

TOTALLY α * CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to define a new class of functions namely totally α * continuous functions and slightly α * continuous functions and study their properties . Additionally, we relate and compare these functions with some other functions in topological spaces.

Keywords and phrases: totally α * continuous and slightly α * continuous.

I. INTRODUCTION

Continuity is an important concept in mathematics and many forms of continuous functions have been introduced over the years. In 1980 S.N Maheswari and S.S.Thakur [4] defined α continuous functions. RC Jain [2] introduced the concept of totally continuous functions and slightly continuous for topological spaces. In this paper, we define totally α * continuous functions and slightly α * continuous functions and basic properties of these functions are investigated and obtained.

II. PRELIMINARIES

Throughout this paper (X, τ) , (Y, σ) and (Z, η) or X, Y, Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure and the interior of A respectively. The power set of X is denoted by $P(X)$. If A is α * open and α * closed , then it is said to be α * clopen.

Definition 2.1: A subset A of a topological space X is said to be a α * open [5] if $A \subseteq \text{int}^*(\text{cl}(\text{int}^*(A)))$.

Definition 2.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **totally continuous** [2] if $f^{-1}(V)$ is clopen set in X for each open set V of Y .

Definition 2.3: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a α * continuous [8] if $f^{-1}(O)$ is a α * open set of (X, τ) for every open set O of (Y, σ) .

Definition 2.4: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **perfectly α * continuous** [6] if the inverse image of every α * open set in (Y, σ) is both open and closed in (X, τ) .

Definition 2.5: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **slightly continuous**[2] if the inverse image of every clopen set in Y is open in X .

Definition 2.6: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **contra continuous** [1] if $f^{-1}(O)$ is closed in (X, τ) for every open set O in (Y, σ) .

Definition 2.7: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **Contra α * continuous functions** [7] if $f^{-1}(O)$ is α * closed in (X, τ) for every open set O in (Y, σ) .

Definition 2.8: A topological space X is called a α * **connected** [9] if X cannot be expressed as a disjoint union of two non-empty α * open sets.

Definition 2.9: A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be **pre α * open** [7] if the image of every α * open set of X is α * open in Y .

Definition 2.10: A topological space X is said to be **connected** [10] if X cannot be expressed as the union of two disjoint nonempty open sets in X .

Definition 2.11: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **strongly α * continuous** [6] if the inverse image of every α * open set in (Y, σ) is open in (X, τ) .

Definition 2.12: A Topological space X is said to be $\alpha^*T_{1/2}$ space or α^* space [8] if every α^* open set of X is open in X .

Definition 2.13: A space (X, τ) is called a **locally indiscrete space** [3] if every open set of X is closed in X .

Theorem 2.14[5]:

- (i) Every open set is α^* - open and every closed set is α^* -closed set

III. Totally α^* continuous functions

Definition 3.1: A function $(X, \tau) \rightarrow (Y, \sigma)$ is called **totally α^* continuous functions** if the inverse image of every open set of (Y, σ) is both α^* open and α^* closed subset of (X, τ) .

Example 3.2: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{abc\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, Y\}$, $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{bc\}, \{ad\}, \{abd\}, \{acd\}, \{abc\}, X\}$ and $\alpha^*C(X, \tau) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{ad\}, \{bc\}, \{bd\}, \{cd\}, \{abd\}, \{acd\}, \{bcd\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = c$, $g(b) = b$, $g(c) = a$, $g(d) = d$. since, $g^{-1}(\{a\}) = \{c\}$, $g^{-1}(\{b\}) = \{b\}$ and $g^{-1}(\{ab\}) = \{bc\}$ is both α^* open and α^* closed in X . Therefore, g is totally α^* continuous.

Theorem 3.2: Every totally α^* continuous functions is α^* continuous.

Proof: Let O be an open set of (Y, σ) . Since, f is totally α^* continuous functions, $f^{-1}(O)$ is both α^* open and α^* closed in (X, τ) . Therefore, f is α^* continuous.

Remark 3.3: The converse of above theorem need not be true.

Example 3.4: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{bc\}, \{abc\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, Y\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a$, $g(b) = c$, $g(c) = b$, $g(d) = d$. $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{bc\}, \{abc\}, \{abd\}, X\}$ and $\alpha^*C(X, \tau) = \{\emptyset, \{c\}, \{d\}, \{ad\}, \{bd\}, \{cd\}, \{abd\}, \{acd\}, \{bcd\}, X\}$. Clearly, g is not totally α^* continuous since $g^{-1}(\{a\}) = \{a\}$ is α^* open in X but not α^* closed. However, g is α^* continuous.

Theorem 3.5: Every totally continuous function is totally α^* continuous.

Proof: Let O be an open set of (Y, σ) . Since, f is totally continuous functions, $f^{-1}(O)$ is both open and closed in (X, τ) . Since every open set is α^* open and every closed set is α^* closed. $f^{-1}(O)$ is both α^* open and α^* closed in (X, τ) . Therefore, f is totally α^* continuous.

Remark 3.6: The converse of above theorem need not be true.

Example 3.7: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{ab\}, X\}$, $\tau^c = \{\emptyset, \{cd\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{bc\}, \{abc\}, Y\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$, $g(d) = d$. $\alpha^*O(X, \tau) = P(X) = \alpha^*C(X, \tau)$. Clearly, g is totally α^* continuous but $g^{-1}(\{a, b\}) = \{a, b\}$ is open in X but not closed in X . Therefore, g is not totally continuous.

Theorem 3.8: Every perfectly α^* continuous map is totally α^* continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a perfectly α^* continuous map. Let O be an open set of (Y, σ) . Then O is α^* open in (Y, σ) . Since f is perfectly α^* continuous, $f^{-1}(O)$ is both open and closed in (X, τ) , implies $f^{-1}(O)$ is both α^* open and α^* closed in (X, τ) . Therefore, f is totally α^* continuous.

Remark 3.9: The converse of above theorem need not be true.

Example 3.10: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{ab\}, X\}$, $\tau^c = \{\emptyset, \{cd\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{bc\}, \{abc\}, Y\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$, $g(d) = d$. $\alpha^*O(X, \tau) = P(X) = \alpha^*C(X, \tau)$. $\alpha^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{bc\}, \{abc\}, \{abd\}, Y\}$. Clearly, g is totally α^* continuous but $g^{-1}(\{a, b\}) = \{a, b\}$ is open in X but not closed in X . Therefore, g is not perfectly α^* continuous.

Remark 3.11: The concept of totally α^* continuous and strongly α^* continuous are independent of each other.

Example 3.12: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{ab\}, X\}$, $\tau^c = \{\emptyset, \{cd\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{bc\}, \{abc\}, Y\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a$, $g(b) = b$, $g(c) = c$, $g(d) = d$. $\alpha^*O(X, \tau) = P(X) = \alpha^*C(X, \tau)$. and $\alpha^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{bc\}, \{abc\}, \{abd\}, Y\}$. Clearly, g is totally α^* continuous but $g^{-1}(\{b\}) = \{b\}$ is not open in X . Therefore, g is not strongly α^* continuous.

Example 3.13: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{ab\}, \{ac\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{b\}, \{ab\}, Y\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a = g(b)$, $g(c) = c$. $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{ab\}, \{ac\}, X\}$, $\alpha^*C(X, \tau) = \{\emptyset, \{bc\}, \{b\}, \{c\}, X\}$ and $\alpha^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{ac\}, Y\}$. Clearly, g is strongly α^* continuous but $g^{-1}(\{a\}) = \{ab\}$ is α^* open in X but not α^* closed. Therefore, g is not totally α^* continuous.

Theorem 3.14: If $f: X \rightarrow Y$ is a totally α^* continuous map, and X is α^* connected, then Y is an indiscrete space.

Proof: Suppose that Y is not an indiscrete space. Let A be a non-empty open subset of Y . Since, f is totally α^* continuous map, then $f^{-1}(A)$ is a non-empty α^* clopen subset of X . Then $X = f^{-1}(A) \cup (f^{-1}(A))^c$. Thus, X is a union of two non-empty disjoint α^* open sets which is contradiction to the fact that X is α^* connected. Therefore, Y must be an indiscrete space

Theorem 3.15: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then $g \circ f: X \rightarrow Z$

- (i) If f is α^* irresolute and g is totally α^* continuous then $g \circ f$ is totally α^* continuous
- (ii) If f is totally α^* continuous and g is continuous then $g \circ f$ is totally α^* continuous.

Proof:

- (i) Let O be an open set in Z . Since g is totally α^* continuous, $g^{-1}(O)$ is α^* clopen in Y . Since f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open and α^* closed in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$. Therefore, $g \circ f$ is totally α^* continuous.
- (ii) Let O be an open set in Z . Since g is continuous, $g^{-1}(O)$ is open in Y . Since, f is totally α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* clopen in X . Hence, $g \circ f$ is totally α^* continuous.

IV. Slightly α^* continuous functions.

Definition 4.1: A function $(X, \tau) \rightarrow (Y, \sigma)$ is called **slightly α^* continuous** at a point $x \in X$ if for each clopen subset V of Y containing $f(x)$, there exists a α^* open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly α^* continuous if f is slightly α^* continuous at each of its points.

Definition 4.2: A function $(X, \tau) \rightarrow (Y, \sigma)$ is said to be **slightly α^* continuous** if the inverse image of every clopen set in Y is α^* open in X .

Example 4.3: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{ab\}, \{abc\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{bcd\}, Y\}$ and $\alpha^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{ad\}, \{bc\}, \{bd\}, \{abc\}, \{abd\}, \{acd\}, \{bcd\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = b$, $g(b) = a$, $g(c) = d$, $g(d) = c$. Clearly, g is slightly α^* continuous.

Proposition 4.4: The definition 4.1 and 4.2 are equivalent.

Proof: Suppose the definition 4.1 holds. Let O be a clopen set in Y and $x \in f^{-1}(O)$. Then $f(x) \in O$ and thus there exists a α^* open set U_x such that $x \in U_x \subseteq f^{-1}(O)$ and $f^{-1}(O) = \bigcup_{x \in f^{-1}(O)} U_x$. Since, arbitrary union of α^* open set is α^* open. $f^{-1}(O)$ is α^* open in X and therefore, f is slightly α^* continuous. Suppose, the definition 4.2 holds. Let $f(x) \in O$ where, O is a clopen set in Y . Since, f is slightly α^* continuous, $x \in f^{-1}(O)$ where $f^{-1}(O)$ is α^* open in X . Let $U = f^{-1}(O)$. Then U is α^* open in X , $x \in X$ and $f(U) \subseteq O$.

Theorem 4.5: For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

- (i) f is slightly α^* continuous.
- (ii) The inverse image of every clopen set O of Y is α^* open in X .
- (iii) The inverse image of every clopen set O of Y is α^* closed in X .
- (iv) The inverse image of every clopen set O of Y is α^* clopen in X .

Proof:

- (i) $\Rightarrow$ (ii): Follows from the proposition 4.4
- (ii) $\Rightarrow$ (iii): Let O be a clopen set in Y which implies O^c is clopen in Y . By (ii), $f^{-1}(O^c) = (f^{-1}(O))^c$ is α^* open in X . Therefore, $f^{-1}(O)$ is α^* closed in X .
- (iii) $\Rightarrow$ (iv): By (ii) and (iii), $f^{-1}(O)$ is α^* clopen in X .
- (iv) $\Rightarrow$ (i): Let O be a clopen set in Y containing $f(x)$, by (iv) $f^{-1}(O)$ is α^* clopen in X . Take $U = f^{-1}(O)$, then $f(U) \subseteq O$. Hence, f is slightly α^* continuous.

Theorem 4.6: Every slightly continuous function is slightly α^* continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a slightly continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is open in X . Since, every open set is α^* open. Hence, f is slightly α^* continuous.

Remark 4.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.8: Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{ab\}, \{abc\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{bcd\}, Y\}$ and $\alpha^* O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{ad\}, \{bc\}, \{bd\}, \{abc\}, \{abd\}, \{acd\}, \{bcd\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a)$

$= b$, $g(b) = a$, $g(c) = d$, $g(d) = c$. Clearly, g is slightly α^* continuous but not slightly continuous. Since, $g^{-1}(a) = b$ where a is clopen in Y but b is not open in X .

Theorem 4.9: Every α^* continuous function is slightly α^* continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a α^* continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is α^* open in X and α^* closed in X . Hence, f is slightly α^* continuous.

Remark 4.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.11: Let $X = \{a, b, c\}$ and $Y = \{a, b\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{ab\}, X\}$, $\sigma = \{\emptyset, \{a\}, Y\}$ and $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{ac\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = b$, $g(b) = g(c) = a$. The function f is slightly α^* continuous but not α^* continuous, since, $g^{-1}(a) = \{bc\}$ is not α^* open in X .

Theorem 4.12: Every contra α^* continuous function is slightly α^* continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a contra α^* continuous function. Let O be a clopen set in Y . Then, $f^{-1}(O)$ is α^* open in X . Hence, f is slightly α^* continuous.

Remark 4.13: The converse of the above theorem need not be true as can be seen from the following example.

Example 4.14: Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{ab\}, X\}$, $\sigma = \{\emptyset, \{a\}, \{c\}, \{ab\}, \{ac\}, Y\}$ and $\sigma^c = \{\emptyset, \{b\}, \{c\}, \{ab\}, \{bc\}, Y\}$ and $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{ab\}, \{ac\}, X\}$. Let $g: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $g(a) = a$, $g(b) = c$, $g(c) = b$. The function f is slightly α^* continuous but not contra α^* continuous, since, $g^{-1}(b) = \{c\}$ is not α^* open in X .

Remark 4.15: Composition of two slightly α^* continuous need not be slightly α^* continuous as it can be seen from the following example.

Example 4.16: Let $X = Y = \{a, b, c, d\}$, $Z = \{a, b, c\}$ and the topologies are $\tau = \{\emptyset, \{a\}, \{ab\}, \{abc\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{bcd\}, Y\}$ and $\eta = \{\emptyset, \{b\}, \{ac\}, Z\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$, $\alpha^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{ab\}, \{ac\}, \{ad\}, \{bc\}, \{abc\}, \{abd\}, \{acd\}, X\}$. Clearly, f is slightly α^* continuous. Define $g: (Y, \sigma) \rightarrow (Z, \eta)$ by $g(a) = a$, $g(b) = b = g(c)$, $g(d) = c$, $\alpha^*O(Y, \sigma) = P(Y)$. Clearly, g is slightly α^* continuous. But $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is not slightly α^* continuous, since $(g \circ f)^{-1}(\{ac\}) = f^{-1}(g^{-1}(\{ac\})) = f^{-1}(\{ad\}) = \{bd\}$ is not a α^* open in (X, τ) .

Theorem 4.17: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be functions. Then the following properties hold:

- (i) If f is α^* irresolute and g is slightly α^* continuous then $(g \circ f)$ is slightly α^* continuous.
- (ii) If f is α^* irresolute and g is α^* continuous then $(g \circ f)$ is slightly α^* continuous.
- (iii) If f is α^* irresolute and g is slightly continuous then $(g \circ f)$ is slightly α^* continuous.
- (iv) If f is α^* continuous and g is slightly continuous then $(g \circ f)$ is slightly α^* continuous.
- (v) If f is strongly α^* continuous and g is slightly α^* continuous then $(g \circ f)$ is slightly continuous.
- (vi) If f is slightly α^* continuous and g is perfectly α^* continuous then $(g \circ f)$ is α^* irresolute.
- (vii) If f is slightly α^* continuous and g is contra continuous then $(g \circ f)$ is slightly α^* continuous.
- (viii) If f is α^* irresolute and g is contra α^* continuous then $(g \circ f)$ is slightly α^* continuous.

Proof:

- (i) Let O be a clopen set in Z . Since, g is slightly α^* continuous, $g^{-1}(O)$ is α^* open in Y . Since, f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open in X . Since, $(g \circ f)^{-1}(O) = f^{-1}(g^{-1}(O))$, $g \circ f$ is slightly α^* continuous.
- (ii) Let O be a clopen set in Z . Since, g is α^* continuous, $g^{-1}(O)$ is α^* open in Y . Since, f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $g \circ f$ is slightly α^* continuous.
- (iii) Let O be a clopen set in Z . Since, g is slightly continuous, $g^{-1}(O)$ is open in Y . Since, f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $g \circ f$ is slightly α^* continuous.
- (iv) Let O be a clopen set in Z . Since, g is slightly continuous, $g^{-1}(O)$ is open in Y . Since, f is α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $g \circ f$ is slightly α^* continuous.
- (v) Let O be a clopen set in Z . Since, g is slightly α^* continuous, $g^{-1}(O)$ is α^* open in Y . Since, f is strongly α^* continuous, $f^{-1}(g^{-1}(O))$ is open in X . Therefore, $g \circ f$ is slightly continuous.
- (vi) Let O be a α^* open in Z . Since, g is perfectly α^* continuous, $g^{-1}(O)$ is open and closed in Y . Since, f is slightly α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $g \circ f$ is α^* irresolute.
- (vii) Let O be a clopen set in Z . Since, g is contra continuous, $g^{-1}(O)$ is open and closed in Y . Since, f is slightly α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $g \circ f$ is slightly α^* continuous.
- (viii) Let O be a clopen set in Z . Since, g is contra α^* continuous, $g^{-1}(O)$ is α^* open and α^* closed in Y . Since, f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open and α^* closed in X . Hence, $g \circ f$ is slightly α^* continuous.

Theorem 4.18: If the function $f: (X, \tau) \rightarrow (Y, \sigma)$ is slightly α^* continuous and (X, τ) is $\alpha^* T_{1/2}$ space, then f is slightly continuous.

Proof: Let O be a clopen set in Y . Since, g is slightly α^* continuous, $f^{-1}(O)$ is α^* open in X . Since, X is $\alpha^* T_{1/2}$ space, $f^{-1}(O)$ is open in X . Hence, f is slightly continuous.

Theorem 4.19: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be functions. If f is surjective and pre α^* open and $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly α^* continuous, then g is slightly α^* continuous.

Proof: Let O be a clopen set in (Z, η) . Since, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* open in X . Since, f is surjective and pre α^* open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is α^* open in Y . Hence, g is slightly α^* continuous.

Theorem 4.20: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be functions. If f is surjective, pre α^* open and α^* irresolute, then $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly α^* continuous if and only if g is slightly α^* continuous.

Proof: Let O be a clopen set in (Z, η) . Since, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly α^* continuous, $f^{-1}(g^{-1}(O))$ is α^* open in X . Since, f is surjective and pre α^* open $f(f^{-1}(g^{-1}(O))) = g^{-1}(O)$ is α^* open in Y . Hence, g is slightly α^* continuous.

Conversely, let g is slightly α^* continuous. Let O be a clopen set in (Z, η) , then $g^{-1}(O)$ is α^* open in Y . Since, f is α^* irresolute, $f^{-1}(g^{-1}(O))$ is α^* open in X . Hence, $(g \circ f): (X, \tau) \rightarrow (Z, \eta)$ is slightly α^* continuous.

Theorem 4.21: If f is a slightly α^* continuous from a α^* connected space (X, τ) onto a space (Y, σ) then Y is not a discrete space.

Proof: Suppose that Y is a discrete space. Let O be a proper non-empty open subset of Y . Since, f is slightly α^* continuous, $f^{-1}(O)$ is a proper non-empty α^* clopen subset of X which is contradiction to the fact that X is α^* connected.

Theorem 4.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a slightly α^* continuous surjection and X is α^* connected, then Y is connected.

Proof: Suppose Y is not connected, then there exists non-empty disjoint open sets U and V such that $Y = U \cup V$. Therefore, U and V are clopen sets in Y . Since, f is slightly α^* continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty disjoint α^* open in X and $X = f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not α^* connected. This is a contradiction and hence, Y is connected.

Theorem 4.23: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a slightly α^* continuous and (Y, σ) is a locally indiscrete space then f is α^* continuous.

Proof: Let O be an open subset of Y . Since, (Y, σ) is a locally indiscrete space, O is closed in Y . Since, f is slightly α^* continuous, $f^{-1}(O)$ is α^* open in X . Hence, f is α^* continuous.

Theorem 4.24: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a slightly α^* continuous and A is an open subset of X then the restriction $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly α^* continuous.

Proof: Let V be a clopen subset of Y . Then $(f|_A)^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is α^* open and A is open, $(f|_A)^{-1}(V)$ is α^* open in the relative topology of A . Hence, $f|_A: (A, \tau_A) \rightarrow (Y, \sigma)$ is slightly α^* continuous.

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