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Finding determinant of high order matrix

Feng Cheng Chang
Allwave Corporation, Torrance, California, USA

ABSTRACT: *A simple, efficient, and direct process is derived to compute the determinant of any square matrix of high order. The approach involves successive application of matrix order condensation algorithm. Computer listing in MATLAB is provided, and a typical example is given to show the merit of the approach presented.*

***KEYWORDS*:** *determinant; matrix inversion; matrix multiplication; matrix order condensation.*

I. INTRODUCTION AND FORMULATION

The determinant of a given matrix [M] of order $N \times N$ can be evaluated as follows by an iterative algorithm relying upon matrix order condensation.

In the k-th step of the iterative process, k = 1, 2, ... , N, the matrix [M_{k-1}] of order $(N-k+1)\times(N-k+1)$, after drawing a horizontal line and a vertical line over any selected non-zero entry located at (r, c), may be expressed in graphic form as

$$[M_{k-1}] \quad = \quad \begin{array}{c} \begin{bmatrix} \cdot & | & \cdot & \cdot & \cdot & \cdot \\ \cdot & | & \cdot & \cdot & \cdot & \cdot \\ - & + & - & - & - & - \\ \cdot & | & \cdot & \cdot & \cdot & \cdot \\ \cdot & | & \cdot & \cdot & \cdot & \cdot \\ \cdot & | & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \leftarrow r \\ \uparrow \\ c \end{array}$$

Here, for illustration, we set N - k = 5, and at (r, c) = (3, 2). The four block items from this matrix [M_{k-1}] are thus generated:

p_k : a pivot element, formed by a entry with cross bar (" + "),
v_k : a vertical array, formed by $(N-k)$ entries with vertical bar (" | "),
u_k : a horizontal array, formed by $(N-k)$ entries with horizontal bar (" − "),
W_k : a square matrix, formed by $(N-k)\times(N-k)$ entries without scratch (" . ").

The condensed matrix [M_k] of order $(N-k)\times(N-k)$ is then easily computed from these four block items:

$$\begin{aligned} [M_k] \quad &= \quad [\, W_k - v_k\, u_k / p_k \,] \\ &= \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} - \begin{bmatrix} | \\ | \\ | \\ | \\ | \end{bmatrix} [- \;\; - \;\; - \;\; - \;\; -] / [+] \end{aligned}$$

Also the sign of pivot element must be adjusted accordingly,

$$z_k \quad = \quad (-1)^{r+c}\, p_k$$

After performing a total of N process steps so that $[M_{N-1}]$ is of order 1 x 1, the determinant of the given matrix $[M] = [M_0]$ is then computed:

$$\det[M] \;=\; z_1 \cdot z_2 \cdots\cdots z_N$$

II. COMPUTER ROUTINE

A computer program in MATLAB routine is presented for easy reference.

```
function detM = det_p(M)
%    Finding determinant of a square matrix
%    by order Condensation
%    Manually select pivot at each iteration.
%    F C Chang     05/15/2015

       Z = [ ]; M,
 for k = 1:size(M,1), k,
       n = size(M,1);
   % disp('Pick an non-zero entry at [r,c]');
       rc = input('[r,c] =  ');
       r = rc(1);  c = rc(2);
       p = M(r,c);
       u = M(r,setdiff(1:n,c));
       v = M(setdiff(1:n,r),c);
       W = M(setdiff(1:n,r),setdiff(1:n,c));
       xM = [p,u; v,W];
       M = [W-v*u/p];
       z = p*(-1)^(r+c);
       Z = [Z,z];
        p,z,xM,M,
 end;
       detM = prod(Z); Z,
```

III. TYPICAL EXAMPLE

Find the determinant of a given 9x9 matrix $[M_0]$ by the order condensation iteration.

$$[M_0] = \begin{bmatrix} 2 & -1 & 5 & 1 & 8 & 0 & 3 & -4 & 5 \\ 0 & -2 & -3 & 1 & 4 & 3 & 0 & 8 & 0 \\ 1 & -3 & 1 & -2 & 5 & 0 & 7 & 1 & -1 \\ 2 & 1 & 0 & 7 & -6 & -1 & 0 & 1 & -1 \\ 4 & 6 & -4 & 1 & 2 & 9 & 0 & 1 & -2 \\ 3 & 3 & 5 & -2 & 4 & -7 & 7 & 0 & 4 \\ 2 & 0 & 1 & 6 & -3 & 2 & 8 & -5 & 3 \\ -1 & -3 & 0 & -5 & 7 & -9 & 1 & 6 & 1 \\ 6 & 9 & -7 & 4 & 0 & 6 & -2 & 1 & 2 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \\ \\ \\ \\ \\ \\ \\ \end{matrix}$$
$$\begin{matrix} & & & & & & & \uparrow & \end{matrix}$$

$k=1,\qquad [r;c]=[3;8],\qquad p_1=+1,\qquad z_1=-1.$

$$[M_1]=\begin{bmatrix} 2 & -1 & 5 & 1 & 8 & 0 & 3 & 5 \\ 0 & -2 & -3 & 1 & 4 & 3 & 0 & 0 \\ 2 & 1 & 0 & 7 & -6 & -1 & 0 & -1 \\ 4 & 6 & -4 & 1 & 2 & 9 & 0 & -2 \\ 3 & 3 & 5 & -2 & 4 & -7 & 7 & 4 \\ 2 & 0 & 1 & 6 & -3 & 2 & 8 & 3 \\ -1 & -3 & 0 & -5 & 7 & -9 & 1 & 1 \\ 6 & 9 & -7 & 4 & 0 & 6 & -2 & 2 \end{bmatrix}-\begin{bmatrix} -4 \\ 8 \\ 1 \\ 1 \\ 0 \\ -5 \\ 6 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -3 & 1 & -2 & 5 & 0 & 7 & -1 \end{bmatrix}/\begin{bmatrix} 1 \end{bmatrix}$$

$$=\begin{bmatrix} 6 & -13 & 9 & -7 & 28 & 0 & 31 & 1 \\ -8 & 22 & -11 & 17 & -36 & 3 & -56 & 8 \\ 1 & 4 & -1 & 9 & -11 & -1 & -7 & 0 \\ 3 & 9 & -5 & 3 & -3 & 9 & -7 & -1 \\ 3 & 3 & 5 & -2 & 4 & -7 & 7 & 4 \\ 7 & -15 & 6 & -4 & 22 & 2 & 43 & -2 \\ -7 & 15 & -6 & 7 & -23 & -9 & -41 & 7 \\ 5 & 12 & -8 & 6 & -5 & 6 & -9 & 3 \end{bmatrix}\leftarrow$$

$$\uparrow$$

$k=2,\qquad [r;c]=[3;3],\qquad p_2=-1,\qquad z_2=-1.$

$$[M_2]=\begin{bmatrix} 6 & -13 & -7 & 28 & 0 & 31 & 1 \\ -8 & 22 & 17 & -36 & 3 & -56 & 8 \\ 3 & 9 & 3 & -3 & 9 & -7 & -1 \\ 3 & 3 & -2 & 4 & -7 & 7 & 4 \\ 7 & -15 & -4 & 22 & 2 & 43 & -2 \\ -7 & 15 & 7 & -23 & -9 & -41 & 7 \\ 5 & 12 & 6 & -5 & 6 & -9 & 3 \end{bmatrix}-\begin{bmatrix} 9 \\ -11 \\ -5 \\ 5 \\ 6 \\ -6 \\ -8 \end{bmatrix}\begin{bmatrix} 1 & 4 & 9 & -11 & -1 & 7 & 0 \end{bmatrix}/\begin{bmatrix} -1 \end{bmatrix}$$

$$=\begin{bmatrix} 15 & 23 & 74 & -71 & -9 & -32 & 1 \\ -19 & -22 & -82 & 85 & 14 & 21 & 8 \\ -2 & -11 & -42 & 52 & 14 & 28 & -1 \\ 8 & 23 & 43 & -51 & -12 & -28 & 4 \\ 13 & 9 & 50 & -44 & -4 & 1 & -2 \\ -13 & -9 & -47 & 43 & -3 & 1 & 7 \\ -3 & -20 & -66 & 83 & 14 & 47 & 3 \end{bmatrix}\begin{matrix} \\ \\ \\ \\ \leftarrow \\ \\ \\ \end{matrix}$$

$$\uparrow$$

$k=3,\qquad [r;c]=[5;6],\qquad p_3=+1,\qquad z_3=-1.$

$$[M_3]=\begin{bmatrix} 15 & 23 & 74 & -71 & -9 & 1 \\ -19 & -22 & -82 & 85 & 14 & 8 \\ -2 & -11 & -42 & 52 & 14 & -1 \\ 8 & 23 & 43 & -51 & -12 & 4 \\ -13 & -9 & -47 & 43 & -3 & 7 \\ -3 & -20 & -66 & 83 & 14 & 3 \end{bmatrix}-\begin{bmatrix} -32 \\ 21 \\ 28 \\ -28 \\ 1 \\ 47 \end{bmatrix}\begin{bmatrix} 13 & 9 & 50 & -44 & -4 & -2 \end{bmatrix}/\begin{bmatrix} 1 \end{bmatrix}$$

$$=\begin{bmatrix} 431 & 311 & 1674 & -1479 & -137 & -63 \\ -292 & -211 & -1132 & 1009 & 98 & 50 \\ -366 & -263 & -1442 & 1284 & 126 & 55 \\ 372 & 275 & 1443 & -1283 & -124 & -52 \\ -26 & -18 & -97 & 87 & 1 & 9 \\ -614 & -443 & -2416 & 2151 & 202 & 97 \end{bmatrix}\begin{matrix} \\ \\ \\ \\ \leftarrow \\ \\ \end{matrix}$$

$$\uparrow$$

$k=4, \quad [r;c]=[5;5], \quad p_4=+1, \quad z_4=+1.$

$$[M_4]=\begin{bmatrix} 431 & 311 & 1674 & -1479 & -63 \\ -292 & -211 & -1132 & 1009 & 50 \\ -366 & -263 & -1442 & 1284 & 55 \\ 372 & 275 & 1443 & -1283 & -52 \\ -614 & -443 & -2416 & 2151 & 97 \end{bmatrix} - \begin{bmatrix} -137 \\ 98 \\ 126 \\ -124 \\ 202 \end{bmatrix} \begin{bmatrix} -26 & -18 & -97 & 87 & 9 \end{bmatrix} / [1]$$

$$= \begin{bmatrix} -3131 & -2155 & -11615 & 10440 & 1170 \\ 2256 & 1553 & 8374 & -7517 & -832 \\ 2910 & 2005 & 10780 & -9678 & -1079 \\ -2852 & -1957 & -10585 & 9505 & 1064 \\ 4638 & 3193 & 17178 & -15423 & -1721 \end{bmatrix} \leftarrow$$

$$\uparrow$$

$k=5, \quad [r;c]=[2;5], \quad p_5=-832, \quad z_5=+832.$

$$[M_5]=\begin{bmatrix} -3131 & -2155 & -11615 & 10440 \\ 2910 & 2005 & 10780 & -9678 \\ -2852 & -1957 & -10585 & 9505 \\ 4638 & 3193 & 17178 & -15423 \end{bmatrix} - \begin{bmatrix} 1170 \\ -1079 \\ 1064 \\ -1721 \end{bmatrix} \begin{bmatrix} 2256 & 1553 & 8374 & -7517 \end{bmatrix} / [-832]$$

$$= \begin{bmatrix} 41.5000 & 28.9063 & 160.9375 & -130.7813 \\ -15.7500 & -9.0469 & -80.0313 & 70.6094 \\ 33.0769 & 29.0481 & 124.0577 & -108.0865 \\ -28.5577 & -19.395 & -143.6995 & 125.9868 \end{bmatrix} \leftarrow$$

$$\uparrow$$

$k=6, \quad [r;c]=[2;1], \quad p_6=-15.7500, \quad z_6=+15.7500$

$$[M_6]=\begin{bmatrix} 28.9063 & 160.9375 & -130.7813 \\ 29.0481 & 124.0577 & -108.0865 \\ -19.3954 & -143.6995 & 125.9868 \end{bmatrix} - \begin{bmatrix} 41.5000 \\ 33.0769 \\ -28.5577 \end{bmatrix} \begin{bmatrix} -9.0469 & -80.0313 & 70.6094 \end{bmatrix} / [-15.750]$$

$$= \begin{bmatrix} 5.0685 & -49.9385 & 55.2688 \\ 10.0485 & -44.0177 & 40.2018 \\ -2.9918 & 1.4121 & -2.0412 \end{bmatrix} \leftarrow$$

$$\uparrow$$

$k=7, \quad [r;c]=[1;2], \quad p_7=-49.9385, \quad z_7=+49.9385.$

$$[M_7]=\begin{bmatrix} 10.0485 & 40.2018 \\ -2.9918 & -2.0412 \end{bmatrix} - \begin{bmatrix} -44.0177 \\ 1.4121 \end{bmatrix} \begin{bmatrix} 5.0685 & 55.2688 \end{bmatrix} / [-49.9385]$$

$$= \begin{bmatrix} 5.5810 & -8.5143 \\ -2.8484 & -0.4784 \end{bmatrix} \leftarrow$$

$$\uparrow$$

$k=8, \quad [r;c]=[1;1], \quad p_8=+5.5810, \quad z_8=+5.5810$

$$[M_8]=[-0.4784]-[-2.8484][-8.5143]/[5.5810]$$

$$=[-4.8239] \leftarrow$$

$$\uparrow$$

$k=9, \quad [r;c]=[1;1], \quad p_9=-4.8239, \quad z_9=-4.8239.$

$$[M_9]=[\quad]$$

The determinant of the given matrix is thus computed:

$$\det[M_0]=(-1.)(-1.)(-1.)(+1.)(+832.)(+15.75)(+49.938)(+5.5810)(-4.8239)$$

$$=+17617891.$$

IV. CONCLUSION

When compared to various approaches available in the literature [1]-[7], the process presented is very compact, efficient, and involves only the simple elementary arithmetical operations of addition, subtraction, multiplication, and division. It is shown that the total number of multiplication/division operations required to compute the determinant of a given N x N matrix is found to be $2/3N^3$, which is fewer than N^3 multiplications needed for the product of two N x N matrices.

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