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On the forward problem of ocean wave-seabed interaction in the shallow continental shelf.

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ABSTRACT: *A rigorous treatment of the multi-layer effects on the transmission of acoustic waves through a layered medium is presented. Therefrom, we calculate the displacement components of the Earth's layers below the seabed in response to the oscillating bottom pressure associated with the low phase velocity component of water waves. Consequently, we compare the displacement components of the medium associated with the seismic events in a homogeneous elastic medium with those oscillations in a multi-layered Earth's structure. Interestingly, it is observed that each type of medium appears to support acoustic waves trapped near the Earth's surface. In the multi-layered case however, the drop in the acoustic wave energy in downward direction is more apparent depicting the effect of wave refraction across the layers. This gives interesting revelations to the solution of the forward problem of ocean wave-seabed interaction in the shallow continental shelf.*

KEYWORDS: *acoustic waves, layered medium, seabed, Earth, continental shelf.*

I. INTRODUCTION

Through variations in hydrodynamic pressure on the seafloor, the propagation of surface gravity waves through intermediate to shallow water zone induce small motions of the seabed sediments. Several investigations have been made on the seismic response of the seafloor to the activities of the ocean waves. Sleath (1984), Yamamoto (1977, 1983, 1986), Okeke (1985) and more recently Trevorrow (1991), Okeke and Asor (2000), Asor (2000), Chanson (2000) have given extensive review of the theorems and data related to this topic formulating convincing models that are backed with realistic data for both the forward and the inverse problems of the wave-seabed interactions.

As regards the generating source, the central figure is the activity of the low phase velocity gravity water waves as they propagate over shallow water areas towards the shoreline. In this process, the amplitude of the low velocity wave components grows considerably. Since the wave energy is proportional to the square of its height, the corresponding energy grows and spreads among all possible range of wave numbers. These include the low wave number components which then contain sufficient energy enough to resonate the elastic modes of the seabed. These are incidentally transmitted through the elastic layer network below the seabed (Hasselmann, 1963; Asor and Okeke, 2000).

In our investigations, we have assumed that the influence of the soil porosity on the transmission of the elastic wave energy is negligible. On the local scale therefore, the earth below the seabed is incidentally elastic and horizontally layered. With these simplifying assumptions, we then calculate the displacement components of Earth's layers below the seabed in response to the oscillating bottom pressure associated with the low phase velocity component of the water waves. The calculations are however confined to the dominant activities of the first order pressure distributions in the shallow water.

II. THE GOVERNING EVOLUTION EQUATIONS OF THE WATER/SOLID INTERACTIONS

In this consideration, the x-axis is horizontal and directed normal to the shoreline. The z-axis is vertical and points downwards from the seafloor which is the origin of the coordinate system in this model. Here, t represents the time in scale to be specified. $\phi(x,z,t)$ and $\Psi(x,z,t)$ are the scalar potentials for the compressional waves with speed α and shear waves with speed β respectively. ρ_s and ρ_w are respectively, the densities of the solid and water; the two of which are in welded contact. Further, the Lame's constants λ and μ are related to α and β as follows:

$$\alpha^2 = \frac{(\lambda + 2\mu)}{\rho_s} \qquad 2.1$$

$$\beta^2 = \frac{\mu}{\rho_s} \qquad 2.2$$

From the above, we formulate the equations governing the evolution of the water/solid interactions. These are expressed in terms of the displacement components (U, W) of the seafloor and beyond. Following Asor (2000), they are

$$P(k) exp ik(x - ct) = \lambda \frac{\partial U}{\partial x} + (\lambda + 2\mu)\frac{\partial W}{\partial z} + \gamma \rho_s \frac{\partial U}{\partial t} \qquad (2.3)$$

$$\mu\left(\frac{\partial W}{\partial x} + \frac{\partial U}{\partial z}\right) + \gamma\rho \frac{\partial W}{\partial t} = 0 \qquad (2.4)$$

$$U = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z} \qquad (2.5)$$

$$W = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \qquad (2.6)$$

where $P(k)$ is the amplitude spectrum of the surface gravity water wave. γ is the layer damping coefficient. k and c are the low wave number and the corresponding high phase velocity components in the spectrum of water waves. Apparently, they are of such magnitude and directions as to resonate the seismic modes of the seabed.

III. THE DISPLACEMENT COMPONENTS OF THE SUBSURFACE LAYER

We attempt to derive the layer matrix and in this consideration, we express the displacement components of the sub-surface layer in the form

$$U(x,z,t) = \overline{U}(z) exp ik(x - ct) \qquad (3.1)$$

$$W(x,z,t) = \overline{W}(z) exp ik(x - ct) \qquad (3.2)$$

Introducing (3.1) and (3.2) into (2.3) and (2.4) respectively, we obtain

$$P(k) = i\lambda k\overline{U} + (\lambda + 2\mu)\frac{d\overline{W}}{dz} - i\gamma\rho kc\overline{U} \qquad (3.3)$$

$$0 = \mu\left(ik\overline{W} + \frac{d\overline{U}}{dz}\right) - i\gamma\rho kc\overline{W} \qquad (3.4)$$

Rearranging (3.3) and (3.4), then,

$$\frac{d\overline{U}}{dz} = ik\left(\frac{c}{\beta^2}\right)\left(\gamma - \frac{\mu}{\rho_s c}\right)\overline{W} \qquad (3.5)$$

$$\frac{d\overline{W}}{dz} = ik\left(\frac{c}{\alpha^2}\right)\left(\gamma - \frac{\lambda}{\rho_s c}\right)\overline{U} + \frac{P(k)}{\lambda + 2\mu} \qquad (3.6)$$

(3.5) and (3.6) combine to give

$$\frac{d}{dz}\begin{bmatrix}\bar{U}\\ \bar{W}\end{bmatrix}=\begin{bmatrix}0 & \frac{ick}{\beta^2}\left(\gamma-\frac{\mu}{\rho_s c}\right)\\ \frac{ick}{\alpha^2}\left(\gamma-\frac{\lambda}{\rho_s c}\right) & 0\end{bmatrix}\begin{bmatrix}\bar{U}\\ \bar{W}\end{bmatrix}+\begin{bmatrix}0\\ \frac{P(k)}{\lambda+2\mu}\end{bmatrix}$$

(3.7)

$$c=c_0+i\delta,\ \delta<<c_0$$

Thus,

$$\frac{1}{c}=\frac{c_0-i\delta}{c_0^2} \quad (3.8)$$

The eigenvalues of the coupling matrix A, where

$$A=\begin{bmatrix}0 & \frac{ick}{\beta^2}\left(\gamma-\frac{\mu}{\rho_s c}\right)\\ \frac{ick}{\alpha^2}\left(\gamma-\frac{\lambda}{\rho_s c}\right) & 0\end{bmatrix} \quad (3.9)$$

are $\pm v$ and

$$v=\frac{ick}{\alpha\beta}\left[\left(\gamma-\frac{\lambda}{\rho_s c}\right)\left(\gamma-\frac{\mu}{\rho_s c}\right)\right]^{\frac{1}{2}}$$

(3.10)

Because of (3.8), v has non-zero real and imaginary part. (3.7) now takes the usual form

$$\frac{df}{dz}=\mathbf{A}f+g_0$$

(3.11)

$$f=\begin{bmatrix}\bar{U}\\ \bar{W}\end{bmatrix},\ g_0=\begin{bmatrix}0\\ \frac{P(k)}{\lambda+2\mu}\end{bmatrix}$$

(3.12)

Using the Poisson's relation $\alpha^2=3\beta^2$, equations (3.8) and (3.10) combine to give

$$Re(v)=\frac{k\lambda\delta^2}{\alpha\beta c_0^2\rho_s}$$

(3.13)

$$Im(v)=\frac{kc_0}{\alpha\beta}\left(\gamma-\frac{\lambda}{\rho_s c_0}\right)$$

(3.14)

We observe the presence of the factor δ^2 in the numerator of (3.13). This depicts the low rate of energy decay in the vertical and the equation further suggests that the material damping coefficient does not affect the vertical decay of the oscillation.

IV. THE LOCAL PATTERN OF THE DECOUPLED COMPRESSIONAL AND SHEAR WAVES

In general, it is assumed that the elastic parameters μ, λ and density ρ_s are constants and functions of the vertical coordinates only. Consequently, in the multi-layered half-space, the region below the Earth's surface is structurally assumed to consist of horizontally parallel slabs in welded contact. The simplified situation implies that the region within each slab is homogeneous and thus satisfies our assumption that the elastic parameters are constant. Equation (3.11) is then a linear first order differential equation with constant matrix coefficients. The solution can be obtained in the form

$$\begin{bmatrix} \overline{U} \\ \\ \overline{W} \end{bmatrix} = \frac{\delta k}{\rho_s c}\left\{ \beta^{-1}\left(\gamma - \frac{\mu}{\rho_s c}\right)\begin{bmatrix} \beta^{-1}\left(\gamma - \frac{\mu}{\rho_s c}\right) \\ \\ 1 \end{bmatrix} e^{+\nu z} + \alpha^{-1}\left(\gamma - \frac{\lambda}{\rho_s c}\right)\begin{bmatrix} 1 \\ \\ \alpha^{-1}\left(\gamma - \frac{\lambda}{\rho_s c}\right) \end{bmatrix} e^{-\nu z} \right\} + g_0 B \tag{4.1}$$

The decaying form of (4.1) suitable for computation is

$$\begin{bmatrix} \overline{U} \\ \overline{W} \end{bmatrix} = \frac{\delta}{k\, c_0}\left\{ \begin{bmatrix} b_1^2 \\ 1 \end{bmatrix} \sin\left(\frac{k\, c_0 b_1 z}{\alpha}\right) + \begin{bmatrix} 1 \\ b_2^2 \end{bmatrix} \mathrm{Cos}\left(\frac{k\, c_0 b_2 z}{\beta}\right) \right\} e^{-b_3 z} + g_0 B \tag{4.2}$$

$$b_1 = \beta^{-1}\left(\gamma - \frac{\mu}{\rho_s c_0}\right), b_2 = \alpha^{-1}\left(\gamma - \frac{\lambda}{\rho_s c_0}\right), b_3 = \mathrm{Re}(\nu), B = A^{-1}.$$

Equation (4.2) seems to have depicted the local pattern of the decoupled compressional and shear waves, each of which is subjected to depth decay. The decay depicted by this model is essentially due to the non-zero imaginary part of the phase velocity c introduced by the damping term γ in equations (2.3) and (2.4).

V. THE DISPLACEMENT COMPONENTS OF THE SEABED

In this consideration, we introduce the propagator matrix $P(z, z_0)$ defined by the relation

$$f(z) = P(z, z_0)\, f(z_0) \tag{5.1}$$

and $f(z_0) = P(z_0, z_0)\, f(z_0)$, thus $P(z_0, z_0) = I$, where I is an identity matrix. Also,

$P(z_1, z_2) = P^{-1}(z_2, z_1)$. Further, $P(z, z_0)$ is determined from the equation (5.1) substituted into (3.11). That is

$$\frac{d}{dz} P(z, z_0) = A(z) P(z, z_0) \tag{5.2}$$

The complete solution of (5.2) is

$$P(z, z_0) = \exp\left[\int_{z_0}^{z} A(z')\,dz'\right] \tag{5.3}$$

since $P(z_0, z_0) = I$

Using (3.11), we note that

$$P^{-1}(z, z_0)\frac{d}{dz} f(z) - P^{-1}(z, z_0) A(z) f(z) = P^{-1}(z, z_0) g(z). \tag{5.4}$$

i.e. $$\frac{d}{dz}[P(z_0, z)f(z)] = P(z_0, z)g(z) \tag{5.5}$$

This can be integrated to obtain

$$P(z_0, z)f(z) = f(z_0) + \int_{z_0}^{z} P(z_0, z')g(z')dz' \tag{5.6}$$

and

$$f(z) = P^{-1}(z_0, z)f(z_0) + P^{-1}(z_0, z)\int_{z_0}^{z} P(z_0, z')g(z')dz' \tag{5.7}$$

But

$$P^{-1}(z_0, z) = P(z, z_0) \text{ an d } P(z, z_0)P(z_0, z') = P(z, z') \tag{5.8}$$

Thus,

$$f(z) = f(z_0)P(z, z_0) + \int_{z_0}^{z} P(z, y)\, g\,(y)dy \tag{5.9}$$

Further development of (5.8) and (5.9) follows from Bullen (1985). $f(z_0)$ is the row vector calculated from the displacement components of the seabed.

In this analysis, using (5.3), (5.8) and (5.9) simplifies to give

$$f(z) = P(z, z_0) f(z_0) + \int_{z_0}^{z} g(y')\exp\left[\int_{z_0}^{y} A(y)dy'\right]dy$$

(5.10)

VI. SEISMIC OSCILLATIONS INDUCED BY THE SHALLOW SEA WAVES

The seismic oscillations induced by the shallow sea waves do not usually penetrate below the depth z=100m measured from the seafloor. We now subdivide this depth into twenty parallel and welded slabs each of thickness 5m. Within each subdivision, λ, μ and ρ_s are assumed to be constant. Regarding $z=z_0$ as the seafloor; the depth of slabs below $z = z_0$ are respectively $z = z_1, z_2, \ldots, z_{20}$. Thus, for
$z_s \le z \le z_{s+1}$, r = 1, 2, ..., 20

$$P(z_r, z_{r+1}) = \exp[-A(z_{r+1} - z_r)] \tag{6.1}$$

which follows from (5.3). In the numerical computations of the surface displacement components of the layer; we have used the Sylvester's interpolation formula (Bullen, 1985) to obtain for each layer

$$\exp[A(z_{r+1} - z_r)] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{6.2}$$

where
$$\begin{aligned} P_{11} &= \cosh[\nu_r h_r], \\ P_{12} &= (\nu_i \mu_i)^{-1}\sinh[\nu_r h_r), \\ P_{21} &= (\nu_r \mu_r)\sinh(\nu_r h_r) \text{ and} \\ P_{22} &= \cosh(\nu_r h_r), h_r = z_{r+1} - z_r. \end{aligned} \tag{6.3}$$

and ν_r is the value of ν in $z_r \le z \le z_{r+1}$ of the r^{th} layer.

Table I

Z (m)	$\overline{W}^{*}$ *(microns)*	$\overline{W}^{**}$ *(microns)*	$\overline{U}^{*}$ *(microns)*	$\overline{U}^{**}$ *(microns)*
0	8.12	7.48	6.27	6.23
10	7.65	7.34	6.04	5.91
20	7.49	7.03	5.83	5.22
30	7.03	6.79	5.62	4.53
40	5.99	5.45	4.81	4.04
50	5.21	4.67	4.23	3.74
60	4.79	4.03	3.74	2.81
70	3.19	2.93	2.14	1.78
80	2.77	1.91	1.33	1.03
90	1.48	1.31	0.06	0.54
100	1.19	0.92	0.01	0.06

Amplitude components for our Earth Model: Homogenous medium (*) and a horizontally layered medium (**) of the seismic noise generated by an 11-second swell.

Discussion

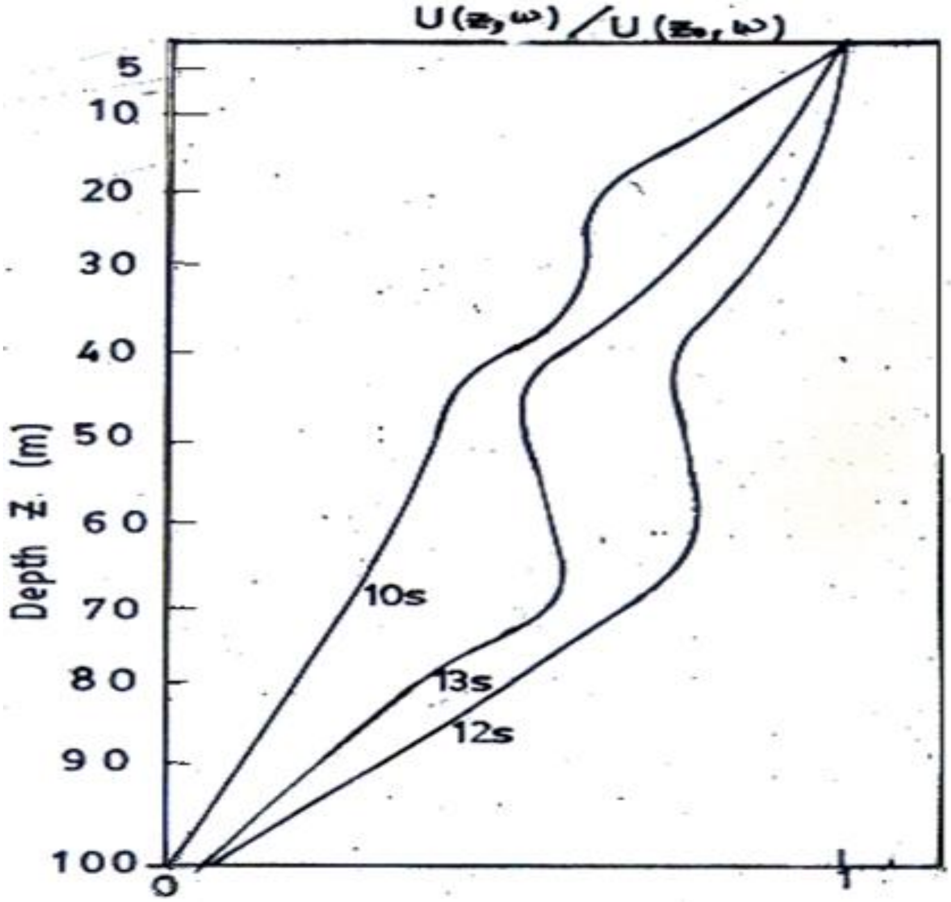
Table I shows the variation of the acoustic wave induced displacement components of the earth's structures below the seabed. Two models are used in this study. These are

(a) homogeneous elastic medium with elastic parameters and density assumed to be constant and
(b) the earth's structures which is horizontally layered.

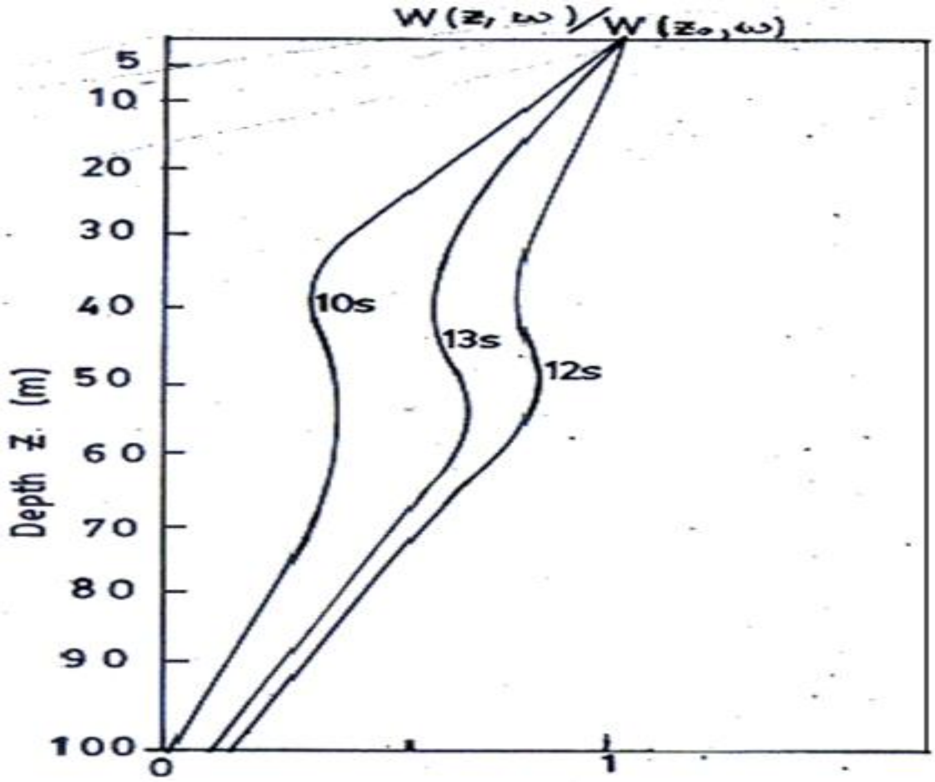
In both models, the displacement components decrease in magnitude with the increasing depth and negligibly small below the depth of 100m. Consequently, the analysis seems to suggest that both models can sustain the progressive acoustic vibrations guided near the earth's surface. Further, considering the vertical profile of the displacement components for each model

(a) gives larger component than
(b) at all depths below the earth's surface.

This depicts the extent of the energy loss as acoustic vibrations are transmitted through a typically multi-layered structure.



The variation of the horizontal ground movements with depth along the Earth's surface.



The variation of the vertical ground movements with depth along the Earth's surface.

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