International Journal Of Mathematics And Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
Www.Ijmsi.Org || Volume 2 Issue 09 || September. 2014 || PP-38-46

Holographic Dark Energy in LRS Bianchi Type-II Space Time

Gitumani Sarma
Research Scholar
Department of Mathematics Gauhati University

ABSTRACT: *In the present paper, we have discussed holographic dark energy in LRS Bianchi type-II space time. The Einstein's field equation are solved by using two assumption. One of them is law off variation for the Hubble parameter, which gives a constant value of deceleration parameter and another is a relation between two metric co-efficient. The law of variation generates power law and exponential form of average scale factor in terms of cosmic time. The physical and kinematical properties of the model are also discussed. The holographic dark energy (DE) EOS parameter behaves like constant,i,e* $\omega_\Lambda = -1$*,which is mathematically equivalent to cosmological constant* Λ *for* $n = 0$*,whereas the holographic dark energy EOS parameter behaves like quintessence for* $n \neq 0$.

I. INTRODUCTION:

Recent cosmological observations indicates that the present universe is undergoing an accelerated expansion. This acceleration of the universe a well established fact that is confirmed by various independent observational data including SNIa, CMB radiation etc. However this discovery can be maintained in general relativity by introducing a mysterious kind of energy source called the dark energy that can generate repulsive gravity [1,2]. It is well known that perfect fluid with a constant equation of state (EOS) parameter lower than $-1/3$. For solution of the Einstein's field equation one can seek by introducing by some kinematical ansatz that are consistent with the observation kinematics of the universe and may investigate the dynamics of the fluid as a possible candidate of the DE. For instance Berman's law [3,4] for the Hubble parameter that yields constant deceleration parameter (DP) has been widely considered for obtaining accelerating cosmological models explicitly in the framework of General relativity. Astronomical observations indicate that our universe currently consists of approximately 70% dark energy, 25% dark matter and 5% baryonic matter and radiation.

Holographic dark energy is the nature of DE can also be studied according to some basic quantum gravitational principle. According to this principle [5], the degrees of freedom in a bounded system should be finite and does not scale by it volume but with its boundary era. Here ρ_Λ is the vacuum energy density. Using this idea in cosmology we take ρ_Λ as DE density. The holographic principle is considered as another alternative to the solution of DE problem. This principle was first considered by G.'t Hooft[6] in the context of blake hole physics. In the context of dark energy problem though the holographic principle proposes a relation between the holographic dark energy density ρ_Λ and the Hubble parameter H as $\rho_\Lambda = H^2$,it does not contribute to the present accelerated expansion of the universe. In [7], Granda and Olivers proposed a holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$,where H is the Hubble parameter and α, β are constants which must satisfy the conditions imposed by the current observational data.

It is known that Bianchi universe anisotropic give rise to CMB anisotropies depending on model type[8].A spatially ellipsoidal geometry of the universe can be described with Bianchi type metrics. However some Bianchi type models isotropize at late times even for ordinary matter and the possible anisotropy of the Bianchi metrices necessarily die away during the inflationary era. In fact this isotropization of the Bianchi metrices is due to the implicit assumption that the DE is isotropic in nature. In this paper, we present general relativistic cosmological models within the framework of spatially homogeneous but totally anisotropic and non flat LRS Bianchi type-II space time. We consider the generalized Berman's law for Hubble parameter and a relation between two metric co-efficient for solving the Einstein's field equations. A similar process was carried out by Singh and Kumar[9] within the framework of LRS Bianchi type-II space time.

II. METRIC AND FIELD EQUATION:

We consider the LRS Bianchi type-II space time in the form

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2 \tag{1}$$

Where A and B are function of time t only.

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + \overline{T}_{ij}) \tag{2}$$

The energy momentum tensor for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \tag{3}$$

And,

$$\overline{T}_{ij} = (\rho_\Lambda + p_\Lambda)u_i u_j - g_{ij}\rho_\Lambda \tag{4}$$

Where ρ_m, ρ_Λ are energy densities of matter and holographic dark energy and p_Λ is the pressure of holographic dark energy.

Field equations are

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{B^2}{4A^4} = \rho_m + \rho_\Lambda \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = -p_\Lambda \tag{6}$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{B^2}{4A^4} = -p_\Lambda \tag{7}$$

For the metric (1) we have the following form

$$V = A^2B = R^3 \tag{8}$$

$$\theta = V^{\mu};_{\mu} = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \tag{9}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{2\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2}\right) - \frac{1}{6}\theta^2 \tag{10}$$

$$H = \frac{\dot{R}}{R} \tag{11}$$

An observed quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 - \frac{\dot{H}}{H^2} \tag{12}$$

The sign of q indicates whether the model inflates or not. The positive sign of q correspond to a standard decelerating model,whreas the negative sign indicates inflation. The average anisotropy parameter (Δ) is defined as-

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{13}$$

Where $\Delta H_i = H_i - H, (i = 1,2,3)$

The holographic dark energy density are given by

$$\rho_\Lambda = \frac{2}{\alpha - \beta}\left(\dot{H} + \frac{3\alpha}{2}H^2\right) \tag{14}$$

The continuity equation can be obtained as

$$\dot{\rho}_m + \dot{\rho}_\Lambda + 3H(\rho_m + \rho_\Lambda + p_\Lambda) = 0 \tag{15}$$

The continuity equation of the matter is

$$\dot{\rho}_m + 3H\rho_m = 0$$
(16)

The continuity equation of the holographic dark energy is

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \tag{17}$$

The barotropic equation of state

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \tag{18}$$

Using (14) and (18) in (17),

$$\omega_\Lambda = -1 - \frac{\left(\ddot{H} + 3\alpha H\dot{H}\right)}{3H\left(\dot{H} + \frac{3\alpha}{2}H^2\right)} \tag{19}$$

III. SOLUTION OF FIELD EQUATION:

There are three equation (5)-(7) with five unknown $A, B, \rho_m, \rho_\Lambda$, and p_Λ. Therefore to solve the field equation we need two condition. For any physically relevant model, the Hubble parameter H and deceleration parameter q are most important observational quantities in cosmology. Hence to solve these equation we firstly assume a law of variation for the generalized Hubble parameter in the anisotropic model which was originally proposed by Berman[3] and Berman and Gomide[4] for FRW model. Later on several authors such as Saha[21], Maharaj and Naidoo[22], Kumar [23] , Pradhan et al.[24], Amirhaschi et al.[24], Baghel and Singh[25], have studied FRW and Bianchi type models by using the law of variation for Hubble parameter. According to the law of variation of generalized Hubble parameter H is related to the average scale factor R of LRS Bianchi type-II space time as

$$H = lR^{-n} = l(A^2B)^{\frac{-n}{3}} \tag{20}$$

Where $l(>0)$ and $n(\geq 0)$ are constant. From equation (8) and (20),

$$\dot{R} = lR^{-n+1} \tag{21}$$

$$\ddot{R} = -l^2(n-1)R^{-2n+1} \tag{22}$$

Using equation (21) and (22) in (12),

$$q = n - 1 \tag{23}$$

We see that the relation (23) gives q as a constant.

Again let us assume

$A = kB^n$, where n is constant.
(24)

From (21) we obtain the average scale factor R as

$$R = (\ln t + c_1)^{\frac{1}{n}} \text{ for } n \neq 0 \tag{25}$$

$$R = \exp\{l(t - c_2)\} \text{ for } n = 0 \tag{26}$$

From equation (16) we have got

$$\rho_m = c_3 R^{-3}, \text{where } c_3 \text{ is a constant of integration} \tag{27}$$

Using (8) and (24),

$$A = \alpha R^{\frac{3}{2n+1}} \tag{28}$$

$$B = \alpha' R^{\frac{3}{n(2n+1)}} \tag{29}$$

Where,

$$\alpha = \frac{k(c_1')^{\frac{1}{2n+1}}}{k^{\frac{2}{2n+1}}}$$

$$\alpha' = \frac{c_1'}{k^2} \text{ where } c_1' \text{ is a constant of integration.}$$

Using (21) and (22) in (14),

$$\rho_\Lambda = \frac{2}{\alpha - \beta}\left[\frac{\ddot{R}}{R} + \left(\frac{3\alpha}{2} - 1\right)\frac{\dot{R}^2}{R^2}\right] \tag{30}$$

From equation (28),

$$p_\Lambda = L\frac{\ddot{R}}{R} + M\frac{\dot{R}^2}{R^2} + NR^{\frac{-6(2n-1)}{N(2n+1)}} \tag{31}$$

Where

$$L = \frac{3(-n-1)}{n(2n+1)}$$

$$M = \frac{3(n+1)\{n(2n+1)-3\}-9(2n^2-n)}{n^2(2n+1)^2}$$

$$N = \frac{(\alpha^{/})^{2-4n}}{4k^4}$$

IV. SPECIAL CASES:

4.1-Behaviour of the model for $n \neq 0$

For $n \neq 0$,we have got $R = (\ln t + c_1)^{\frac{1}{n}}$

Equation (28),(29) gives

$$A = \alpha(\ln t + c_1)^{\frac{3}{n(2n+1)}} \tag{32}$$

$$B = \alpha^{/}(\ln t + c_1)^{\frac{3}{n^2(2n+1)}} \tag{33}$$

Equation (27),(30) and (31) gives

$$\rho_\Lambda = \frac{2}{\alpha-\beta}\left[\frac{2l^2(1-n)+(3\alpha-2)l^2}{2(\ln t + c_1)^2}\right] \tag{34}$$

$$p_\Lambda = L(\ln t + c_1)^{-2} + L^{/}(\ln t + c_1)^{\frac{-6(2n-1)}{n(1+2n)}} \tag{35}$$

Where

$$L^{//} = M^{/}(--n-1) + N^2(n-2n^2)$$

$$L^{/} = \frac{-(\alpha^{/})^{2-4n}}{4k^4},\ M^{/} = \frac{M}{\alpha^{/}},$$

$$M = \frac{3n\alpha^{/}l^2\{3-n(2n+1)\}}{n(2n+1)^2},\ N = \frac{3l}{2n+1}$$

And,

$$\rho_m = c_3(\ln t + c_1)^{\frac{-3}{n}} \tag{36}$$

4.2:Behaviour of the Model for $n = 0$

For $n = 0$ we have got

$$R = \exp\{l(t-c_2)\}$$

Equation (28) and (29) gives

$$A = \alpha\left[\exp\{l(t-c_2)\}\right]^{\frac{3}{(2n+1)}} \tag{37}$$

$$B = \alpha^{/}\left[\exp\{l(t-c_2)\}\right]^{\frac{3}{n(2n+1)}} \tag{38}$$

Again equation (27),(30) and (31) gives

$$\rho_m = c_3\left[\exp\{l(t-c_2)\}\right]^{-3} \tag{39}$$

$$\rho_\Lambda = \frac{3\alpha}{\alpha-\beta} l^2 \tag{40}$$

$$p_\Lambda = W + N\left[\exp\{l(t-c_2)\}\right]^{\frac{-6(2n-1)}{n(2n+1)}} \tag{41}$$

Where $W = (L+M)l^2$

Equation (40) shows that holographic dark energy density is constant for $n = 0$.

V. SOME PHYSICAL PROPERTIES OF THE MODEL:

The physical and kinematical parameter of the model have the following expressions

$$V = \alpha\alpha^{/} R^{\frac{3}{n}} \tag{42}$$

$$\theta = \frac{3}{n}\frac{\dot{R}}{R} \tag{43}$$

Equation (43) shows that for $n = 0$ the expansion scalar is constant whereas for $n \neq 0$

$$\theta = \frac{3l}{n(\ln t + c_1)}$$

$$\sigma^2 = \frac{\alpha^{//}}{(\ln t + c_1)^2} \tag{44}$$

where $\alpha^{//} = \dfrac{l^2\left(50n^2 - 4n + 26\right)}{6n^2(2n+1)^2}$

$$H = \frac{l}{\ln t + c_1} \tag{45}$$

Relation (45) shows that for $n = 0$, Hubble parameter is constant.

$$q = n - 1, 0 \le n < 1 \tag{46}$$

From equation (46) we observed that when $n > 1$,deceleration parameter is positive which shows the decelerating behaviour of the cosmological model. It is worthwhile to mention the work of Vishwakarma[], where he has shown that the decelerating model is also consistent with recent CMB observations model by WNAP , as well as with the high redshift supernova Ia. But for $n = 0$, decelerating parameter is -1. So we have examined that n must be lies between 0 and 1 for decelerating model. The matter density parameter (Ω_m) and holographic dark energy density parameter Ω_Λ are given by

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} \tag{47}$$

Using (24) in (5),

$$\frac{\alpha^{////}}{(\ln t + c_1)^2} - \frac{\beta^{////}}{(\ln t + c_1)^{\frac{6(2n-1)}{n^2(2n+1)}}} = \rho_m + \rho_\Lambda$$

Where $\alpha^{////} = \dfrac{9n(n+1)l^2}{n^2(2n+1)^2}$, $\beta^{////} = \dfrac{(\alpha^{/})^{2-4n}}{4k^4}$

Now,

$$\Omega_m + \Omega_\Lambda = \frac{1}{3l^2}\left[\alpha^{////} - \beta^{////}(\ln t + c_1)^{-\{12n-6-4n^3-2n^2\}/n^2(2n+1)}\right] \text{ for } n \neq 0 \tag{49}$$

Equation (49) gives the sum of the energy density parameter approaches to $\dfrac{\alpha^{////}}{3l^2}$ at late time. So at late time the universe becomes flat. At large time, this model give that the anisotropy of the universe will dampout and universe will become isotropic. This result also shows that in the very early stage of the universe i,e during the radiation and matter dominated era the universe was anisotropic and the universe is approaches to isotropy as dark energy starts to dominate the energy density of the universe.

Using (25) and (26) in (14)

$$\rho_\Lambda = \frac{l^2}{\alpha - \beta}\left\{\frac{3\alpha - 2(n-1)}{(\ln t + c_1)^2}\right\} \text{ for } n \neq 0 \tag{50}$$

From equation (50) we have observed that for $n = 0$ the holographic dark energy density is constant.

Using (25) and (26) in (19),we obtain

$$\omega_\Lambda = -1 + I + \frac{2Q}{3l^3\{3\alpha - 2(n-1)\}(\ln t + c_1)} \tag{51}$$

Where

$$P = l^3(1-n)(1-2n),\ Q = l^3(1-n) + l^2$$

$$I - \frac{6\alpha(n-1)}{3\{3\alpha - 2(n-1)\}} - \frac{2P}{3l^3\{3\alpha - 2(n-1)\}}$$

From (51) we have observed that for $n = 0$, the holographic dark energy EOS parameter constant.i,e

$\omega_\Lambda = -1$

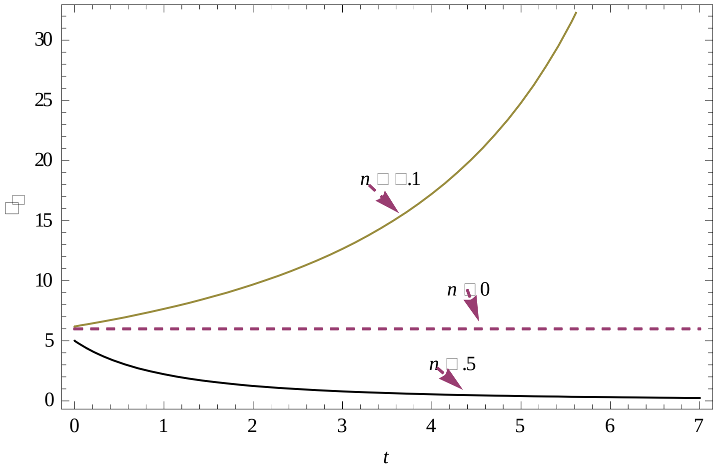

Fig-1The plot of holographic energy density vs. time.

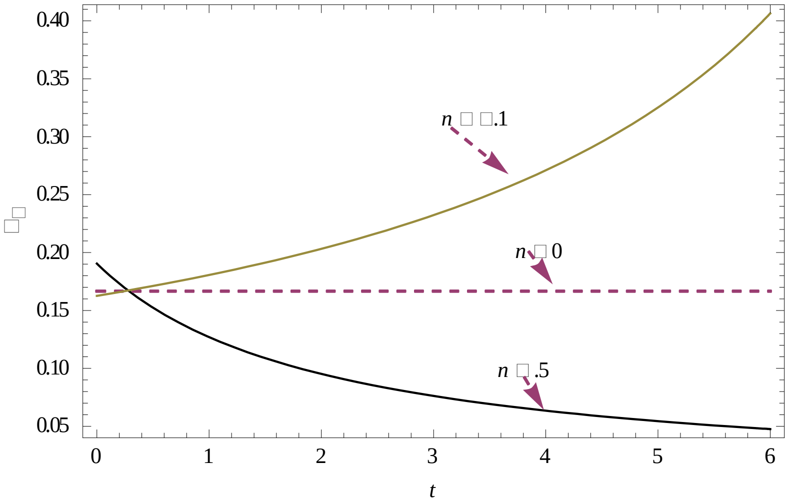

Fig-2 The plot of holographic dark energy EOS parameter vs. time

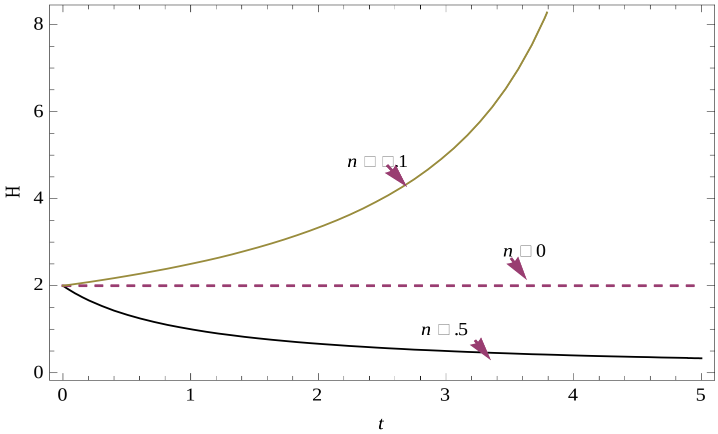

Fig-3The plot of Hubble parameter vs.time

VI. DISSCUSION:

The exact solution of the Einstein's field equationshave been obtained by assuming the law of variation for the generalized Hubble parameter. There is a cosmological view that the universe might be inhomogeneous and anisotropic in the very early stage of the universe. Observational data also suggest that dark energy is responsible for gearing up the universe some five billion years ago. But at the time the universe need not to be isotropic. So we have assumed the universe to be anisotropic and consider the LRS Bianchi type-II universe filled with matter and holographic dark energy. In this paper we have examined the present universe is accelerating under certain condition. We have also observed that the behaviour of the model for $n = 0$ and $n \neq 0$. For $n = 0$, the EOS parameter of the holographic dark energy is constant.i,e $\omega_\Lambda = -1$,which in mathematically equivalent to cosmological constant(Λ).

REFERENCES:

[1]. Copeland,E.J.,Sami,M.,Tsujikawa,S.:Int.J.Mod.Phys. D 15, 1753-1935(2006).
[2]. Li,M., et al.: Commun. Theor.Phys. 56 525-604(2011)
[3]. Berman, M.S.: Nuovo Cimento B 74,182-186(1983)
[4]. Berman,M.S.,Gomide,F.M.: Gen. Relativ.Gravi. 20,191-198(1988)
[5]. Susskind,L.:J.Math.Phys. 36, 6377(1995)
[6]. ‘t Hooft,G.:gr-qc/9310026
[7]. Granda,L.N.,Oliveros,A.:Phys.Lett. B 669,275(2008)
[8]. G.F.R. Ellis: Modern Cosmology, Institute of Physics Publishing, Bristol, pp138-139(2002)
[9]. Singh,C.P.,Kumar,S.:Int.J.Mod.Phys. D 15,419-438(2006)
[10]. Pradhan, A., Amirhaschi, H.: arxiv: 1110.1019v1 [physics.gen-ph].5 oct.2011
[11]. Debnath,U.:Class.Quantum Gravity 25,205019(2008)
[12]. Huang,J.Z.,et al.:Astrophysics.space sci.315,175(2008)
[13]. Shao,Y.,Gui,Y.:Phys.Lett.A 23,65(2008)
[14]. Pradhan,A.,et al.:Int.J.Theor.Phys.(2011).doi:10.1007/s10773-0100793-z.arXiv:1010.1121[gr-qc]
[15]. Perlmutter, S., et al.:Nature(London) 391,51(1998)
[16]. Riess,A.G.,et al.:Astron.J.116,1009(1998)
[17]. Sarkar, S., Mahanta, C.R.: Int.J.Theor.Phys.52, i482 (2013)
[18]. Samanta, G.C.: Int.J.Theor.Phys.52; 4389(2013)
[19]. Adhav, K.S.:Eur.Phys.J.Plus 126, 52(2011)
[20]. Adhav,K.S.:Astrophys.Space sci.335,611(2011)
[21]. B.Saha,Asrophys.Space sci.302,83(2006).
[22]. S.D.Maharaj,R.Naidoo,Astrophys.space sci.208,261(1993)
[23]. S.Kumar,Astrophys.space sci.332,449(2011)
[24]. A.Pradhan,H.Amirhaschi,B.Saha,Int.J.Theor.Phys.50,2923(2011)
[25]. P.S.Baghel,J.P.Singh,Research in Astron.Astrophys 11,1457(2012)