Tx_{n_k},Tx_{n_k},Tx_{m_k-1})\\&<0+0+\varepsilon\end{aligned}.$$

Thus by (10), we get

$$\lim_{k\to\infty}S(Tx_{m_k},Tx_{m_k},Tx_{n_k})\leq\varepsilon\tag{13}$$

Similarly, we get

$$\lim_{k\to\infty}S(Ty_{m_k},Ty_{m_k},Ty_{n_k})\leq\varepsilon\tag{14}$$

Also,

$$\begin{aligned}S(Tx_{m_k-1},Tx_{m_k-1},Tx_{n_k-1})&\leq S(Tx_{m_k-1},Tx_{m_k-1},Tx_{m_k-2})+S(Tx_{m_k-1},Tx_{m_k-1},Tx_{m_k-2})+S(Tx_{n_k-1},Tx_{n_k-1},Tx_{m_k-2})\\&\leq 2S(Tx_{m_k-1},Tx_{m_k-1},Tx_{m_k-2})+\varepsilon\end{aligned}$$

Using (10), we get

$$\lim_{k\to\infty}S(Tx_{m_k-1},Tx_{m_k-1},Tx_{n_k-2})\leq\varepsilon\tag{15}$$

Similarly, we get

$$\lim_{k\to\infty}S(Ty_{m_k-1},Ty_{m_k-1},Ty_{m_k-2})\leq\varepsilon\tag{16}$$

Using (11) and (13)-(16), we get

$$\begin{aligned}&\lim_{n\to\infty}\max\{S(Tx_{m_k},Tx_{m_k},Tx_{n_k}),S(Ty_{m_k},Ty_{m_k},Ty_{n_k})\}\\&=\lim_{n\to\infty}\max\{S(Tx_{m_k-1},Tx_{m_k-1},Tx_{n_k-1}),S(Ty_{m_k-1},Ty_{m_k-1},Ty_{n_k-1})\}\\&=\varepsilon\end{aligned}\tag{17}$$

Now, by inequality (1), we get

$$\begin{aligned}S(Tx_{m_k},Tx_{m_k},Tx_{n_k})&=S(TF(x_{m_k-1},y_{m_k-1}),TF(x_{m_k-1},y_{m_k-1}),TF(x_{n_k-1},y_{n_k-1}))\\&\leq\phi[\max\{S(Tx_{m_k-1},Tx_{m_k-1},Tx_{n_k-1}),S(Ty_{m_k-1},Ty_{m_k-1},Ty_{n_k-1})\}]\end{aligned}\tag{18}$$

and

$$\begin{aligned} S(Ty_{m_k}, Ty_{m_k}, Ty_{n_k}) &= S(TF(y_{m_k-1}, x_{m_k-1}), TF(y_{m_k-1}, x_{m_k-1}), TF(y_{n_k-1}, x_{n_k-1})) \\ &\le \phi[\max\{S(Ty_{m_k-1}, Ty_{m_k-1}, Ty_{n_k-1}), S(Tx_{m_k-1}, Tx_{m_k-1}, Tx_{n_k-1})\}] \end{aligned} \tag{19}$$

Now, by (18) and (19), we obtain that

$$\begin{aligned} &\max\{S(Tx_{m_k}, Tx_{m_k}, Tx_{n_k}), S(Ty_{m_k}, Ty_{m_k}, Ty_{n_k})\} \\ &\qquad \le \phi[\max\{S(Tx_{m_k-1}, Tx_{m_k-1}, Tx_{n_k-1}), S(Ty_{m_k-1}, Ty_{m_k-1}, Ty_{n_k-1})\}] \end{aligned} \tag{20}$$

Assuming $k \to \infty$, in (20), and using (17), we obtain

$$0 < \varepsilon \le \lim_{t \to \varepsilon^+} \phi(t) < \varepsilon$$

which is a contradiction. Hence $\{Tx_n\}$ and $\{Ty_n\}$ are Cauchy sequences in (X, S). Since X is complete S-metric space, $\{Tx_n\}$ and $\{Ty_n\}$ are convergent sequences.

Since T is an ICS mapping, there exist $x, y \in X$ such that

$$\lim_{n\to\infty} x_n = x \text{ and } \lim_{n\to\infty} y_n = y \tag{21}$$

Sicne T is continuous, we get

$$\lim_{n\to\infty} Tx_n = Tx \text{ and } \lim_{n\to\infty} Ty_n = Ty \tag{22}$$

Now, assume that the assumption (a) holds, that is F is continuous. By (3), (21) and (22) we have

$$\begin{aligned} x &= \lim_{n\to\infty} x_{n+1} \\ &= \lim_{n\to\infty} F(x_n, y_n) \\ &= F\left[\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\right] \\ &= F(x, y) \end{aligned}$$

and

$$\begin{aligned} y &= \lim_{n\to\infty} y_{n+1} \\ &= \lim_{n\to\infty} F(y_n, x_n) \\ &= F\left[\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n\right] \\ &= F(y, x) \end{aligned}$$

Thus we have show that F has a coupled fixed point. Suppose, now the assumption (b) holds. Since $\{x_n\}$ is non decreasing with $x_n \to x$ and also $\{y_n\}$ is non increasing with $y_n \to y$. Then by assumption (b), we get $x_n \le x$ and $y_n \ge y$ for all n.

Now,

$$\begin{aligned} S(Tx, Tx, TF(x, y)) &\le S(Tx, Tx, Tx_{n+1}) + S(Tx, Tx, Tx_{n+1}) + S(TF(x, y), TF(x, y), Tx_{n+1}) \\ &= 2S(Tx, Tx, Tx_{n+1}) + S(TF(x, y), TF(x, y), TF(x_n, y_n)) \\ &\le 2S(Tx, Tx, Tx_{n+1}) + \phi(\max\{S(Tx, Tx, Tx_n), S(Ty, Ty, Ty_n)\}) \end{aligned}$$

taking $\lim_{n\to\infty}$ and using (22), right hand side of this inequality is equal to zero.

Hence $S(Tx, Tx, TF(x, y)) = 0$.

So $Tx = TF(x, y)$ and sine T is injective so $x = F(x, y)$.

Similarly, we can show that $y = F(y, x)$.

Thus, we have shown F has a coupled fixed point in X.

Corollary 1. Let $(X, \le)$ be a partially ordered set and suppose those is a metric S on X such that (X, S) is a complete S-metric space. Suppose $T : X \to X$ is an ICS mapping and $F : X \times X \to X$ is such that F has the mixed monotone property.

Assume that there exists $\phi \in \Phi$ such that

$$S(TF(x,y),TF(x,y),TF(u,v)) \le \phi\left(\frac{S(Tx,Tx,Tu)+S(Ty,Ty,Tv)}{2}\right)$$

for any $x,y,u,v \in X$ for which $x \le u$ and $v \le y$. Suppose either

(a) F is continuous or

(b) X has the following properties:

(i) if non-decreasing sequence $x_n \to x$, then $x_n \le x$ for all n.

(ii) if non-increasing sequence $y_n \to y$, then $y \le y_n$ for all n.

If there exist $x_0, y_0 \in X$ such that

$x_0 \le F(x_0, y_0)$ and $F(y_0, x_0) \le y_0$

then there exist $x, y \in X$ such that

$F(x,y) = x$ and $F(y,x) = y$,

that is F has a coupled fixed point.

Proof. If can be easily shown as

$$\frac{S(Tx,Tx,Tu)+S(Ty,Ty,Tv)}{2} \le \max\{S(Tx,Tx,Tu), S(Ty,Ty,Tv)\}.$$

Then apply Theorem 1, because that ϕ is non-decreasing.

Corollary 2. Let $(X,\le)$ be a partially ordered set and suppose that there is a metric S on X such that (X,S) is a complete S-metric space. Suppose $T: X \to X$ is an ICS mapping and $F: X \times X \to X$ is such that F has the mixed monotone property. Assume that there exist $k \in [0,1)$ such that

$$S(TF(x,y),TF(x,y),TF(u,v)) \le k\max\{S(Tx,Tx,Tu), S(Ty,Ty,Tv)\}$$

for any $x,y,u,v \in X$ for which $x \le u$ and $v \le y$. Suppose either

(a) F is continuous or

(b) X has the following properties:

(i) if non-decreasing sequence $x_n \to x$, then $x_n \le x$ for all n.

(ii) if non-decreasing sequence $y_n \to y$, then $y \le y_n$ for all n.

If there exist $x_0, y_0 \in X$ such that

$x_0 \le F(x_0, y_0)$ and $F(y_0, x_0) \le y_0$

then there exist $x, y \in X$ such that

$F(x,y) = x$ and $F(y,x) = y$,

that is F has a coupled fixed point.

Proof. It follows by $\phi(t) = kt$ in Theorem 1.

Corollary 3. Let $(X,\le)$ be a partially ordered set and suppose that there is a metric S on X such that (X,S) is a complete S-metric on X. Suppose $T: X \to X$ is an ICS mapping and $F: X \times X \to X$ is such that F has the mixed monotone property. Assume that there exist $k \in [0,1)$ such that

$$S(TF(x,y),TF(x,y),TF(u,v)) \le \frac{k}{3}(S(Tx,Tx,Tu), S(Ty,Ty,Tv))$$

for any $x,y,u,v \in X$ for which $x \le u$ and $v \le y$. Suppose either

(a) F is continuous or

(b) X has the following properties:

(i) if non-decreasing sequence $x_n \to x$, then $x_n \leq x$ for all n.

(ii) if non-increasing sequence $y_n \to y$, then $y \leq y_n$ for all n.

If there exist $x_0, y_0 \in X$ such that

$$x_0 \leq F(x_0, y_0) \text{ and } F(y_0, x_0) \leq y_0$$

then there exist $x, y \in X$ such that

$$F(x,y) = x \text{ and } F(y,x) = y,$$

that is F has a coupled fixed point.

Proof. It follows by $\phi(t) = kt$ in Theorem 1.

Theorem 2. In addition to the condition of Theorem 1, suppose that for all $(x,y),(u,v) \in X \times X$, there exist $(a,b) \in X \times X$ such that $(F(a,b), F(b,a))$ is comparable to $(F(x,y), F(y,x))$ and $(F(u,v), F(v,u))$ then F has a unique coupled fixed point (x,y).

Proof. We know that the set of coupled fixed poit of F is not empty by Theorem 1. Suppose, now (x,y) and (u,v) are two coupled fixed point of F, that is

$$F(x,y) = x;\ F(y,x) = y \text{ and } F(u,v) = u,\ F(v,u) = v.$$

We shall prove that (x,y) and (u,v) are equal.

By our suppositon there exist $(a,b) \in X \times X$ such that $(F(a,b), F(b,a))$ is comparable to $(F(x,y), F(y,x))$ and $(F(u,v), F(v,u))$. Now construct sequences $\{a_n\}$ and $\{b_n\}$ such that $a_0 = a$, $b_0 = b$ and for any $n \geq 1$.

$$a_n = F(a_{n-1}, b_{n-1}),\ b_n = F(b_{n-1}, a_{n-1}) \quad \text{for all } n \tag{23}$$

Again set $x_0 = x$, $y_0 = y$ and $u_0 = u$, $v_0 = v$ and on the same way construct the sequence $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$. Then

$$x_n = F(x,y),\ y_n = F(y,x),\ u_n = F(u,v),\ v_n = F(v,u) \quad \text{for all } n \geq 1 \tag{24}$$

Since $(F(x,y), F(y,x)) = (x_1, y_1) = (x,y)$ is comparable to $(F(a,b), F(b,a)) = (a_1, b_1)$ then it is easy to show $(x,y) \geq (a_1, b_1)$.

Similarly, we have that

$$(x,y) \geq (a_n, b_n) \quad \text{for all } n \tag{25}$$

From (25) and (1), we get

$$\begin{aligned} S(Tx,Tx,Ta_{n+1}) &\leq S(TF(x,y), TF(x,y), TF(a_n,b_n)) \\ &\leq \phi(\max\{S(Tx,Tx,Ta_n), S(Ty,Ty,Tb_n)\}) \end{aligned} \tag{26}$$

and

$$\begin{aligned} S(Ty,Ty,Tb_{n+1}) &= S(Tb_{n+1}, Tb_{n+1}, Ty) \\ &= S(TF(b_n,a_n), TF(b_n,a_n), TF(y,x)) \\ &\leq \phi(\max\{S(Tb_n,Tb_n,Ty), S(Ta_n,Ta_n,Tx)\}) \end{aligned} \tag{27}$$

It follows, from (26) and (27), that

$$\phi\{(Tx,Tx,Ta_{n+1}), S(Ty,Ty,Tb_{n+1})\} \leq \phi[\max\{S(Tx,Tx,Ta_n), S(Ty,Ty,Tb_n)\}]$$

So for all $n \geq 1$

$$\max\{S(Tx,Tx,Ta_n), S(Ty,Ty,Tb_n)\} \leq \phi^n \max\{S(Tx,Tx,Ta_0), S(Ty,Ty,Tb_0)\} \tag{28}$$

But we know that $\phi(t) < t$ and $\lim_{r \to t^+} \phi(r) < t$ implies

$\lim_{n \to \infty} \phi^n(t) = 0$ for all $t > 0$.

Hence from (28), we get

$$\lim_{n\to\infty} \max\{S(Tx,Tx,Ta_n), S(Ty,Ty,Tb_n)\} = 0$$

This gives

$$\lim_{n\to\infty} \{S(Tx,Tx,Ta_n)\} = 0 \tag{29}$$

and

$$\lim_{n\to\infty} \{S(Ty,Ty,Tb_n)\} = 0 \tag{30}$$

Similarly we can show that

$$\lim_{n\to\infty} \{S(Tu,Tu,Ta_n)\} = 0 \tag{31}$$

and

$$\lim_{n\to\infty} \{S(Tv,Tv,Tb_n)\} = 0 \tag{32}$$

Adding (29)-(32), we get (Tx,Ty) and (Tu,Tv) are equal. The fact that T is injective gives us $x=u$ and $y=v$.

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