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Two warehouses deteriorating items inventory model under partial backlogging, inflation and permissible delay in payments

Shital S. Patel
Department of Statistics, Veer Narmad South Gujarat University, Surat, INDIA

ABSTRACT: *A deteriorating items inventory model with two warehouses under time varying holding cost and linear demand with inflation and permissible delay in payments is developed. Shortages are allowed and partially backlogged. A rented warehouse (RW) is used to store the excess units over the capacity of the own warehouse. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.*

KEYWORDS: *Deterioration, Two-warehouse, Inventory, partial backlogging, Inflation, Permissible delay in payment*

I. INTRODUCTION

In past few decays deteriorating items inventory models were widely studied. An inventory model with constant rate deterioration was developed by Ghare and Schrader [12]. The model was further extended by considering variable rate of deterioration by Covert and Philip [10]. Shah [30] further extended the model by considering shortages. The related work are found in (Nahmias [23], Raffat [25], Goyal and Giri [14], Mandal [21]), Mishra et al. [22]).

The classical inventory model generally deal with single storage facility with the assumption that the available warehouse of the organization has unlimited capacity. But in actual practice, to take advantages of price discount for bulk purchases, a rented warehouse (RW) is used to store the excess units over the fixed capacity W of the own warehouse (OW). The cost of storing items at the RW is higher than that at the OW but provides better preserving facility with a lower rate of deterioration.

Hartley [15] was the first one to consider a two warehouse model. A two warehouse deterministic inventory model with infinite rate of replenishment was developed by Sarma [29]. Pakkala and Achary [24] extended the two-warehouse inventory model for deteriorating items with finite rate of replenishment and shortages. Yang [31] considered a two warehouse inventory model for deteriorating items with constant rate of demand under inflation in two alternatives when shortages are completely backordered. Dye et al. [11] considered a two warehouse inventory model with partially backlogging. Jaggi et al. [16] developed an inventory model with linear trend in demand under inflationary conditions and partial backlogging rate in a two warehouse system.Related work is also find in (Benkherouf [3], Bhunia and Maiti [4], Kar et al. [19], Rong et al. [26], Sana et al. [28], Agarwal and Banerjee [1], Bhunia et al. [5]).

Goyal [13] first considered the economic order quantity model under the condition of permissible delay in payments. Aggarwal and Jaggi [2] extended Goyal’s [13] model to consider the deteriorating items. Aggarwal and Jaggi’s [2] model was further extended by Jamal et al. [17] to consider shortages. The related work are found in (Chung and Dye [7], Jamal et al. [18], Salameh et al. [27], Chung et al. [8], Chang et al. [6]). Chung and Huang [9] proposed a two warehouse inventory model for deteriorating items under permissible delay in payments, but they assumed that the deterioration rate of the two warehouse were same. Liao and Huang [20] considered an order level inventory model for deteriorating items with two warehouse and a permissible delay in payments.

In this paper we have developed a two-warehouse inventory model under time varying holding cost and linear demand with inflation and permissible delay in payments. Shortages are allowed and partially backlogged. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

$D(t)$: Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

A : Replenishment cost per order for two warehouse system

c : Purchasing cost per unit

p : Selling price per unit

c_2 : Shortage cost per unit

c_3 : Cost of lost sales per unit

HC(OW): Holding cost per unit time is a linear function of time t (x_1+y_1t, $x_1>0$, $0<y_1<1$) in OW

HC(RW): Holding cost per unit time is a linear function of time t (x_2+y_2t, $x_2>0$, $0<y_2<1$) in RW

I_e : Interest earned per year

I_p : Interest charged per year

M : Permissible period of delay in settling the accounts with the supplier

T : Length of inventory cycle

$I(t)$: Inventory level at any instant of time t, $0 \leq t \leq T$

W : Capacity of owned warehouse

$I_0(t)$: Inventory level in OW at time t

$I_r(t)$: Inventory level in RW at time t

Q_1 : Inventory level initially

Q_2 : Shortage of inventory

Q : Order quantity

R : Inflation rate

t_r : Time at which the inventory level reaches zero in RW in two warehouse system

$\theta_1 t$: Deterioration rate in OW, $0< \theta_1<1$

$\theta_2 t$: Deterioration rate in RW, $0< \theta_2<1$

TC_i : Total relevant cost per unit time (i=1,2,3)

ASSUMPTIONS:

The following assumptions are considered for the development of two warehouse model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and partially backlogged.
- OW has a fixed capacity W units and the RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory costs per unit in the RW are higher than those in the OW.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, a lot size of certain units enter the system. W units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the interval $[0,t_r]$, the inventory in RW gradually decreases due to demand and deterioration and it reaches to zero at $t=t_r$. In OW, however, the inventory W decreases during the interval $[0,t_r]$ due to deterioration only, but during $[t_r, t_0]$, the inventory is depleted due to both demand and deterioration. By the time to t_0, both warehouses are empty. Shortages occur during (t_0,T) of size Q_2 units. The figure describes the behaviour of inventory system.

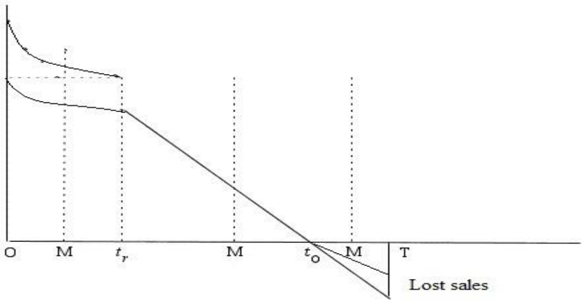


Figure 1

. Hence, the inventory level at time t at RW and OW are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} + \theta_2 t I_r(t) = -(a+bt), \qquad 0 \le t \le t_r$$

(1)

with boundary conditions $I_r(t_r) = 0$ and

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = 0, \qquad 0 \le t \le t_r$$

(2)

with initial condition $I_0(0) = W$, respectively.

While during the interval (t_r, t_0), the inventory in OW reduces to zero due to the combined effect of demand and deterioration both. So the inventory level at time t at OW, $I_0(t)$, is governed by the following differential equation:

$$\frac{dI_0(t)}{dt} + \theta_1 t I_0(t) = -(a+bt), \qquad t_r \le t \le t_0$$

(3)

with the boundary condition $I_0(t_0)=0$.

Similarly during (t_0, T) the shortage level at time t, $I_s(t)$ is governed by the following differential equation:

$$\frac{dI_s(t)}{dt} = -e^{-\delta(T-t)}(a+bt), \qquad t_0 \le t \le T, \qquad (4)$$

with the boundary condition $I_s(t_0)=0$.

The solutions to equations (1) to (4) are given by:

$$I_r(t) = \begin{bmatrix} a\left(t_r - t\right) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{1}{6}a\theta_2\left(t_r^3 - t^3\right) \\ + \frac{1}{8}b\theta_2\left(t_r^4 - t^4\right) - \frac{1}{2}a\theta_2 t^2\left(t_r - t\right) - \frac{1}{4}b\theta_2 t^2\left(t_r^2 - t^2\right) \end{bmatrix} \qquad 0 \le t \le t_r \qquad (5)$$

$$I_o(t) = W\left(1 - \theta_1 t^2\right), \qquad 0 \le t \le t_r \qquad (6)$$

$$I_o(t) = \begin{bmatrix} a\left(t_0 - t\right) + \frac{1}{2}b\left(t_0^2 - t^2\right) + \frac{1}{6}a\theta_1\left(t_0^3 - t^3\right) \\ + \frac{1}{8}b\theta_1\left(t_0^4 - t^4\right) - \frac{1}{2}a\theta_1 t^2\left(t_0 - t\right) - \frac{1}{4}b\theta_1 t^2\left(t_0^2 - t^2\right) \end{bmatrix} \qquad t_r \le t \le t_0 \qquad (7)$$

$$I_s(t) = \begin{bmatrix} -\frac{1}{3}b\delta t^3 - \frac{1}{2}[a\delta + b(1-\delta T)]t^2 - at(1-\delta T) \\ + \frac{1}{3}b\delta t_0^3 + \frac{1}{2}a\delta t_0^2 + \frac{1}{2}bt_0^2 - \frac{1}{2}b\delta T t_0^2 - a\delta t_0 T + at_0 \end{bmatrix}, \qquad t_0 \le t \le T$$

(8)

(by neglecting higher powers of θ_1, θ_2)

Using the condition $I_r(t) = Q_1 - W$ at t=0 in equation (5), we have

$$Q_1 - W = \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right],$$

$$\therefore\ Q_1 = W + \left[at_r + \frac{1}{2}bt_r^2 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{8}b\theta_2 t_r^4\right].$$

(9)

Using the condition $I_s(t) = Q - Q_1$ at t=T in equation (8), we have

$$Q - Q_1 = -\left[a(T - t_0) + \frac{1}{2}b(T^2 - t_0^2)\right]$$

$$\therefore\ Q = Q_1 - \left[a(T - t_0) + \frac{1}{2}b(T^2 - t_0^2)\right].$$

(10)

Using the continuity of $I_0(t)$ at t=tr in equations (6) and (7), we have

$$I_o(t_r) = W\left(1 - \theta_1 t^2\right) = \left[\begin{array}{l} a\left(t_0 - t_r\right) + \frac{1}{2}b\left(t_0^2 - t_r^2\right) + \frac{1}{6}a\theta_1\left(t_0^3 - t_r^3\right) \\ + \frac{1}{8}b\theta_1\left(t_0^4 - t_r^4\right) - \frac{1}{2}a\theta_1 t_r^2\left(t_0 - t\right) - \frac{1}{4}b\theta_1 t_r^2\left(t_0^2 - t_r^2\right) \end{array}\right] \qquad (11)$$

which implies that

$$t_0 = \frac{-a + \sqrt{a^2 + 2bW - bW\theta_1 t_r^2 + b^2 t_r^2 + 2abt_r}}{b}$$

(12)

(by neglecting higher powers of tr and t_0)

From equation (12), we note that t_0 is a function of t_r, therefore t_0 is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant costs TC_i, include the following elements:

(i) Ordering cost (OC) = A

(13)

$$\text{(ii)}\ \ HC(RW) = \int_0^{t_r}(x_2+y_2 t)I_r(t)e^{-Rt}dt = \int_0^{t_r}(x_2+y_2 t)\left[\begin{array}{l} a\left(t_r - t\right) + \frac{1}{2}b\left(t_r^2 - t^2\right) + \frac{1}{6}a\theta_2\left(t_r^3 - t^3\right) \\ + \frac{1}{8}b\theta_2\left(t_r^4 - t^4\right) - \frac{1}{2}a\theta_2 t^2\left(t_r - t\right) - \frac{1}{4}b\theta_2 t^2\left(t_r^2 - t^2\right) \end{array}\right]e^{-Rt}dt$$

$$= -\frac{1}{56}y_2R\theta_2 bt_r^7 + \frac{1}{6}\left(\frac{1}{8}\left(y_2 - x_2R\right)\theta_2 b - \frac{1}{3}y_2R\theta_2 a\right)t_r^6 + \frac{1}{5}\left(\frac{1}{8}x_2\theta_2 b + \frac{1}{3}\left(y_2 - x_2R\right)\theta_2 a - y_2R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^5$$

$$+ \frac{1}{4}\left(\frac{1}{3}x_2\theta_2 a + \left(y_2 - x_2R\right)\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) + y_2Ra\right)t_r^4$$

$$+ \frac{1}{3}\left(x_2\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right) - \left(y_2 - x_2R\right)a - y_2R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^3$$

$$+ \frac{1}{2}\left(-x_2 a + \left(y_2 - x_2R\right)\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^2 + x_2\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)t_r$$

(14)

(by neglecting higher powers of R)

$$\text{(iii)}\ \ HC(OW) = \int_0^{t_0}(x_1+y_1 t)I_0(t)e^{-Rt}dt = \int_0^{t_r}(x_1+y_1 t)I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0}(x_1+y_1 t)I_0(t)e^{-Rt}dt$$

$$= \int_0^{t_r}(x_1+y_1 t)W\left(1 - \theta_1 t^2\right)e^{-Rt}dt + \int_{t_r}^{t_0}(x_1+y_1 t)\left[\begin{array}{l} a\left(t_0 - t\right) + \frac{1}{2}b\left(t_0^2 - t^2\right) + \frac{1}{6}a\theta_1\left(t_0^3 - t^3\right) \\ + \frac{1}{8}b\theta_1\left(t_0^4 - t^4\right) - \frac{1}{2}a\theta_1 t^2\left(t_0 - t\right) - \frac{1}{4}b\theta_1 t^2\left(t_0^2 - t^2\right) \end{array}\right]e^{-Rt}dt$$

$$= W\left(\frac{1}{10}y_1Rb\theta_1t_r^5 - \frac{1}{8}(y_1\text{-}x_1R)\theta_1t_r^4 + \frac{1}{3}\left(-\frac{1}{2}x_1\theta_1 - y_1R\right)t_0^3 + \frac{1}{2}(y_1\text{-}x_1R)t_0^2\ x_1t_r\right)$$

$$+\left(\begin{array}{l} -\frac{1}{56}y_1Rb\theta_1t_0^7 + \frac{1}{6}\left(\frac{1}{8}(y_1\text{-}x_1R)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)t_0^6 \\ +\frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}(y_1\text{-}x_1R)a\theta_1\text{-}y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)\right)t_0^5 \\ +\frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + (y_1\text{-}x_1R)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)+y_1Ra\right)t_0^4 \\ +\frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)-(y_1\text{-}x_1R)a+y_1R\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_0^3 \\ +\frac{1}{2}\left(-x_1a-(y_1\text{-}x_1R)\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_0^2+x_1\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)t_0 \end{array}\right)$$

$$+\left(\begin{array}{l} \frac{1}{56}y_1Rb\theta_1t_r^7 - \frac{1}{6}\left(\frac{1}{8}(y_1\text{-}x_1R)b\theta_1 - \frac{1}{3}y_1Ra\theta_1\right)t_r^6 \\ -\frac{1}{5}\left(\frac{1}{8}x_1b\theta_1 + \frac{1}{3}(y_1\text{-}x_1R)a\theta_1 - y_1R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)\right)t_r^5 \\ -\frac{1}{4}\left(\frac{1}{3}x_1a\theta_1 + (y_1\text{-}x_1R)\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)+y_1Ra\right)t_r^4 \end{array}\right)$$

$$+\left(\begin{array}{l} -\frac{1}{3}\left(x_1\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)-(y_1\text{-}x_1R)a - y_1R\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_r^3 \\ -\frac{1}{2}\left(-x_1a+(y_1\text{-}x_1R)\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_r^2 - x_1\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)t_r \end{array}\right) \quad (15)$$

(iv) Deterioration cost:

The amount of deterioration in both RW and OW during $[0,t_0]$ are:

$$\int_0^{t_r}\theta_2tI_r(t)dt \ \text{ and } \int_0^{t_0}\theta_1tI_0(t)dt$$

So deterioration cost

$$DC = c\left[\int_0^{t_r}\theta_2tI_r(t)e^{-Rt}dt + \int_0^{t_0}\theta_1tI_0(t)e^{-Rt}dt\right]$$

$$= c\left[\int_0^{t_r}\theta_2tI_r(t)e^{-Rt}dt + \int_0^{t_r}\theta_1tI_0(t)e^{-Rt}dt + \int_{t_r}^{t_0}\theta_1tI_0(t)e^{-Rt}dt\right]$$

$$= c\theta_2\left[\begin{array}{l} -\frac{1}{56}R\theta_2bt_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_2 - \frac{1}{3}Ra\theta_2\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_2 - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b\right)\right)t_r^5 \\ +\frac{1}{4}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2+at_r\right)-\frac{1}{2}b+Ra\right)t_r^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_2t_r^4+\frac{1}{6}a\theta_2t_r^3+\frac{1}{2}bt_r^2+at_r\right)\right)t_r^3 \\ +\frac{1}{2}\left(\frac{1}{8}b\theta_2t_r^4+\frac{1}{6}a\theta_2t_r^3+\frac{1}{2}bt_r^2+at_r\right)t_r^2 + c\theta_1W\left[\frac{1}{10}R\theta_1t_r^5 - \frac{1}{8}\theta_1t_r^4 - \frac{1}{3}Rt_r^3 + \frac{1}{2}t_r^2\right] \end{array}\right]$$

$$+ c\theta_1\left[\begin{array}{l} -\frac{1}{56}R\theta_1bt_0^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_0^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b\right)\right)t_0^5 \\ +\frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2+at_0\right)-\frac{1}{2}b+Ra\right)t_0^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)\right)t_0^3 \\ +\frac{1}{2}\left(\frac{1}{8}b\theta_1t_0^4+\frac{1}{6}a\theta_1t_0^3+\frac{1}{2}bt_0^2+at_0\right)t_0^2 \end{array}\right]$$

$$- c\theta_1 \begin{bmatrix} -\frac{1}{56}R\theta_1 b t_r^7 + \frac{1}{6}\left(\frac{1}{8}b\theta_1 - \frac{1}{3}Ra\theta_1\right)t_r^6 + \frac{1}{5}\left(\frac{1}{3}a\theta_1 - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_r^5 \\ + \frac{1}{4}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_r^4 + \frac{1}{3}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_r^3 \\ + \frac{1}{2}\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)t_r^2 \end{bmatrix}$$

(16)

(v) Shortage cost:

$$SC = -c_2\int_{t_0}^{T} I(t)e^{-Rt}dt = -c_2\int_{t_0}^{T}\begin{bmatrix} -\frac{1}{3}b\delta t^3 - \frac{1}{2}[a\delta + b(1-\delta T)]t^2 - at(1-\delta T) \\ + \frac{1}{3}b\delta t_0^3 + \frac{1}{2}a\delta t_0^2 + \frac{1}{2}bt_0^2 - \frac{1}{2}b\delta T t_0^2 - a\delta t_0 T + at_0 \end{bmatrix} e^{-Rt}dt$$

$$= c_2\begin{bmatrix} \frac{1}{15}b\delta R T^5 + \frac{1}{4}\left(-\left(-\frac{1}{2}a\delta - \frac{1}{2}b(1-\delta T)\right)R - \frac{1}{3}b\delta\right)T^4 \\ + \frac{1}{3}\left(a(1-\delta T)R - \frac{1}{2}a\delta - \frac{1}{2}b(1-\delta T)\right)T^3 \\ + \frac{1}{2}\left(-\left(\frac{1}{2}a\delta t_0^2 + \frac{1}{2}bt_0^2 - \frac{1}{2}b\delta T t_0^2 + \frac{1}{3}b\delta t_0^3 - a\delta T t_0 + at_0\right)R - a(1-\delta T)\right)T^2 \\ + \frac{1}{2}a\delta T t_0^2 + \frac{1}{2}bTt_0^2 - \frac{1}{2}b\delta T^2 t_0^2 + \frac{1}{3}b\delta T t_0^3 - a\delta T^2 t_0 + aTt_0 \end{bmatrix}$$

$$+ c_2\begin{bmatrix} \frac{1}{15}b\delta R t_0^5 + \frac{1}{4}\left(-\left(-\frac{1}{2}a\delta - \frac{1}{2}b(1-\delta T)\right)R - \frac{1}{3}b\delta\right)t_0^4 \\ + \frac{1}{3}\left(a(1-\delta T)R - \frac{1}{2}a\delta - \frac{1}{2}b(1-\delta T)\right)t_0^3 \\ + \frac{1}{2}\left(-\left(\frac{1}{2}a\delta t_0^2 + \frac{1}{2}bt_0^2 - \frac{1}{2}b\delta T t_0^2 + \frac{1}{3}b\delta t_0^3 - a\delta T t_0 + at_0\right)R - a(1-\delta T)\right)t_0^2 \\ + \frac{1}{2}a\delta t_0^3 + \frac{1}{2}bt_0^3 - \frac{1}{2}b\delta T t_0^3 + \frac{1}{3}b\delta t_0^4 - a\delta T t_0^2 + at_0^2 \end{bmatrix}.$$

(17)

(vi) Cost due to lost sales:

$$LS = c_3\int_{t_2}^{T}(a+bt)[1-(1-\delta(T-t)]e^{-Rt}dt = c_3\delta\begin{bmatrix} aT(T-t_0) + \frac{1}{2}(bT - a - aRT)(T^2 - t_0^2) \\ + \frac{1}{3}(-b - bRT + aR)(T^3 - t_0^3) + \frac{1}{4}bR(T^4 - t_2^4) \end{bmatrix}.$$

(18)

(vii) Interest Earned: There are two cases:

Case I : ($M \le t_r \le T$):

In this case interest earned is:

$$IE_1 = pI_e\int_0^M (a+bt)te^{-Rt}dt = pI_e\left[-\frac{1}{4}bRM^4 + \frac{1}{3}(-Ra+b)M^3 + \frac{1}{2}aM^2\right]$$

(19)

Case II : ($t_0 \le M \le T$):

In this case interest earned is:

$$IE_2 = pI_e\left(\int_0^{t_0}(a+bt)te^{-Rt}dt + (a+bt_0)t_0(M-t_0)\right)$$

$$= pI_e\left[-\frac{1}{4}bRt_0^2 + \frac{1}{3}(-Ra+b)t_0^3 + \frac{1}{2}at_0^2 + (a+bt_0)t_0(M-t_0)\right] \qquad (20)$$

(viii) Interest Payable: There are three cases described as in figure:

Case I : ($M \le t_r \le T$):
In this case, annual interest payable is:

$$IP_1 = cI_p\left[\int_M^{t_r} I_r(t)e^{-Rt}dt + \int_M^{t_r} I_0(t)e^{-Rt}dt + \int_{t_r}^{t_0} I_0(t)e^{-Rt}dt\right]$$

$$= cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bt_r^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)t_r^5 + \frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)t_r^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)t_r^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)t_r^2 \\ + \frac{1}{8}\theta_2 bt_r^5 + \frac{1}{6}a\theta_2 t_r^4 + \frac{1}{2}bt_r^3 + at_r^2 \end{array}\right]$$

$$- cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_2 bM^6 + \frac{1}{5}\left(\frac{1}{8}\theta_2 b - \frac{1}{3}R\theta_2 a\right)M^5 + \frac{1}{4}\left(\frac{1}{3}\theta_2 a - R\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b\right)\right)M^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_2\left(\frac{1}{2}bt_r^2 + at_r\right) - \frac{1}{2}b + Ra\right)M^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_2 t_r^4 + \frac{1}{6}a\theta_2 t_r^3 + \frac{1}{2}bt_r^2 + at_r\right)\right)M^2 \\ + \frac{1}{8}\theta_2 bt_r^4 M + \frac{1}{6}a\theta_2 t_r^3 M + \frac{1}{2}bt_r^2 M + at_r M \end{array}\right]$$

$$+ cI_p W\left[t_r + \frac{1}{8}R\theta_1 t_r^4 - \frac{1}{6}\theta_1 t_r^3 - \frac{1}{2}Rt_r^2\right] - cI_p W\left[M + \frac{1}{8}R\theta_1 M^4 - \frac{1}{6}\theta_1 M^3 - \frac{1}{2}RM^2\right]$$

$$+ cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bt_0^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_0^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_0^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_0^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_0^2 \\ + \frac{1}{8}\theta_1 bt_0^5 + \frac{1}{6}a\theta_1 t_0^4 + \frac{1}{2}bt_0^3 + at_0^2 \end{array}\right]$$

$$- cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bt_r^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_r^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_r^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_r^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)t_r^2 \\ + \frac{1}{8}\theta_1 bt_0^4 t_r + \frac{1}{6}a\theta_1 t_0^3 t_r + \frac{1}{2}bt_0^2 t_r + at_0 t_r \end{array}\right]$$

(21)

Case II : ($t_r \le M \le T$):
In this case interest payable is:

$$IP_2 = cI_p\int_M^{t_0} I_0(t)e^{-Rt}dt$$

$$= cI_p\left[\begin{array}{l} -\frac{1}{48}R\theta_1 bt_0^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3}R\theta_1 a\right)t_0^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)t_0^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)t_0^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + aT\right)\right)t_0^2 \\ + \frac{1}{8}\theta_1 bt_0^5 + \frac{1}{6}a\theta_1 t_0^4 + \frac{1}{2}bt_0^3 + at_0^2 \end{array}\right]$$

$$- cI_p \left[\begin{array}{l} - \frac{1}{48} R\theta_1 bM^6 + \frac{1}{5}\left(\frac{1}{8}\theta_1 b - \frac{1}{3} R\theta_1 a\right)M^5 + \frac{1}{4}\left(\frac{1}{3}\theta_1 a - R\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b\right)\right)M^4 \\ + \frac{1}{3}\left(-\frac{1}{2}\theta_1\left(\frac{1}{2}bt_0^2 + at_0\right) - \frac{1}{2}b + Ra\right)M^3 + \frac{1}{2}\left(-a - R\left(\frac{1}{8}b\theta_1 t_0^4 + \frac{1}{6}a\theta_1 t_0^3 + \frac{1}{2}bt_0^2 + at_0\right)\right)M^2 \\ + \frac{1}{8}\theta_1 bt_0^4 M + \frac{1}{6}a\theta_1 t_0^3 M + \frac{1}{2}bt_0^2 M + at_0 M \end{array} \right] \tag{22}$$

Case III : ($t_0 \le M \le T$):
In this case, no interest charges are paid for the item. So,

$IP_3 = 0$. (23)

The retailer's total cost during a cycle, $TC_i(t_r,T)$, i=1,2,3 consisted of the following:

$$TC_i = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + SC + LS + IP_i - IE_i\right]$$
(24)

and t_0 is approximately related to t_r through equation (12).
Substituting values from equations (13) to (18) and equations (19) to (23) in equation (24), total costs for the three cases will be as under:

$$TC_1 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + SC + LS + IP_1 - IE_1\right]$$
(25)

$$TC_2 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + SC + LS + IP_2 - IE_1\right]$$
(26)

$$TC_3 = \frac{1}{T}\left[A + HC(OW) + HC(RW) + DC + SC + LS + IP_3 - IE_2\right]$$
(27)

The optimal value of tr = tr*, T=T* (say), which minimizes TCi can be obtained by solving equation (25), (26) and (27) by differentiating it with respect to t_r and T and equate it to zero i.e.

i.e. $\frac{\partial TC_i(t_r,T)}{\partial t_r} = 0, \ \frac{\partial TC_i(t_r,T)}{\partial T} = 0, \ i=1,2,3,$ (28)

provided it satisfies the condition

$\frac{\partial^2 C_i(t_r,T)}{\partial^2 t_r} > 0, \ \frac{\partial^2 C_i(t_r,T)}{\partial^2 T} > 0$ and $\left[\frac{\partial^2 C_i(t_r,T)}{\partial^2 t_r}\right]\left[\frac{\partial^2 C_i(t_r,T)}{\partial^2 T}\right] - \left[\frac{\partial^2 C_i(t_r,T)}{\partial t_r \partial T}\right]^2 > 0, \ i=1,2,3.$ (29)

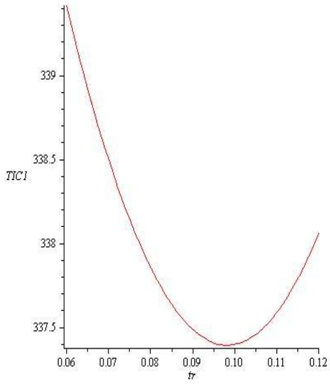
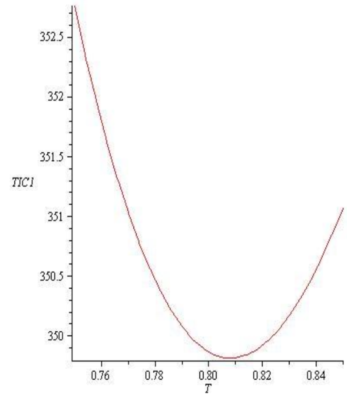
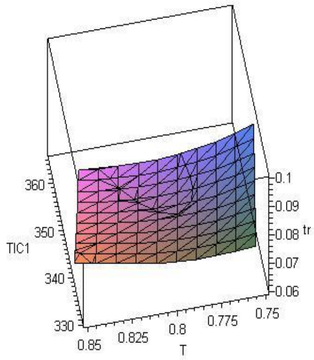
IV. NUMERICAL EXAMPLES

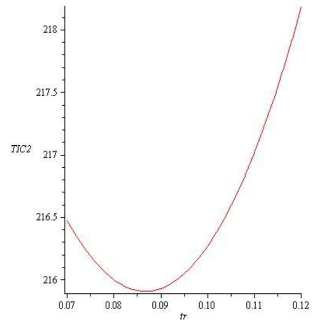
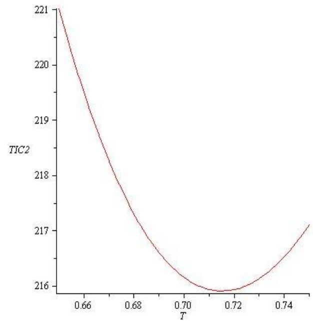
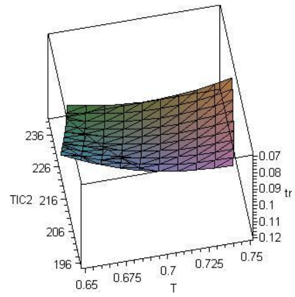
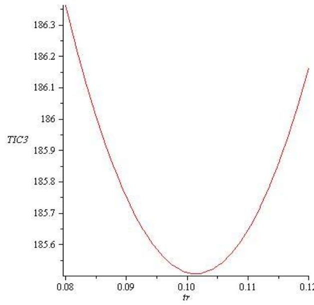
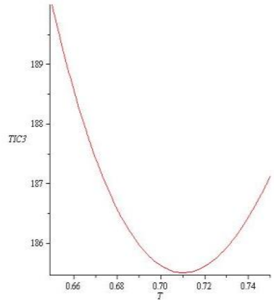
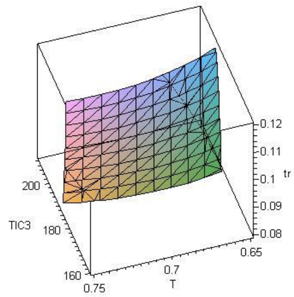
Case I: Considering A= Rs.150, W = 100, a = 200, b=0.05, c=Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1 = Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, R = 0.06, c_2 = Rs. 8, c_3 = Rs. 2, δ = 0.8, M=0.01 year, in appropriate units. The optimal value of t_r^* =0.0831, T*=0.8081 and TC_1^* = Rs. 349.8128.

Case II: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie = Rs. 0.12, c_2 = Rs. 8, c_3 = Rs. 2, δ = 0.8, M=0.55 year, in appropriate units. The optimal value of t_r^* =0.0867, T*=0.7151 and TC_2^* = Rs. 215.9081.
Case III: Considering A= Rs.150, W = 100, a = 200, b=0.05, c = Rs. 10, p= Rs. 15, θ_1=0.1, θ_2 =0.06, x_1= Rs. 1, y_1=0.05, x_2= Rs. 3, y_2=0.06, Ip= Rs. 0.15, Ie= Rs. 0.12, c_2 = Rs. 8, c_3 = Rs. 2, δ = 0.8, M = 0.65 year, in appropriate units. The optimal value of t_r^* =0.1014, T*=0.7101 and TC_1^* = Rs. 185.5066.

The second order conditions given in equation (29) are also satisfied. The graphical representation of the convexity of the cost functions for the three cases are also given.

Case I		
t_r and cost	T and cost	t_r, T and cost

 Graph 1	 Graph 2	 Graph 3

Case II		
t_r and cost	T and cost	t_r, T and cost
 Graph 4	 Graph 5	 Graph 6
Case III		
t_r and cost	T and cost	t_r, T and cost
 Graph 7	 Graph 8	 Graph 9

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

		Case I			Case II			Case III		
		($M \le t_r \le T$)			**($t_r \le M \le T$)**			**($t_0 \le M \le T$)**		
Para-meter	%	t_r	T	Cost	t_r	T	Cost	t_r	T	Cost
a	+10%	0.0951	0.7620	367.8316	0.1068	0.6783	218.6641	0.1149	0.6672	185.5466
	+5%	0.0895	0.7841	358.9034	0.0973	0.6959	217.3366	0.1086	0.6878	185.5075
	-5%	0.0758	0.8344	340.5512	0.0749	0.7362	214.3719	0.0932	0.7345	185.3120
	-10%	0.0674	0.8633	331.1092	0.0616	0.7596	212.7207	0.0838	0.7613	184.9143
x_1	+10%	0.0774	0.8055	353.8748	0.0830	0.7144	220.5564	0.0969	0.7087	190.3883
	+5%	0.0802	0.8068	351.8509	0.0848	0.7148	218.2381	0.0991	0.7094	187.9540
	-5%	0.0860	0.8094	347.7605	0.0885	0.7154	213.5663	0.1036	0.7108	183.0461
	-10%	0.0888	0.8107	345.6942	0.0904	0.7157	211.2129	0.1058	0.7115	180.5725
x_2	+10%	0.0786	0.8037	350.0558	0.0833	0.7119	216.2113	0.0965	0.7054	185.9206

	+5%	0.0808	0.8059	349.9375	0.0850	0.7134	216.0626	0.0989	0.7077	185.7184
	-5%	0.0856	0.8105	349.6812	0.0885	0.7168	215.7475	0.1040	0.7127	185.2846
	-10%	0.0882	0.8131	349.5419	0.0904	0.7186	215.5806	0.1068	0.7153	185.0514
θ_1	+10%	0.0806	0.8062	350.6601	0.0850	0.7140	216.8554	0.0991	0.7085	186.5307
	+5%	0.0819	0.8072	350.2374	0.0858	0.7145	216.3824	0.1003	0.7093	186.0197
	-5%	0.0844	0.8091	349.3862	0.0876	0.7157	215.4322	0.1025	0.7109	184.9916
	-10%	0.0857	0.8101	348.9576	0.0885	0.7162	214.9549	0.1037	0.7118	184.4746
θ_2	+10%	0.0831	0.8081	349.8143	0.0867	0.7151	215.9099	0.1013	0.7101	185.5096
	+5%	0.0831	0.8081	349.8135	0.0867	0.7151	215.9090	0.1014	0.7101	185.5081
	-5%	0.0831	0.8082	349.8121	0.0867	0.7151	215.9071	0.1014	0.7102	185.5051
	-10%	0.0832	0.8082	349.8114	0.0867	0.7151	215.9062	0.1014	0.7102	185.5037
δ	+10%	0.0835	0.8083	350.2002	0.0869	0.7141	216.1404	0.1016	0.7092	185.6874
	+5%	0.0833	0.8082	350.0065	0.0868	0.7146	216.0246	0.1015	0.7097	185.5973
	-5%	0.0829	0.8081	349.6190	0.0866	0.7156	215.7909	0.1013	0.7106	185.4152
	-10%	0.0827	0.8080	349.4251	0.0865	0.7160	215.6730	0.1012	0.7111	185.3232
R	+10%	0.0836	0.8098	349.4392	0.0865	0.7156	215.9521	0.1011	0.7104	185.5842
	+5%	0.0834	0.8090	349.6263	0.0866	0.7153	215.9302	0.1013	0.7103	185.5455
	-5%	0.0829	0.8073	349.9988	0.0868	0.7149	215.8857	0.1015	0.7100	185.4675
	-10%	0.0826	0.8065	350.1843	0.0869	0.7146	215.8630	0.1017	0.7098	185.4282
A	+10%	0.0832	0.8080	349.5957	0.1004	0.7179	199.1456	0.1044	0.7005	165.5004
	+5%	0.0832	0.8081	349.7043	0.0935	0.7164	207.4508	0.1029	0.7053	175.5502
	-5%	0.0831	0.8082	349.9214	0.0800	0.7141	224.5186	0.0998	0.7148	195.3707
	-10%	0.0830	0.8082	350.0298	0.0734	0.7133	233.2833	0.0981	0.7195	205.1433
M	+10%	0.1018	0.8418	367.9956	0.1017	0.7440	236.4685	0.1205	0.7427	206.1560
	+5%	0.0926	0.8251	358.9970	0.0943	0.7297	226.2903	0.1110	0.7266	195.9471
	-5%	0.0735	0.7910	340.4325	0.0789	0.7003	205.3104	0.0915	0.6934	174.8192
	-10%	0.0636	0.7735	332.2985	0.0710	0.6853	194.4848	0.0814	0.6763	163.8678

From the table we observe that as parameter a increases/ decreases average total cost increases/ decreases in case I and case II and there is very slight increase/ decrease in case III with respective increase/ decrease in parameter a..

From the table we observe that with increase/ decrease in parameters x_1 and θ_1, there is corresponding increase/ decrease in total cost for case I, case II and case III respectively.

Also, we observe that with increase and decrease in the value of x_2, θ_2, δ and R, there is corresponding very slight increase/ decrease in total cost for case I, case II and case III. From the table we observe that with increase/ decrease in M, there is corresponding decrease/ increase in total cost for case I, case II and case III respectively. Moreover, we observe that with increase/ decrease in the value of A, there is corresponding increase/ decrease in total cost in cases I, II and III.

VI. CONCLUSION

We have developed a two warehouse inventory model for deteriorating items with linear demand and partial backlogging under inflationary conditions and permissible delay in payments in this model. It is assumed that rented warehouse holding cost is greater than own warehouse holding cost but provides a better storage facility and there by deterioration rate is low in rented warehouse. Sensitivity with respect to parameters have been carried out. The results show that there is increase/ decrease in cost when there is increase/ decrease in the parameter values.

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