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Volume Discount/Price Regime Implementation on Microsoft Excel Platform

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ABSTRACT : *Models were developed for volume discounts / price regime deterministic demand processes, incorporating uncapacitated and capacitated supply systems. Then, the solutions were implemented on Microsoft Excel platform by skillfully exploiting the composition of the minimum and maximum values concepts. Finally, the Goal Seek procedure was exposed to determine demand levels that would achieve specified revenue targets, with a note on the vexed issue of feasibility.*

KEYWORDS: *Code, Demand, Excel, Feasibility, Price, Revenue*

I. INTRODUCTION

Consider a class of inventory problems in which the unit price is variable and depends on the quantity to be purchased. Interest is usually focused on the revenue to be earned at given demand levels, how often purchases are to be made and how many units should be ordered during any ordering period. Winston [1] determined the economic order quantity (EOQ) for various time breaks without shortages and with backlogging respectively; Hillier and Lieberman [2] determined the first economic lot-size model; Taha[3] considered the case of EOQ with two price breaks; Verma[4] determined the optimal ordering policy for a four price-break model; Gupta and Hira[5] examined two inventory model instances with three and four price break conditions in which there are no optimal ordering policies corresponding to certain price breaks due to the infeasibility of the Economic order quantities. Unfortunately thus far, there is no unifying single generic formula for the revenue function and its implementation needed for the economic order quantity (EOQ). This article fills this yawning gap.

II. PRELIMINARIES

We investigate the revenue accruable in the problem of a vendor offering price breaks based on quantities demanded of a particular product for various demand and ordering policy contingencies. For definiteness, let the demand process with thecorresponding price regime in arbitrary monetary units be as tabulated below:

Given product	Unit price
First q_1 units	p_1
Next q_2 units	p_2
Next q_3 units	p_3
…	…
Next q_{k-1} units	p_{k-1}
Next further units	P_k

Table 1:Price Breaks

Two cases are examined.

Case 1: Determine the revenue accruing to the vendor when actual demand level is v and the supply capacity is unconstrained,

Case 2: Determine the revenue when actual demand level is v and the supply capacity is constrained.

A note on case 1: The feasibility of case 1 is established if backlogging of demand is allowed, when supply is exceeded. The main results of this paper are collected into the five sections3 through7. Section 3 deals with the formulation and derivation of the revenue formula for specified demand levels with respect to uncapacitated and capacitated supplies.Section 4 is concerned with the translation of above revenue formula into appropriate mathematical format needed for the implementation of the revenue formulas on Microsoft Excel platform.Section 5provides and implements the codeforthe revenue formula of section 4on Excel platform. Section 6 furnishes illustrative examples of above supply cases. Finally, in section 7, the Goal Seek procedure is used to determine demand levels that yield specified revenue targets. The results obtained in this procedure are optimal if they are nonnegative and discrete, or infeasible otherwise, in which case a satisficing solution obtained by rounding up or down the nearest integer may be considered, depending on the financial sensitivity of the given problem.

III. RESULTS AND DISCUSSIONS

3.1. Theorem on Revenue arising from case 1: unconstrained availability

Denote the revenue from *v* units by $R(v)$. Then $R(v)$ is given by:

$$R(v) = \begin{cases} p_1 v, \quad \text{if} \quad 0 \le v \le q_1 & (1) \\ p_1 q_1 + p_2 (v - q_{1)} \ \text{if} \ q_1 \le v \le q_1 + q_2 & (2) \\ p_1 q_1 + p_2 q_2 + p_3 (v - (q_1 + q_2)) \ \text{if} \ q_1 + q_2 \le v \le q_1 + q_2 + q_3 & (3) \\ \vdots & \\ \sum_{j=1}^{k-2} p_j q_j + p_{k-1}\left(v - \sum_{j=1}^{k-2} p_j q_j \right) \ \text{if} \ \sum_{j=1}^{k-2} q_j \le v \le \sum_{j=1}^{k-1} q_j & (k-1) \\ \sum_{j=1}^{k-1} p_j q_j + p_k \left(v - \sum_{j=1}^{k-1} q_j \right) \ \text{if} \ v \ge \sum_{j=1}^{k-1} q_j & (k) \end{cases}$$

Proof

Proof is by mathematical induction on *v*. If $0 \le v \le q_1$, then the unit price p_1 is applicable. The obtainable revenue is the product of the number of units demanded and the unit price on offer. Thus, $R(v) = p_1 v,$ for $0 \le v \le q_1$. Assume that the relation is true for *v* such that $\sum_{j=1}^{i-1} q_j \le v \le \sum_{j=1}^{i} q_j$, for some $2 \le i < k$. Then by the induction hypothesis, the revenue corresponding to *v* is $\sum_{j=1}^{i-1} p_j q_j + p_i \left(v - \sum_{j=1}^{i-1} q_j . \right)$

Suppose $i \le k-2$. If $\sum_{j=1}^{i} q_j \le v \le \sum_{j=1}^{i+1} q_j$, then the demand $\sum_{j=1}^{i} q_j$ is first satisfied, with corresponding revenue $\sum_{j=1}^{i} p_j q_j = \sum_{j=1}^{i+1-1} p_j q_j$. The revenue corresponding to the left-over units is $p_{i+1}\left(v - \sum_{j=1}^{i} q_j \right)$. Hence the total revenue is $\sum_{j=1}^{i+1-1} p_j q_j + p_{i+1}\left(v - \sum_{j=1}^{i+1-1} q_j \right)$.

Finally, if $i = k-1$ and $v \ge \sum_{j=1}^{i} q_j$, then $R(v) = \sum_{j=1}^{i+1-1} p_j q_j + p_{i+1}\left(v - \sum_{j=1}^{i+1-1} q_j \right)$.

Therefore, the relation is true with respect to the index *i* +1. This proves the theorem.

3.2. Corollary on Revenue arising from case 2: constrained availability/supply

Denote the supply capacity by s, set $q_k = \max\left\{ s - \sum_{j=1}^{k-1} q_j . \ 0 \right\}$, replace the expression $p_k\left(v - \sum_{j=1}^{k-1} q_j \right)$ in equation (k) of theorem 3.1 by $p_k q_k$. Replace v by $\min\{s, v\}$. Then $R(v)$ in the formula theorem 3.1 modifies to $R\left(\min\{s, v\}\right)$. The proof is immediate from table 1 and the proof of theorem 3.1.

The unsatisfied demand is given by $\max(\{v - s, 0\})$; this demand may be backlogged bymutual agreement.

IV. FORMAT OF REVENUE FORMULAS FOR EXCEL IMPLEMENTATION

In this section, the formulas obtained in section 3 will be translated into appropriate formats for Excel implementation, by manipulating the max / min concept. The utility / elegance of this translation can be appreciated by the fact that $R(v)$ collapses to only one expression. Compare this with the horrendous looking piece – wise expressions for $R(v)$ and $R(\min\{s,v\})$ in section 3. The embedded price break conditions called for a high degree of mathematical insight. Both cases will now be revisited.

Case 1

Suppose that v is the actual demand. Then v can be distributed among the price breaks $p_1, p_2, \cdots, p_k,$ to yield the following table:

Given product	Unit price	Demand
First q_1 units	p_1	v_1
Next q_2 units	p_2	v_2
Next q_3 units	p_3	v_3
…	…	…
Next q_{k-1} units	p_{k-1}	v_{k-1}
Further units	p_k	v_k

Table 2. Price breaks with actual demands

where $v = \sum_{j=1}^{k} v_j,\ v_j \geq 0;\ v_1 > 0,$ for otherwise $v=0,$ a vacuous case.

Above insight triggers the translation of $R(v)$ into the format given bythe theorembelow

Theorem 4.1: For any demand level v,

$$R(v) = \sum_{j=1}^{k-1} p_j \max\left\{\min\left\{v - \sum_{i=1}^{j-1} q_i,\ q_j\right\},\ 0\right\} + p_k \max\left\{v - \sum_{l=1}^{k-1} q_l,\ 0\right\},\ k \geq 1,$$

where $\sum_{j=1}^{0} f_j = 0.$

Proof

For any finite demand level v, the expression for the corresponding revenue, $R(v)$ is

$$\begin{aligned} R(v) = \quad & p_1 \min\{v,\ q_1\} + p_2 \max\{\min\{v - q_1,\ q_2\},\ 0\} + p_3 \max\{\min\{v-(q_1+q_2),\ q_3\},\ 0\} \\ & + \cdots + p_{k-1} \max\{\min\{\{v-(q_1+q_2+\cdots+q_{k-2}),\ q_{k-1}\},\ 0\} \\ & + p_k \max\{v-(q_1+q_2+\cdots+q_{k-2}+q_{k-1}),\ 0\}. \end{aligned}$$

Note that $\max\{\min\{v,\ q_1\}\} = \min\{v,\ q_1\}$.Therefore, $R(v)$ can be expressed more succintly and compactly in the form stated in the theorem.

Case 2

Recall that the modifications on case 1, yielding case 2 assure that

$$R(\min\{s,\ v\}) = \sum_{j=1}^{k} p_j q_j, \text{ where } q_k = \max\left\{s - \sum_{j=1}^{k-1} q_j,\ 0\right\}.$$

Above expression translates to the relation:

$$R(\min\{s,\ v\}) = \sum_{j=1}^{k} p_j \max\left\{\min\left\{\min\{s,\ v\} - \sum_{i=1}^{j-1} q_i,\ q_j\right\},\ 0\right\}, \text{ for any specified demand level } v.$$

V. IMPLEMENTATIONOF RESULTS

This section deals with mathematical implementation of the results of section four on the Microsoft Excel platform and the Goal Seek procedure.

5.1 Mathematical Implementation of Results

Case 1: The data can be organized in an Excel worksheet as shown below:

	A	B	C	D
1	**Volume Discounts Model Implementation**			
2				
3	Incremental volume for price breaks		Case :	Actual Demand = v
4		Unit price		Incr_Rev.
5				
6				
7	q_1	p_1		
8	q_2	p_2		
9	q_3	p_3		
.	.	.		
.	.	.		
.	.	.		
$n-1$	q_{k-1}	p_{k-1}		
n		p_k		
$n+1$				
$n+2$	Actual Demand =	v		
$n+3$	Supply cap. =	infinity	Total_Rev. =	

Table 3:General Excel implementations of revenues

Let $R_j \equiv R_j(v)$ denote the incremental revenue (Incr_Rev.) corresponding to the price break $p_j;\ j \in \{1, 2, \cdots, k\}$, for a given demand level v.

Implementation

Step 1: Type =$B7*max (min ($B$n+2-sum ($A$6:$A6), $A7), 0) in cell D7. <ENTER> to obtain the revenue R_1 corresponding to the demand min $\{v, q_1\}$.

Step 2: Place the cursor on the right boundary of cell D7; it immediately changes to a crosshair. Drag the crosshair down to cell Dn-1. <ENTER> to secure the incremental revenues corresponding to all price breaks except the last price break.

Step 3: Type = Bn*max (Cn+2 – sum ($A7:$An-1), 0) in cell Dn.

<ENTER> to obtain the revenue for the demand $q_k = \max\left\{ s - \sum_{j=1}^{k-1} q_j.\ \ 0 \right\}.$

corresponding to the last price break.

Step 4: Type = sum ($D7:$Dn) in cell Dn+2. <ENTER> to obtain the total revenue $R(v)$.

Case 2

Step 1: Write “Supplycap.= ” in cell An+3. Suppose that the capacity is s. Type in the value of s in cell Bn+3. Type the following in cell An+2:

= max (min (Bn+2, Bn+3) – sum ($A$7:An-1), 0). <ENTER> to secure q_k.

Step 2: Follow steps 1 and 2 of case 1 implementation with the crosshair dragging process extended to cell Dn. <ENTER> to generate all incremental revenues.

Step 3: Use step 4 of case 1 implementation to obtain the total revenue $R(\min\{s, v\})$.

Note that the demand v – min$\{s, v\}$ is unsatisfied.

VI. ILLUSTRATIVE EXAMPLES

Two examples of case1 are displayed below, illustrating the scenario $v < \sum_{j=1}^{k-1} q_j$ and the scenario $v \geq \sum_{j=1}^{k-1} q_j$. In the first scenario, note the placements of the zeros in the D column, reflecting the fact that $R(v) = 0$ for all $l \geq j+1$, whenever

$$\sum_{i=1}^{j-1} q_i \leq v \leq \sum_{i=1}^{j} q_i \text{, for some } j.$$

Volume Discount Model Implementation				
Incremental				
Volume for		Case v = :	2000	3000
Price Breaks	Unit Price		Incr_rev	Incr_rev
500	500		250000	250000
460	470		216200	216200
430	450		193500	193500
390	420		163800	163800
330	400		88000	132000
270	380		0	102600
220	360		0	79200
200	330		0	66000
170	310		0	52700
	300		0	9000
Demand =	2000	Total_rev=	911,500.00	1,265,000.00
Supply cap =				

Table 4: Excel implementations of revenues for given data..

	A	B	C	D	E
1	**Volume Discounts Model Implementation**				
2					
3	Incremental				
4	Volume for		Case:	v=2000	v=3000
5	Price breaks	Unit price		Incr_rev	Incr_rev
6					
7	500	500		250000	250000
8	460	470		216200	216200
9	430	450		193500	193500
10	390	420		163800	163800
11	330	400		88000	132000
12	270	380		0	102600
13	220	360		0	79200
14	200	330		0	66000
15	170	310		0	52700
16		300		0	9000
17					
18	Demand =	2000	Total_rev =	911,500	1,265,000
19	Supply cap =				

Table 5:Full view of Excel implementations of revenues for given data in Table 4.

6.2 Unconstrained supply Implementation Example

Using the demand v = 3000, the outputs are as shown in column E, with Total revenue equal to 1, 265,000.

6.3 Constrained supply Implementation Example

Step 1: Type in 1500 in cell B19. This is the supply capacity. Type 2000 in cell B18 for the actual demand. Type the following code in cell A16:
= max (min ($B$19, $B$18) – sum ($A$7:$A$15), 0). This is the value of q_{10}. Here, k = 10.
Step 2: Type = $B7*max (min ($C$19, $C$18) – sum ($A$6:$A6), $A7), 0) in cell D7; <ENTER> to obtain the revenue R_1 corresponding to the demand min {min {v, s}, q_1}.

Step 3: Place the cursor on the right boundary of cell D7 and drag the crosshair down to cell D16 to secure R_2 through R_{10} The results are $R_1 = 250,000,\ R_2 = 216,200, R_3 = 193,500,$

$R_4 = 46,200, R_5 = R_6 = \cdots, R_{10} = 0.$ Total revenue = 705,900.

6.4 Constrained supply Example

Now use the values 3,200 and 3,500 for the demand and supply capacity to secure the outputs

$R_1 = 250,000,\ R_2 = 216,200, R_3 = 193,500,\ R_4 = 163,800, R_5 = 132,00, R_6 = 102,600,$

$R_7 = 79,200, R_8 = 66,000, R_9 = 527,000, R_{10} = 69,000.$ Total revenue = 1,325,000.

VII. THE GOAL SEEK PROCEDURE

Goal Seek is a sensitivity analysis procedure. It adjusts a value in a specified cell until a dependent formula achieves the desired goal. For example, Goal Seek can be used to find the sales figures needed for a salesperson to achieve a desired total monthly wage based on a fixed commission rate, bonus qualification requirement and its associated payment rate. To make this mathematically precise, suppose that the commission rate is r_1% of the total sales figures x_1 and the bonus rate is r_2, based on a minimum sales figure x_2. Let $f(x_1, x_2, r_1, r_2)$ denote the total monthly wage of a salesperson with above sales figure. Then

$$y = f(x_1,\ x_2,\ r_1,\ r_2) = r_1 x_1 + \begin{cases} r_2 x_2, & \text{if } \ x_1 \ge x_2 \\ 0, & \text{otherwise} \end{cases}$$

or $y = f(x_1,\ x_2,\ r_1,\ r_2) = r_1 x_1 + r_2 x_2 \operatorname{sgn}\left(\max\{x_1 - x_2 + \varepsilon, 0\}\right), \varepsilon$ infinitesimally positive.

Suppose that the cell references for the values x_1, x_2, r_1%, r_2%, and the total monthly wage are C10, C11, C12, C13 and C14 respectively. Then this formula would be translated to the following Excel code in cell C14**:**

= $C12*$C10 + $C13*IF($C10 >= $C11, $C10, 0). Pressing the ENTER key implements the code with the output in cell C14. Subsequently, to get the sales figure corresponding to a desired monthly wage, you would click on the target cell, C14 and then invoke the GOAL SEEK feature.

GOAL SEEK will now be applied first to the infinite supply capacity case.
Click on the target cell, E18. Invoke GOAL SEEK on the Tools menu:

Tools ⟶ ↡ ⟶ GOAL SEEK. The following GOAL SEEK dialog box appears.

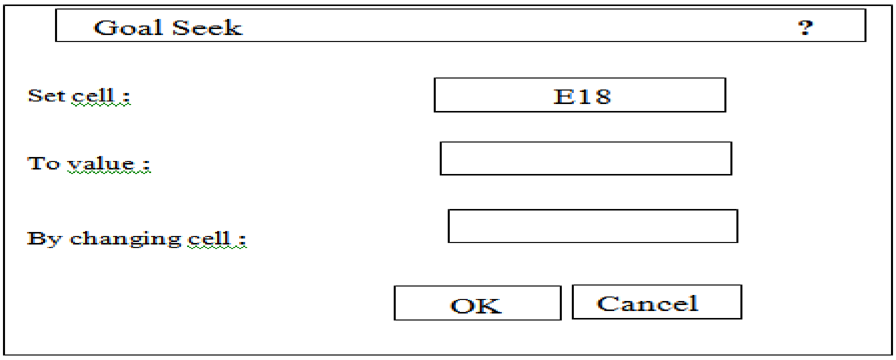

Figure 1: Goal seek dialogue box

7.1 The Goal Seek Implementation Example: Uncapacitated Supply Case.

Changing the value of cell E18 under the Goal Seek: Type the desired target revenue, 900000 say, in the To value text box. Type in the cell reference of the cell to be changed, C18 in the By changing cell text box. Click OK to implement GOAL SEEK (Cancel to abort the procedure). The following dialog box appears.

Goal Seek Status ? x

Goal Seeking with cell C18 **:** Ok

Found a solution Cancel

Target value:900,000 Stop

Current value **:**900,000 Pause

The Stop and Pause buttons are ghosted to indicate that those options are unavailable. Click Ok to accept the result. Click Cancel to abort the GOAL SEEK activity. If Ok is clicked, the incremental revenues change automatically to reflect the change in the total revenue and demand

The Goal Seek output demand and incremental revenues are 1971.25; $R_1 = 250,000,$ in cell D7, $R_2 = 216,200,$ in cell D8, $R_3 = 193,500,$ in cell D9, $R_4 = 163,800,$ in cell D10, $R_5 = 76,500,$ in cell D11, $R_6 = 0,$ in cell D12, $R_7 = 0,$ in cell D13, $R_8 = 0,$ in cell D14, $R_9 = 0,$ in cell D15,and $R_{10} = 0,$ in cell 16.

Unfortunately, the new demand generated by Goal Seek is non-integral. A satisficing demand can be generated from

argmin {abs(R(1971) – 900,000), abs(R(1972) – 900,000). Type in 1971 and 1972 in cell C18, followed by the execution of "<ENTER>" in each case, to obtain the revenues 899,900 and 900,300 respectively.

Clearly, argmin {abs(R (1971) – 900,000), abs(R (1972) – 900,000) = 1971. This new demand may be accepted, depending on the financial sensitivity of the problem, as well as the company objectives.

7.2The Goal Seek Implementation Example: Capacitated Supply Case.

Apply the Goal Seek procedure to table with supply capacity = 3,500, demand = 3,200 and total revenue of 1,000,000.

VIII. RESULTS

R_1 = 250,000, in cell D7, R_2 = 216,200, in cell D8, R_3 =193,500, in cell D9, R_4 = 163,800, in cell D10, R_5 = 132,000, in cell D11, R_6= 44,500, in cell D12, R_7 = 0, in cell D13, R_8 = 0, in cell D14, R_9 = 0, in cell D15,and R_{10} = 0, in cell 16.

Generated demand = 2,227.105. The reader should verify that

Argmin {abs(R(2,227) – 1,000,000), abs(R(2,228) – 1,000,000) = 2,227.

Observe even after the Goal Seek activity has been completed, the results could still be discarded and the original results reverted to by the use of <CTRL_Z>; <CRTL_Y> reverts to the Goal Seek results.

IX. CONCLUSION

This paper, with its novel approach has been motivated by the need to expose the reader to the multi-stage process of model development and implementation / automation, as applied to aspects of volume discount problems.The insight gained from the unique and broad techniques of presentation and strategy of proofs can be leveraged to solve related problems and may even be extended to appropriate model building situations. Furthermore, most readers who do not have access to customized inventory software can readily exploit the established results to solve this class of problems on Excel platform, which is widely and readily available.

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