International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 2 Issue 5 || May. 2014 || PP-30-35

Necessary and Sufficient Conditions For Controllability Of Single-Delay Autonomous Neutral Control Systems And Applications

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ABSTRACT : *This article formulated and proved necessary and sufficient conditions for the Euclidean controllability of single-delay autonomous linear neutral control systems, in terms of rank conditions on some concatenated determining matrices.The proof was achieved by the exploitation of the structure of the determining matrices developed in Ukwu [1], the relationship among the determining matrices, the indices of control systems and system's coefficients of the relevant system,obtained in [1]and an appeal to Taylor's theorem , Knowles [2],as applied to vector functions.*

KEYWORDS:*Controllable, Euclidean, Rank, System, Taylor.*

I. INTRODUCTION

Controllability results for multifarious and specific types of hereditary systems with diversity in treatment approaches are quite prevalent in control literature. Angell [3] discussed controllability of nonlinear hereditary systems, using a fixed-point approach, Balachandran[4] discussed controllability of nonlinear systems with delays in both state and control using a constructive control approach and an appeal to Arzela-Ascoli and Shauder fixed point theorems to guarantee the existence and admissibility of such controls. Balachandran [5], Balachandran and Balasubramaniam[6]studied controllability of VolterraIntegro-differential systems. Bank and Kent [7] discussed Controllability of functional differential equations of retarded and neutral types to targets in function space; Dauer and Gahl[8] looked at controllability of nonlinear delay systems; Jacobs and Langenhop[9] obtained some criteria for function space controllability of linear neutral systems; Onwuatu [10] studied null controllability in function space of nonlinear neutral differential systems with limited controls; Underwood and Chukwu[11] investigated null controllability of nonlinear neutral differential equations, to mention just a few.

Gabasov and Kirilova formulated a necessary and sufficient condition for Euclidean controllability of system $\dot{x}(t) = Ax(t) + Bx(t-h) + Cu(t)$ with piecewise continuous controls using a sequence determining matrices for the free part of the above restricted system. Unfortunately, the investigation of the dependence of the controllability matrix for infinite horizon on that for finite horizon very crucial for his proof was not fully addressed.

Ukwu [12], developed computational criteria for the Euclidean controllability of the above delay system using the determining matrices witha very simple structure, effectively eliminating the afore-mentioned drawback.However,a major drawback of Ukwu's major result is that it relied on Manitius[13] for the necessary and sufficient conditions for the Euclidean controllability of the delay system , stated in terms of the transition matrices, which until [1] were a herculean or almost impossible task to obtain. Definitely, It would be a positive contribution to obtain computational criteria for Euclidean controllability of the more complex systems under consideration.Herein lies the justification for this investigation.

II. SYSTEM OF INTEREST, PRELIMINARY DEFINITIONS AND WORKING TOOLS

2.1 System of interest

Consider the autonomous linear differential – difference control system of neutral type:

$$\dot{x}(t) = A_{-1}\dot{x}(t-h) + A_0 x(t) + A_1 x(t-h) + B\,u(t);\ t \ge 0 \quad (1)$$

$$x(t) = \phi(t),\ t \in [-h, 0],\ h > 0 \quad (2)$$

where A_{-1}, A_0, A_1 are $n \times n$ constant matrices with real entries and B is an $n \times m$ constant matrix with the real entries. The initial function ϕ is in $C([-h, 0], \mathbf{R}^n)$ equipped with sup norm. The control u is in $\Omega \subseteq L_\infty([0, t_1], \mathbf{R}^n)$. Such controls will be called admissible controls. $x(t), x(t-h) \in \mathbf{R}^n$ for $t \in [0, t_1]$. If $x \in C([-h, t_1], \mathbf{R}^n)$, then for $t \in [0, t_1]$ we define $x_t \in C([-h, 0], \mathbf{R}^n)$ by

. $x_t(s) = x(t+s), \; s \in [-h, 0]$

Let:

$$\hat{Q}_n(t_1) = [Q_0(s)B, Q_1(s)B, \cdots, Q_{n-1}(s)B : s \in [0, t_1), s = 0, h, \cdots, (n-1)h], \tag{3}$$

where $Q_k(s)$ is a determining matrix for the uncontrolled part of (1) and satisfies

$$Q_k(s) = A_{-1}Q_k(s-h) + A_0\, Q_{k-1}(s) + A_1\, Q_{k-1}(s-h)$$

for $k = 0, 1, ...; \; s = 0, h, 2h, ...$ subject to $Q_0(0) = I_n$, the $n \times n$ identity matrix

and $\quad Q_k(s) = 0 \quad$ for $\; k < 0 \;$ or $\; s < 0$.

It is proved in [1], among other alternative expressionsfor $Q_k(jh)$, that

$$\begin{aligned} \mathrm{Q}_k(jh) \; = & \sum_{(v_1,\cdots,v_{j+k}) \in P_{-1(j),0(k)}} A_{v_1} \cdots A_{v_{j+k}} \\ & + \left[\sum_{(v_1,\cdots,v_j) \in P_{-1(j-k),1(k)}} A_{v_1} \cdots A_{v_j} + \sum_{r=1}^{k-1} \sum_{(v_1,\cdots,v_{j+r}) \in P_{-1(r+j-k),0(r),1(k-r)}} A_{v_1} \cdots A_{v_{j+r}} \right] \mathrm{sgn}(\max\{0, j+1-k\}), \\ & + \left[\sum_{(v_1,\cdots,v_k) \in P_{0(k-j),1(j)}} A_{v_1} \cdots A_{v_k} + \sum_{r=1}^{j-1} \sum_{(v_1,\cdots,v_{k+r}) \in P_{-1(r),0(r+k-j),1(j-r)}} A_{v_1} \cdots A_{v_{k+r}} \right] \mathrm{sgn}(\max\{0, k-j\}) \end{aligned}$$

for positive integral j and k.

2.2 Definition of global Euclidean controllability

The system (1) is said to be Euclidean controllable if for each $\phi \in C([-h, 0], \mathbf{R}^n)$ defined by:

$$\phi(s) = g(s), s \in [-h, 0), \phi(0) = g(0) \in \mathbf{R}^n \tag{4}$$

and for each $x_1 \in \mathbf{R}^n$, there exists a t_1 and an admissible control $u \in \Omega$ such that the solution(response) $x(t, \phi, u)$ of (1) satisfies $x_0(\phi, u) = \phi$ and $x(t_1; \phi, u) = x_1$.

2.3 Definition of Euclidean controllability on an interval

Let $x(t, \phi, u)$ denote the solution of system (1) with initial function ϕ and admissible control u at time t. System (1) is said to be Euclidean controllable on the interval $[0, t_1]$, if for each ϕ in $C([-h, 0], \mathbf{R}^n)$ and $x_1 \in \mathbf{R}^n$, there is an admissible control $u \in L_\infty([0, t_1], \mathbf{R}^n)$ such that $x_0(\phi, u) = \phi$ and $x(t_1, \phi, u) = x_1$. System (1) is Euclidean controllable if it is Euclidean controllable on every interval $[0, t_1]$, $t_1 > 0$.

Our objective

We wish to establish necessary and sufficient conditions for the Euclidean controllability of system (1) on the interval $\left[0, t_1\right]$.

III. RESULT

3.1 Theorem on rank condition for Euclidean controllability of system (1)

Let $\hat{Q}_n(t_1)$ be defined as in (3). Then system (1) is Euclidean controllable $\left[0, t_1\right]$ if and only if $\operatorname{rank}\left[\hat{Q}_n(t_1)\right] = n.$ Moreover $\hat{Q}_n(t_1)$ and dim $\hat{Q}_n(t_1)$ are expressible in the form

$$\hat{Q}(t_1) = \left[\; Q_k(s)B : k \in \{0,1,\cdots,n-1, s \in \{0,h,\cdots,\left(\min\left\{(n-1), \left[\left[\left[\frac{t_1-h}{h}\right]\right]\right]\right\}\right)h\;\right]$$

$$\mathrm{Dim}\left[\hat{Q}_n(t_1)\right] = n \times mn\left(\min\left\{\left[\left[\left[\frac{t_1}{h}\right]\right]\right],\, n\right\}\right) = n \times mn\left(1 + \min\left\{\left[\left[\left[\frac{t_1-h}{h}\right]\right]\right],\, n-1\right\}\right).$$

Here $[[[.]]]$ denotes the least integer function, otherwise referred to as the ceiling function in Computer Science.

Proof

By Chukwu([14] pp. 341-345), system (1) is Euclidean controllability on $\left[0, t_1\right]$ if and only if $c^{\mathrm{T}} X\left(\tau, t_1\right) B \neq 0$ for any $c \in \mathbf{R}^n$, $c \neq 0$, where $\tau \to X\left(\tau, t_1\right)$ denotes the control index matrix of (1) for fixed t_1.

Sufficiency**:** First we prove that if $\hat{Q}_n(t_1) = n,$ then (1) is Euclidean controllable on $\left[0, t_1\right]$. Equivalently we prove that if (1) is not Euclidean controllable on $\left[0, t_1\right]$, then rank $\hat{Q}_n(t_1) < n$ because $\hat{Q}_n(t_1)$ has n rows and therefore has rank at most n. Suppose that system (1) is not Euclidean controllable on $\left[0, t_1\right]$. Then these exists a nonzero column vector $c \in \mathbf{R}^n$ such that:

$$c^{\mathrm{T}} X\left(\tau, t_1\right) B \equiv 0; \tau \in [0, t_1] \tag{5}$$

But:

$$X\left(\tau, t_1\right) \equiv 0 \text{ , on} \left(t_1, \infty\right) \tag{6}$$

Therefore:

$$c^{\mathrm{T}} X\left(\tau, t_1\right) B \equiv 0, \text{ on } \;\tau \in \left[0, \infty\right] \tag{7}$$

yielding:

$$c^{\mathrm{T}} X^{(k)}\left(\left(t_1 - jh\right)^{-}, t_1\right) B = 0,\;\; c^{\mathrm{T}} X^{(k)}\left(\left(t_1 - jh\right)^{-}, t_1\right) B = 0 \tag{8}$$

for all integers $j : t_1 - jh > 0,\, k \in \{0,1,2,...\}$.

Now:

$$\Delta X^{(k)}\left(t_1 - jh, t_1\right) = \left(-1\right)^k Q_k\left(jh\right), \tag{9}$$

for $j : t_1 - jh > 0,\, k \in \{0,1,2,...\}$, by theorem 3.1 of [15].

From (8) and (9) we deduce that:

$$\left(-1\right)^k c^{\mathrm{T}} Q_k\left(jh\right) = 0 \tag{10}$$

for some $c \in \mathbf{R}^n$, $c \neq 0$, for all $j : t_1 - jh > 0,\, k \in \{0,1,2,...\}$.

By virtue of (3) and theorem 3.6 of [15], condition (10) implies that the nonzero vector c is orthogonal to all columns of $\hat{Q}_\infty(t_1)$ and hence orthogonal to all columns of $\hat{Q}_n(t_1)$. Thus $\hat{Q}_n(t_1)$ does not have full rank. Since $\hat{Q}_n(t_1)$ has n rows, we deduce that:

$$\operatorname{rank} \hat{Q}_n(t_1) < n \tag{11}$$

(11) proves the contra-positive statement: $\operatorname{rank} \hat{Q}_n(t_1) = n \Rightarrow$ (1) is Euclidean controllable on $[0, t_1]$.

Necessity: Suppose that $\operatorname{rank} \hat{Q}_\infty(t_1) < n$. Then $\exists\, c \in \mathbf{R}^n, c \neq 0$ such that:

$$c^{\mathrm{T}} Q_k(s) B = 0, \quad \text{for all} \quad s \in [0,\ t_1] \text{ and } k \in \{0,1,2,...\}. \tag{12}$$

From theorem 3.1 of [15],

$$\begin{aligned} 0 &= (-1)^k c^{\mathrm{T}}\, Q_k(jh) B = c^{\mathrm{T}}\, \Delta\, X^{(k)}(t_1 - jh, t_1) B \\ &= c^{\mathrm{T}} \left[X^{(k)}\left((t_1 - jh)^-, t_1\right) - X^{(k)}\left((t_1 - jh)^+, t_1\right) \right] B, \end{aligned} \tag{13}$$

for nonnegative integral j: $t_1 - jh > 0$. From (13), we deduce that:

$$c^{\mathrm{T}}\, X^{(k)}\left((t_1 - jh)^-, t_1\right) B = c^{\mathrm{T}} X^{(k)}\left((t_1 - jh)^+, t_1\right) B \tag{14}$$

for $k \in \{0,1,2,....\}$ and $j : t_1 - jh > 0$. (14) is equivalent to:

$$\psi^{(k)}\left(c, (t_1 - jh)^-\right) = \psi^{(k)}\left(c, (t_1 - jh)^+\right) \tag{15}$$

for $k \in \{0,1,2,....\}$ and $j : t_1 - jh > 0$. In particular, if $j = 0,$ then (15) yields:

$$\psi^{(k)}\left(c, t_1^-\right) = \psi^{(k)}\left(c, t_1^+\right) \tag{16}$$

But:

$$\psi^{(k)}\left(c, t_1^+\right) = \lim_{\substack{\tau \to t_1 \\ \tau \in (t_1, t_1 + h)}} \psi^{(k)}(c, \tau) = \lim_{\substack{\tau \to t_1 \\ \tau \in (t_1, t_1 + h)}} c^{\mathrm{T}} X^{(k)}(\tau, t_1) B = 0 \tag{17}$$

since $\mathrm{X}^{(k)}(t_1, \tau) \equiv 0$ for all $\tau \in (\mathrm{t}_1, \infty)$ and $k = 0,1,2,...$

Therefore:

$$\psi^{(k)}\left(c,\ t_1^-\right) = \psi^{(k)}\left(c,\ t_1^+\right) = 0 \tag{18}$$

for $k \in \{0,1,2,....\}$. In particular the left continuity of $X(t_1, \tau)$ at $\tau = t_1$ implies that of $\psi(c, \tau)$ at $\tau = t_1$.
Hence:

$$\psi(c, t_1) = \psi\left(c, t_1^-\right) \tag{19}$$

But:

$$\psi\left(c,\ t_1^-\right) = \psi\left(c,\ t_1^+\right) \tag{20}$$

$$\psi(c, t_1) = \psi\left(c, t_1^-\right) = \psi\left(c, t_1^+\right) = 0 \tag{21}$$

Since $\tau \to \psi(c, \tau)$ is piecewise analytic for $\tau \in (t_1 - (j+1)h, t_1 - jh)$, for all $j : t_1 - (j+1)h > 0,$ we may apply Taylor's theorem to each component of $\psi\ (c,\ \tau)$ for the rest of the proof.

Set $a = t_1$. Now each component of the m-vector function $\psi(c, \tau)$ satisfies the hypothesis of Taylor's theorem, with $a = t_1$, because $\psi\ (c,\ \tau)$ is analytic on $(t_1 - (j+1)h,\ t_1 - jh)$, $j \in \{0,1,...\}$ such that $t_1 - (j+1)h > 0$.

Denote the i^{th} component of $\psi^{(k)}(c, \tau)$ by $\psi_i^{(k)}(c, \tau)$; $i \in \{1, 2, ..., m\}$

Then by Taylor's theorem,

$$\psi_i(c,\tau) = \sum_{k=0}^{\infty} \frac{\psi_i^{(k)}(c,t_1^-)(\tau - t_1)^k}{k!} \tag{22}$$

for all $\tau \in (t_1 - h, t_1)$. From (21) we deduce that:

$$\psi_i(c,\tau) = 0 \tag{23}$$

for all $\tau \in (t_1 - h,\ t_1];\ \ i = 1,\ 2, \ldots, m$

Now set $a = t_1 - h,\ a - h = t_1 - 2h$. By (15) and (23) we deduce that:

$$\psi_i^{(k)}\left(c,(t_1 - h)^-\right) = \psi_i^{(k)}\left(c,(t_1 - h)^+\right) = 0 \tag{24}$$

By Taylor's theorem, applied on the τ - interval $(t_1 - 2h,\ t_1 - h)$:

$$\psi_i(c,\tau) = \sum_{k=0}^{\infty} \frac{\psi_i^{(k)}\left(c,(t_1 - h)^-\right)\left(\tau - (t_1 - h)\right)^k}{k!} \tag{25}$$

for $i \in \{1, 2, \cdots, m\}$, for all $\tau \in (t_1 - 2h, t_1 - h)$.But $\psi_i\left(c,(t_1 - h)^-\right) = \psi_i(c, t_1 - h)$.
Hence $\psi_i(c,\tau) \equiv 0$ on $(t_1 - 2h, t_1 - h)$. Continuing in the above fashion we get $\psi_i(c.\tau) = 0$, for all $\tau \in (0, h]$ for $i \in \{1, 2, \cdots, m\}$. Finally we use the fact that $X(0,t_1) = X(0^+, t_1)$ to deduce that $\psi(c,0) = \psi(c,0^+) = 0$. Hence $\psi(c,\tau) = 0$, for all $t \in [0, t_1]$; that is, $\exists\ c \in \mathbf{R}^n,\ c \neq 0$ such that:

$$c^{\mathrm{T}} X(\tau, t_1) B \equiv 0 \ \text{ on } \ [0, t_1] \tag{26}$$

We immediately invoke Chukwu to deduce that system (1) is not Euclidean controllable on $[0, t_1]$ for any $t_1 > 0$. This proves that if the system (1) is Euclidean controllable on $[0, t_1]$ then $\hat{Q}_\infty(t_1)$ attains its full rank, *n*. By theorem 3.6 of [m?], $\operatorname{rank} \hat{Q}_\infty(t_1) = \operatorname{rank} \hat{Q}_n(t_1)$.
Hence:

$$\operatorname{rank} \hat{Q}_n(t_1) = n \tag{27}$$

Observe that for any given $t_1 > 0, \exists$ a non-negative integer $p : t_1 = ph + \sigma,$ for some $0 \le \sigma < h$;

thus $\left[\left[\left[\dfrac{t_1 - h}{h}\right]\right]\right] = \begin{cases} p, & \sigma \neq 0 \\ p - 1, & \sigma = 0 \end{cases}$, proving the computable expression for $\left[\hat{Q}_n(t_1)\right]$.

The expression for the dimension follows from the fact that there are altogether

$n\left(1 + \min\left\{\left[\left[\left[\dfrac{t_1 - h}{h}\right]\right]\right],\ n - 1\right\}\right)$ column-wise concatenated matrices in $\hat{Q}_n(t_1)$, each of dimension $n \times m$.

This completes the proof of the theorem.

IV. CONCLUSION

This article pioneered the introduction of the least integer function in the statement and proof of the necessary and sufficient conditions for the Euclidean controllability of linearhereditary systems; this makes the controllability matrix in (3) quite computable and eliminates any ambiguity that could arise in its application. The proof relied on the results in [1,15], incorporated the characterization of Euclidean controllability in terms of the indices of control systems and appropriated Taylor's theorem as an indispensible tool.

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