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Comparision between various entering vector criteria with quick simplex algorithm for optimal solution to the linear programming problem.

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ABSTRACT: *In this paper, a new approach is suggested while solving linear programming problems using simplex method. The method sometimes involves less iteration than in the simplex method or at the most an equal number because the method attempts to replace more than one basic variable simultaneously. In this paper we compared quick simplex method with other methods of introducing vectors with various criteria to reach the optimum solution.*

KEY WORDS AND PHRASES*: basic feasible solution, optimum solution, simplex method, key determinant.*

I. INTRODUCTION

The linear programming has its own importance in obtaining the solution of a problem where two or more activities complete for limited resources.
Mathematically we have to maximize the objective function cx
subject to $Ax = b$, $x \geq 0$
where

$x = n \times 1$ column vector

$A = m \times n$ coefficient matrix

$b = m \times 1$ column vector

$C = 1 \times n$ row vector

and the columns of A are denoted by $P_1, \cdots, P_n$.

There are **two methods** to obtain the solution of the above problem. These methods can be classified as :

(**i**) the graphical method

(**ii**) simplex method.

The simplex method is the most general and powerful. We now give a brief account of the simplex method as below:

Consider a non-degenerate basic feasible solution

$$x_0 = (x_{10}, x_{20}, \cdots, x_{m0}, 0 \cdots 0)$$

The corresponding value of the objective function is

$$x_{10}c_1 + x_{20}c_2 + \cdots + x_{m_0}c_m = z_0 \text{ (say)}$$

It follows from the study of linear programming that for any fixed j, a set of feasible solutions can be constructed such that $z < z_0$ for any member of the set where net evaluation $z_j - c_j > 0$. The condition imposed on θ is

$$\theta = \min_i \frac{x_{i0}}{x_{ij}} > 0, \; x_{ij} > 0 \text{ for fixed } j.$$

II. SIMPLEX ALGORITHM

Step 1: Check whether the objective functions of the given LPP is maximized or minimized .If it is minimized, then we convert it into a problem of maximizing by using the result,
Minimum z = -Maximum(-z)

Step 2: Check Whether all b_i(i=1,2…..,m) are non-negative.If any one b_i is negative,then multiply the corresponding inequalities of the constraints by (-1),so as to get all b_i (i=1,2,…….m) non-negative.

Step 3: Convert all the inequalities of the constraints into equations by introducing slack/surplus variables in the constraints. Put the costs of these variables equal to zero.

Step 4: Obtain an initial basic feasible solution to the problem in the form $X_B=B^{-1}b$ and put it in the first column of the simplex table.

Step 5: Compute the net evaluations Z_j-C_j,(j=1,2,…,n) by using the relation

$Z_j - C_j = C_B Y_j - C_j$

Examine the sign of $Z_j - C_j$

1.If all $(Z_j - C_j) \geq 0$ then the initial basic feasible solution X_B is an optimum basic feasible solution.

2 If at least one $(Z_j-C_j) < 0$, proceed on to the next step.

Step 6: If there are more one negative Z_j-C_j, then choose the most negative of them. Let it be Z_r-C_r for some j = r

1. If all $y_{ir} \leq 0$ (i = 1,2,…..m) then there is an unbounded solution to the given problem

2. If atleast one $y_{ir} > 0$ (i= 1,2,….m) then the corresponding vector y_r enters the basis y_B.

Step 7: Compute the ratios($\frac{X_{Bi}}{y_{ir}}$, $y_{ir} > 0$, i=1,2,…..m) and choose the minimum of them.Let the minimum of the ratios $\frac{X_{Bk}}{y_{kr}}$.Then the vector y_k will leave the basis.The common element y_{kr} which is in the kth row and ith column is known as the leading element (or pivotal element) of the table.

Step 8: Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its columns to zeros by making use of the relations.

$$\bar{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir} \quad , i = 1,2 \ldots (m+1), i \neq k$$

$$\bar{y}_{kj} = \frac{y_{kj}}{y_{kr}} \,, \qquad j = 1,2 \ldots..n$$

Step 9: Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

III. CRITERIA :I

Dantzig's [**1**] suggestion is to choose that entering vector corresponding to which $z_j - c_j$ is most negative. **Khobragade's** [**4**] suggestion is to choose that entering vector corresponding to which $\frac{(z_j - c_j)\theta_j}{c_j}$ is most negative. It is shown that if we choose the vector y_j such that $\frac{(z_j - c_j)\theta_j}{c_j \sum y_{ij}}$, $(c_j > 0,\ y_{ij} \geq 0)$ is most negative, then the iterations required are fewer in some problems. This has been illustrated by giving the solution of a problem. We also show that either the iterations required are the same or less but iterations required are never more than those of the simplex method.
We shall illustrate the problem where the iterations are less (our method) than the simplex method.

IV. CRITERIA:II

Dantzig's [**1**] suggestion is to choose that entering vector corresponding to which $z_j - c_j$ is most negative. **Khobragade's** [**4**] suggestion is to choose that entering vector corresponding to which $\frac{(z_j - c_j)}{c_j}$ is most negative. It is shown that if we choose the vector y_j such that $\frac{(z_j - c_j)}{c_j \sum y_{ij}}$, $(c_j > 0,\ y_{ij} \geq 0)$ is most negative, then the iterations required are fewer in some problems. This has been illustrated by giving the solution of a problem. We also show that either the iterations required are the same or less but iterations required are never more than those of the simplex method.
We shall illustrate the problem where the iterations are less (our method) than the simplex method.

V. QUICK SIMPLEX METHOD

1.Replacement of n variables simultaneously and obtaining simplex table after such replacement.
2. Simultaneous replacement of n variables is possible only when **pivotal** elements in the entering vectors are in different rows.
3. We Define key determinant R, which is of order n.
4.R is the determinant of submatrix of matrix A.
5. This submatrix is obtained by using rows and columns containing pivotal elements.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
Pivot a_1	b_1	c_1	d_1	**1**	**0**	**0**	**0**
a_2	**Pivot**b_2	c_2	d_2	**0**	**1**	**0**	**0**
a_3	b_3	**Pivot** c_3	d_3	**0**	**0**	**1**	**0**
a_4	b_4	c_4	d_4	**0**	**0**	**0**	**1**

Here a_1, b_2 *and* c_3 are the pivotal elements when P_1, P_2, P_3 are the entering vectors in initial simplex table ,then R=

Pivot a_1	b_1	c_1
a_2	**Pivot**b_2	c_2
a_3	b_3	**Pivot** c_3

Is of order 3 as 3 variables are entered simultaneously.
6. We are giving formula to obtain simplex table after replacement of such n variables.
7. We shall call elements in the new simplex table as * elements.
8. Star elements are obtained by ratio of two determinants.
9. Denominator is nothing but the determinant R.
10. In the rows containing a pivotal element numerator is of order n.
11.Numerators are obtained as follows:
Here column of pivotal element is replaced by the column for which * elements are to be obtained.
Numerator of star elements in the rows in which no pivotal element is there is a determinant of order (n+1)
12. It is obtained by adding the corresponding column in the current simplex table and the row of the element to R.
13. In previous case , simplex table after replacing P_5, P_6, P_7 by P_1, P_2, P_3 will be as follows.

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
1	0	0	$d_1^{***} = \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} / \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$				0

0	1	0	$d_2^{***} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$				0
0	0	1	$d_3^{***} == \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$				0
0	0	0	$d_4^{***} = \frac{\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$				1

14. One must evaluate X_B Column first and nth iteration table should be evaluated only,when all the entries in X_B column comes to be non negative because it may indicate whether n variables can be entered.If any entry is negative then try by entering (n-1) variables instead of n variables.

VI .QUICK SIMPLEX METHOD FORMULAE FOR ENTERING *TWO* VARIABLES SIMULTANEOUSLY

Step1.Use new criteria for entering vector as above.

Step2. Simultaneous exchange of two variables instead of exchanging one basic variable at a time (as done in classical simplex method)

Step3. Both the Pivotal element should not be in same row.Otherwise we can not use our method.

Step 4. Development of the formula to go to third simplex table from first simplex table.

Consider Initial simplex table as following:

Step 1: Initial simplex table

P_1	P_2	P_3
Pivot a_1	b_1	c_1
a_2	Pivotb_2	c_2
a_3	b_3	c_3

Here P_1 and P_2 are entering vectors.

Step 2: Second Simplex table

P_1	P_2	P_3
1	$b_1^* = \frac{b_1}{a_1}$	$c_1^* = \frac{c_1}{a_1}$
0	$b_2^* = b_2 - \frac{a_2 b_1}{a_1}$	$c_2^* = c_2 - \frac{a_2 c_1}{a_1}$
0	$b_3^* = b_3 - \frac{a_3 b_1}{a_1}$	$c_3^* = c_3 - \frac{a_3 c_1}{a_1}$

Step 3:Third Simplex table

P_1	P_2	P_3
1	0	$c_1^{**} = c_1^* - \frac{b_1^* c_2^*}{b_2^*}$
0	1	$c_2^{**} = \frac{c_2^*}{b_2^*}$
0	0	$c_3^{**} = c_3^* - \frac{b_3^* c_2^*}{b_2^*}$

Here we can find c_1^{**}, c_2^{**} and c_3^{**} using following formula.

$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{R}$	$c_2^{**} = \frac{(-1)^2 \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{R}$	$c_3^{**} = \frac{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}{R}$

$c_4^{**} = \frac{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_4 & b_4 & c_4 \end{vmatrix}}{R}$	$R = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$

VII. STATEMENT OF THE PROBLEM-I

Solve the following LPP

Maximize $z = \frac{3}{4}x_1 - 150x_2 + \frac{1}{50}x_3 - 6x_4 + x_5 + x_6 + x_7$

Subject to the constraints :

$$\frac{1}{2}x_1 - 90x_2 - \frac{1}{50}x_3 + 3x_4 \leq 0$$

$$x_3 + x_6 \leq 1$$

$$\frac{1}{4}x_1 - 60x_2 - \frac{1}{25}x_3 + 9x_4 \leq 0$$

$$x_1, \cdots, x_7 \geq 0$$

VIII. SOLUTION OF THE PROBLEM

Maximize $z = \frac{3}{4}x_1 - 150x_2 + \frac{1}{50}x_3 - 6x_4$

Subject to the constraints :

$$\frac{1}{2}x_1 - 90x_2 - \frac{1}{50}x_3 + 3x_4 + x_5 = 0$$

$$x_3 + x_6 = 1$$

$$\frac{1}{4}x_1 - 60x_2 - \frac{1}{25}x_3 + 9x_4 + x_7 = 0$$

$$x_1, \cdots, x_7 \geq 0$$

(where $x_5, x_6, x_7 \rightarrow$ slack variables)

We now employ the conventional simplex method where we choose the entering vector for which $z_j - c_j$ is most negative.

Step (1): (Initial table)

		3/4	–150	1/50	-6	0	0	0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B	
0	x_5	1/4	–60	–1/25	9	1	0	0	0	0
0	x_6	1/2	–90	–1/50	3	0	1	0	0	0
0	x_7	0	0	1	0	0	0	1	1	Not defined
	$z_j - c_j$	–3/4	150	–1/50	6	0	0	0		
		↑				↓				

Step (2) : Introduce P_1 and drop P_5

		3/4	–150	1/50	-6	0	0	0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B	
3/4	x_1	1	–240	–4/25	36	4	0	0	0	-ve
0	x_6	0	**30**	3/50	-15	-2	1	0	0	0
0	x_7	0	0	1	0	0	0	1	**1**	**Not defined**
	$z_j - c_j$	0	-30	–7/50	33	3	0	0		
			↑				↓			

Step (3) : Introduce P_2 and drop P_6

		3/4	–150	1/50	-6	0	0	0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B	
3/4	x_1	1	0	8/25	-84	-12	8	0	0	0
-150	x_2	0	1	**1/500**	-1/2	-1/15	1/30	0	0	0

0	x_7	0	0	1	0	0	0	1	**1**	**1**
	$z_j - c_j$	0	0	–2/25	18	1	1	0		
				↑				↓		

Step (4) : Introduce P_3 and drop P_7

		3/4	–150	1/50	-6	0	0	0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B	
3/4	x_1	1	-160	1	-4	-4/3	8/3	0	0	-ve
1/50	x_3	0	500	1	-250	-100/3	50/3	0	0	-ve
0	x_7	0	-500	0	250	100/3	-50/3	1	**1**	**1/250**
	$z_j - c_j$	0	40	3/4	-2	-5/3	7/3	0		
					↑			↓		

Step (5) : Introduce P_4 and drop P_7

		3/4	–150	1/50	-6	0	0	0		Ratio
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B	
3/4	x_1	1	-168	1	0	-4/5	36/15	4/250	4/250	-ve
1/50	x_3	0	0	1	0	0	0	1	1	-ve
-6	x_4	0	-2	0	1	**2/15**	-1/15	1/250	**1/250**	**1/250**
	$z_j - c_j$	0	780	3/4	0	-7/5	11/5	2/250		
					↓	↑				

Step (5) : Introduce P_4 and drop P_7

		3/4	–150	1/50	-6	0	0	0	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B
3/4	x_1	1	-180	1	6	0	2	1/25	1/25
1/50	x_3	0	0	1	0	0	0	1	1
0	x_5	0	-15	0	15/2	**1**	-1/2	3/100	**3/100**
	$z_j - c_j$	0	15	0	21/2	0	3/2	1/20	

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached

IX. SOLUTION OF THE PROBLEM APPLYING CRITERIA I:

We now employ the simplex method where we choose the entering vector for which $\left(\frac{(z_j - c_j)\theta_j}{c_j \sum y_{ij}} \right)$ is most negative.

Step (1): (Initial table)

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
x_5	0	1/4	–60	–1/25	9	1	0	0	0
x_6	0	1/2	–90	–1/50	3	0	1	0	0
x_7	0	0	0	**1**	0	0	0	1	1
$z_j - c_j$		–3/4	150	–1/50	6	0	0	0	
			↑			↓			

In this step, we have

$$z_1 - c_1 = -3/4 \text{ and } z_3 - c_3 = -1/50$$

$$\therefore \frac{(z_1 - c_1)\theta_1}{c_1 \sum y_{i1}} = -\frac{3}{4} \times \frac{4}{3} \times \frac{4}{3} = -\frac{4}{3} \quad \text{and} \quad \frac{(z_3 - c_3)\theta_3}{c_3 \sum y_{i3}} = \frac{-1}{50} \times \frac{50}{1} = -1, \ \theta_1 = 0 \text{ and } \theta_3 = 1 > 0.$$

Therefore the entering vector is P_1.

Step (2) : Introduce P_1 and drop P_6

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
x_5	0	1/4	–60	0	9	1	0	1/25	1/25
x_6	0	**1/2**	–90	0	3	0	1	1/50	1/50
x_3	1/50	0	0	1	0	0	0	1	1
$z_j - c_j$		-3/4	150	0	6	0	0	1/50	
		↑					↓		

Step (3) :

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
x_5	0	0	–15	0	15/2	1	–1/2	3/100	3/100
x_1	3/4	1	–180	0	6	0	2	1/25	1/25
x_3	1/50	0	0	1	0	0	0	1	1
$z_j - c_j$		0	15	0	21/2	0	3/2	1/20	

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

X. SOLUTION OF THE PROBLEM APPLYING CRITERIA II:

We now employ the simplex method where we choose the entering vector for which $\left(\dfrac{z_j - c_j}{c_j \sum y_{ij}}\right)$ is most negative.

Step (1): (Initial table)

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
x_5	0	1/4	–60	–1/25	9	1	0	0	0
x_6	0	**1/2**	–90	–1/50	3	0	1	0	0
x_7	0	0	0	1	0	0	0	1	1
$z_j - c_j$		–3/4	150	–1/50	6	0	0	0	
		↑					↓		

In this step, we have

$z_1 - c_1 = -3/4$ and $z_3 - c_3 = -1/50$

$\therefore \dfrac{z_1 - c_1}{c_1 \sum y_{i1}} = -\dfrac{3}{4} \times \dfrac{4}{3} \times \dfrac{4}{3} = -\dfrac{4}{3}$ and $\dfrac{z_3 - c_3}{c_3 \sum y_{i3}} = \dfrac{-1}{50} \times \dfrac{50}{1} = -1$, $\theta_1 = 0$ and $\theta_3 = 1 > 0$.

Therefore the entering vector is P_1.

Step (2) : Introduce P_1 and drop P_6

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X_B
x_5	0	0	–15	-3/100	9/2	1	-1/2	0	1/25
x_1	3/4	1	–180	-1/25	6	0	2	0	1/50
x_7	0	0	0	**1**	0	0	0	1	1

$z_j - c_j$	0	15	-1/20	21/2	0	3/2	0	
			↑				↓	

The entering vector is P_3

Step (3) : Introduce P_3 and drop P_7

		3/4	–150	1/50	–6	0	0	0	
Basis	c_j	P_1	P_2	P_3	P_4	P_5	P_6	P_7	X$_B$
x_5	0	0	–15	0	9/2	1	–1/2	3/100	3/100
x_1	3/4	1	–180	0	6	0	2	1/25	1/25
x_3	1/50	0	0	1	0	0	0	1	1
$z_j - c_j$		0	15	0	21/2	0	3/2	1/20	

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

XI. HERE WE APPLY QUICK SIMPLEX METHOD:

Step (1): (Initial table)

		3/4	–150	1/50	-6	0	0	0		Ratio1	Ratio 3
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B		
0	x_5	$\boldsymbol{a_3} = \mathbf{1/4}$	–60	b_3 =–1/25	9	1	0	0	0	0	-ve
0	x_6	a_1 =1/2	–90	b_1 =–1/50	3	0	1	0	0	0	-ve
0	x_7	a_2 =0	0	$\boldsymbol{b_2} = \mathbf{1}$	0	0	0	1	**1**	**Not defined**	**1**
	$z_j - c$	–3/4	150	–1/50	6	0	0	0			
		↑		↑		↓	↓				

Here we introduce P_1, P_3 Simultaneously and outgoing vectors are P_5 , P_6 .

To find new values in X_B column.

Here we can find c_1^{**}, c_2^{**} and c_3^{**} using following formula.

$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} -1/50 & 0 \\ 1 & 1 \end{vmatrix}}{1/2} = 1/25$	$c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 1/2 & 0 \\ 0 & 1 \end{vmatrix}}{1/2} = 1$

$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & 0 \\ 0 & 1 & 1 \\ 1/4 & -1/25 & 0 \end{vmatrix}}{1/2} = 3/100$	$R = \begin{vmatrix} 1/2 & -1/50 \\ 0 & 1 \end{vmatrix} = 1/2$

New $\boldsymbol{X_B} = \begin{bmatrix} 1/25 \\ 1 \\ 3/100 \end{bmatrix}$

To find other entries in third simplex table.

Column $P_2 = \begin{bmatrix} -90 \\ 0 \\ -60 \end{bmatrix} :-$

$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} -1/50 & -90 \\ 1 & 0 \end{vmatrix}}{1/2} = -180$	$c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 1/2 & -90 \\ 0 & 0 \end{vmatrix}}{1/2} = 0$

$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & -90 \\ 0 & 1 & 0 \\ 1/4 & -1/25 & -60 \end{vmatrix}}{1/2} = -15$	$R = \begin{vmatrix} 1/2 & -1/50 \\ 0 & 1 \end{vmatrix} = 1/2$

New$\boldsymbol{P_2} = \begin{bmatrix} -180 \\ 0 \\ -15 \end{bmatrix}$

To find other entries in third simplex table.

Column $P_4 = \begin{bmatrix} 9 \\ 3 \\ 0 \end{bmatrix} :-$

$c_1^{**} = \frac{(-1)^1 \begin{vmatrix} -1/50 & 3 \\ 1 & 0 \end{vmatrix}}{1/2} = 6$	$c_2^{**} = \frac{(-1)^2 \begin{vmatrix} 1/2 & 3 \\ 0 & 0 \end{vmatrix}}{1/2} = 0$

$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & 3 \\ 0 & 1 & 0 \\ 1/4 & -1/25 & 9 \end{vmatrix}}{1/2} = 15/2$

New Column $P_4 = \begin{bmatrix} 6 \\ 0 \\ 15/2 \end{bmatrix}$

Column $P_5 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$:−

$c_1^{**} = \dfrac{(-1)^1 \begin{vmatrix} -1/50 & 1 \\ 1 & 0 \end{vmatrix}}{1/2} = 2$	$c_2^{**} = \dfrac{(-1)^2 \begin{vmatrix} 1/2 & 1 \\ 0 & 0 \end{vmatrix}}{1/2} = 0$

$$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & 1 \\ 0 & 1 & 0 \\ 1/4 & -1/25 & 0 \end{vmatrix}}{1/2} = -1/2$$

New Column $P_5 = \begin{bmatrix} 2 \\ 0 \\ -1/2 \end{bmatrix}$

Column $P_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$:−

$c_1^{**} = \dfrac{(-1)^1 \begin{vmatrix} -1/50 & 0 \\ 1 & 1 \end{vmatrix}}{1/2} = 1/25$	$c_2^{**} = \dfrac{(-1)^2 \begin{vmatrix} 1/2 & 0 \\ 0 & 1 \end{vmatrix}}{1/2} = 1$

$$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & 0 \\ 0 & 1 & 1 \\ 1/4 & -1/25 & 0 \end{vmatrix}}{1/2} = 3/100$$

New Column $P_6 = \begin{bmatrix} 1/25 \\ 1 \\ 3/100 \end{bmatrix}$

Column $P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$:−

$c_1^{**} = \dfrac{(-1)^1 \begin{vmatrix} -1/50 & 0 \\ 1 & 0 \end{vmatrix}}{1/2} = 0$	$c_2^{**} = \dfrac{(-1)^2 \begin{vmatrix} 1/2 & 0 \\ 0 & 0 \end{vmatrix}}{1/2} = 0$

$$c_3^{**} = \frac{\begin{vmatrix} 1/2 & -1/50 & 0 \\ 0 & 1 & 0 \\ 1/4 & -1/25 & 1 \end{vmatrix}}{1/2} = 1$$

New Column $P_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

So we get direct third simplex table using above formulae

STEP :5

		3/4	–150	1/50	-6	0	0	0	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	P_7	x_B
3/4	x_1	1	-180	1	6	0	2	1/25	1/25
1/50	x_3	0	0	1	0	0	0	1	1
0	x_5	0	-15	0	15/2	**1**	-1/2	3/100	**3/100**
	$z_j - c_j$	0	15	0	21/2	0	3/2	1/20	

Since all $z_j - c_j \geq 0$, an optimum basic feasible solution has been reached.

In this problem it is observed that we reached to solution by using conventional simplex method in fifth step And by using criteria-I it reached to two step and by criteria-II it reached to two step while by using quick simplex algorithm we reached to solution in one step only. Since this is a cyclic problem ,here we got advantage of our method.

XII. STATEMENT OF THE PROBLEM-II

Solve the LPP: Max $Z = 22x_1 + 30x_2 + 25x_3$

Subject to the constraints

$2x_1 + 2x_2 \leq 100$, $2x_1 + x_2 + x_3 \leq 100$, $x_1 + 2x_2 + 2x_3 \leq 100$, $x_1, x_2, x_3 \geq 0$

XIII. SOLUTION OF THE PROBLEM:

Max $Z = 22x_1 + 30x_2 + 25x_3$

Subject to the constraints

$2x_1 + 2x_2 + x_4 = 100$, $2x_1 + x_2 + x_3 + x_5 = 100$, $x_1 + 2x_2 + 2x_3 + x_6 = 100$, $x_1, x_2, x_3 \geq 0$

(where $x_4, x_5, x_6 \rightarrow$ slack variables)

We now use the simplex method where we choose the entering vector for which $z_j - c_j$ is most negative.

Step (1). Initial Simplex Table

		22	30	25	0	0	0		
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	X_B	Ratio2
0	X_4	2	2	0	1	0	0	100	50
0	X_5	2	1	1	0	1	0	100	100
0	X_6	1	**2**	2	0	0	**1**	100	**50**

	Z^*	–22	–30	–25	0	0	0	0	
			↑				↓		

Step (2) : Introduce P_2 and drop P_6

		22	30	25	0	0	0		
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B	Ratio
0	X_4	**1**	0	-2	**1**	0	-1	0	**0**
0	X_5	3/2	0	0	0	1	–1/2	50	100/3
30	X_2	1/2	1	1	0	0	1/2	50	100
	z_j-c_j	–7	0	5	0	0	15		
		↑			↓				

In this step, we have

Step (3) : Introduce P_1 and drop P_4

		22	30	25	0	0	0		
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B	Ratio
22	X_1	1	0	-2	1	0	-1	0	-ve
0	X_5	0	0	**3**	-3/2	1	1	50	**50/3**
30	X_2	0	1	2	-1/2	0	1	50	50/2
		0	0	-9	7	0	8		
				↑		↓			

Step (4) : Introduce P_3 and drop P_5

		22	30	25	0	0	0	
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	X_B
22	X_1	1	0	0	0	2/3	-1/3	100/3
25	X_3	0	0	1	-1/2	1/3	1/3	50/3
30	X_2	0	1	0	1/2	-2/3	1/3	50/3
		0	0	0	5/2	3	11	1650

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 100/3$, $x_2 = x_3 = 50/3$ and Max. $Z = 1650$.

XIV. SOLUTION OF THE PROBLEM APPLYING CRITERIA-I

We now use the simplex method where we choose the entering vector for which $\left(\frac{z_j - c_j}{c_j \sum y_{ij}}\right)\theta_j$ is most negative.

Step (1). Initial Simplex Table

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	X_4	2	2	0	1	0	0	100
0	X_5	2	1	1	0	1	0	100
0	X_6	1	2	**2**	0	0	1	100
	Z^*	–22	–30	–25	0	0	0	0
				↑			↓	

In this step, we have $\frac{(z_1 - c_1)\theta_1}{c_1 \sum y_{i1}} = -\frac{22}{22} \times \frac{1}{5} \times 50 = -10$, $\frac{(z_2 - c_2)\theta_2}{c_2 \sum y_{i2}} = -\frac{30}{30} \times \frac{1}{5} \times 50 = -10$ and $\frac{(z_3 - c_3)\theta_3}{c_3 \sum y_{i3}} = \frac{-25}{25} \times \frac{1}{3} \times 50 = -\frac{50}{3}$.

Therefore the entering vector is P_3.

Step (2) : Introduce P_3 and drop P_6

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	x_4	2	2	0	1	0	0	100
0	x_5	**3/2**	0	0	0	1	–1/2	50
25	x_3	1/2	1	1	0	0	1/2	50
	$z_j - c_j$	–19/2	-5	0	0	0	25/2	
		↑				↓		

Step (3) : Introduce P_1 and drop P_5

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	x_4	0	**2**	0	1	-4/3	2/3	100/3
22	x_1	1	0	0	0	2/3	-1/3	100/3
25	x_3	0	1	1	0	-1/3	2/3	100/3

		0	-5	0	0	19/3	7	
			↑		↓			

Step (4) : Introduce P_2 and drop P_4

			22	30	25	0	0	0
C_B	x_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6
30	x_2	50/3	0	1	0	1	-2/3	1/3
22	x_1	100/3	1	0	0	0	2/3	-1/3
25	x_3	50/3	0	0	1	-1	1/3	1/3
		1650	0	0	0	5	3	11

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 100/3$, $x_2 = x_3 = 50/3$ and Max. $Z = 1650$.

XV. SOLUTION OF THE PROBLEM APPLYING CRITERIA II

We now use the simplex method where we choose the entering vector for which $\left(\dfrac{z_j - c_j}{c_j \sum y_{ij}}\right)$ is most negative

Step (1). Initial Simplex Table

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	x_4	2	2	0	1	0	0	100
0	x_5	2	1	1	0	1	0	100
0	x_6	1	2	**2**	0	0	1	100
	Z^*	–22	–30	–25	0	0	0	0
				↑			↓	

In this step, we have

$$\therefore \frac{z_1 - c_1}{c_1 \sum y_{i1}} = -\frac{22}{22} \times \frac{1}{5} = -\frac{1}{5}, \frac{z_2 - c_2}{c_2 \sum y_{i2}} = -\frac{30}{30} \times \frac{1}{5} = -\frac{1}{5} \text{ and } \frac{z_3 - c_3}{c_3 \sum y_{i3}} = \frac{-25}{25} \times \frac{1}{3} = -\frac{1}{3}$$

Therefore the entering vector is y_3.

Step (2) : Introduce P_3 and drop P_6

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	x_4	2	2	0	1	0	0	100
0	x_5	**3/2**	0	0	0	1	–1/2	50
25	x_3	1/2	1	1	0	0	1/2	50
	$z_j - c_j$	–19/2	-5	0	0	0	25/2	
		↑				↓		

Step (3) : Introduce P_1 and drop P_5

		22	30	25	0	0	0	
C_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B
0	x_4	0	**2**	0	1	-4/3	2/3	100/3
22	x_1	1	0	0	0	2/3	-1/3	100/3
25	x_3	0	1	1	0	-1/3	2/3	100/3
		0	-5	0	0	19/3	7	
			↑		↓			

Step (4) : Introduce P_2 and drop P_4

			22	30	25	0	0	0
C_B	x_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6
30	x_2	50/3	0	1	0	1	-2/3	1/3
22	x_1	100/3	1	0	0	0	2/3	-1/3
25	x_3	50/3	0	0	1	-1	1/3	1/3
		1650	0	0	0	5	3	11

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 100/3$, $x_2 = x_3 = 50/3$ and Max. $Z = 1650$.

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XVI. QUICK SIMPLEX METHOD

Step (1). Initial Simplex Table

		22	30	25	0	0	0				
C_B	X_B	P_1	P_2	P_3	P_4	P_5	P_6	x_B	Ratio 1	Ratio2	Ratio3
0	X_4	$\boldsymbol{a_1}$= 2	c_1=2	b_1=0	1	0	0	100	50	50	---
0	X_5	a_2=2	c_2=1	$\boldsymbol{b_2}$=1	0	1	0	100	50	100	100
0	X_6	a_3=1	$\boldsymbol{c_3}$=2	b_3=2	0	0	1	100	100	50	50
	Z^*	−22	−30	−25	0	0	0	0			
		↑	↑	↑	↓	↓	↓				

Here we introduce simultaneously P_1 , P_2 , P_3 three vectors and outgoing vectors are P_4 , P_6 , P_5 , So we get direct fourth simplex table using above formulae. Introduction is such that Pivotal element should be in different row. To find new values in X_B column.

$$x_1 = \frac{\begin{vmatrix} 0 & 2 & 100 \\ 1 & 1 & 100 \\ 2 & 2 & 100 \end{vmatrix}}{R} = \frac{100}{3}, \quad x_2 = \frac{\begin{vmatrix} 2 & 100 & 2 \\ 2 & 100 & 1 \\ 1 & 100 & 2 \end{vmatrix}}{R} = \frac{50}{3}, \quad x_3 = \frac{\begin{vmatrix} 2 & 0 & 100 \\ 2 & 1 & 100 \\ 1 & 2 & 100 \end{vmatrix}}{R} = \frac{50}{3},$$

where $R=\begin{vmatrix} 2 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 6$

Since all the values are positive, non-negativity constraint is satisfied and it indicates that such transformation is allowed. To find other entries in third simplex table.

Column $P_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} :-$

$$x_1 = \frac{\begin{vmatrix} 0 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{vmatrix}}{R} = 0, \quad x_2 = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{vmatrix}}{R} = \frac{-1}{2}, \quad x_3 = \frac{\begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix}}{R} = \frac{1}{2}$$

New $P_4 = \begin{bmatrix} 0 \\ -1/2 \\ 1/2 \end{bmatrix}$

Column $P_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} :-$

$$\mathbf{x_1} = \frac{\begin{vmatrix} 0 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 0 \end{vmatrix}}{\mathbf{R}} = \frac{2}{3}, \quad \mathbf{x_2} = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix}}{\mathbf{R}} = \frac{1}{3}, \quad \mathbf{x_3} = \frac{\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 0 \end{vmatrix}}{\mathbf{R}} = \frac{-2}{3}$$

$$\text{New } \mathbf{P_5} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{-2}{3} \end{bmatrix}$$

$$\text{Column } P_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} :-$$

$$\mathbf{x_1} = \frac{\begin{vmatrix} 0 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{vmatrix}}{\mathbf{R}} = \frac{-1}{3}, \quad \mathbf{x_2} = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix}}{\mathbf{R}} = \frac{1}{3}, \quad \mathbf{x_3} = \frac{\begin{vmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix}}{\mathbf{R}} = \frac{1}{3}$$

$$\text{New } P_6 = \begin{bmatrix} \frac{-1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\text{New } P_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \text{New } P_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{New } P_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Step (4) :

			22	30	25	0	0	0
C_B	x_B	x_B	P_1	P_2	P_3	P_4	P_5	P_6
22	x_1	100/3	1	0	0	0	2/3	-1/3
25	x_3	50/3	0	0	1	-1/2	1/3	1/3
30	x_2	50/3	0	1	0	1/2	-2/3	1/3
		1650	0	0	0	5/2	3	11

Since all $Z_j - C_j \geq 0$ hence an optimum basic feasible solution has been reached.

$\therefore$ optimum solution is $x_1 = 100/3$, $x_2 = x_3 = 50/3$ and Max. $Z = 1650$

In this problem it is observed that we reached to solution by using conventional simplex method in third step And by using criteria-I it reached to third step and by criteria-II it reached to third step while by using quick simplex algorithm we reached to solution in one step only. Here also we got advantage of our method

XVII. CONCLUSION

In the first problem it is observed that we reached to solution by using conventional simplex method in fifth step And by using criteria-I it reached to two step and by criteria-II it reached to two step while by using quick simplex algorithm we reached to solution in one step only. Since this is a cyclic problem ,here we got advantage of our method

In the second problem it is observed that we reached to solution by using conventional simplex method in third step And by using criteria-I it reached to third step and by criteria-II it reached to third step while by using quick simplex algorithm we reached to solution in one step only. Here also we got advantage of our method.

Hence the number of iterations required is reduced by Quick simplex algorithm methodology. This method found to be more convenient when introducing more than one vector simultaneously.

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