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MISHRA DISTRIBUTION

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Abstract: *We name this distribution as "Mishra distribution" in honor of Prof. A.Mishra, Department of Statistics, Patna University, Patna. It is a new member of a family of exponential probability distributions. This continuous probability distribution has a single parameter. Its probability density function has been obtained. Its different characteristics such as moments about origin, Co-efficient of Variation, moment generating function and probability distribution have been obtained. .Estimation of Parameter of this distribution has been discussed by the method of moments as well as maximum likelihood method. The distribution has been fitted to some data-sets to test its goodness of fit and it has been found that this distribution gives better fit than the Exponential distribution and in some cases it gives better fit than the Lindley distribution. .*

Key-words: *Exponential distribution, Lindley distribution, moments, estimation of parameters, goodness of fit.*

I. INTRODUCTION

The exponential distribution occupies a central place among the continuous probability distributions and plays an important role in statistical theory with many interesting properties and having a wide range of applications in various fields. Its probability density function (pdf) is given by

$$f(x;\phi) = \phi e^{-\phi x};\ x > 0,\ \phi > 0 \qquad \ldots \qquad \ldots \qquad (1.1)$$

One of the interesting properties of this distribution is its 'memory loss' which is expressed as

$$P(X \geq s+t) = P(X \geq s)\,.\,P(X \geq t) \qquad \ldots \qquad \ldots \qquad (1.2)$$

Its rth moment about origin is given by

$$\mu_r^{/} = \frac{\Gamma(r+1)}{\phi^r},\ r = 1,2,3,\ldots \qquad \ldots \qquad \ldots \qquad (1.3)$$

Lindley (1958) introduced a one-parameter distribution, known as Lindley distribution, given by its probability density function

$$f(x;\phi) = \frac{\phi^2}{\phi+1}(1+x)e^{-\phi x}\ ;\ x > 0,\ \phi > 0 \qquad \ldots \qquad (1.4)$$

The first four moments about origin of the Lindley distribution have been obtained as

$$\mu_1^{/} = \frac{2(\phi+2)}{\phi(\phi+1)},\ \mu_2^{/} = \frac{2(\phi+3)}{\phi^2(\phi+1)},\ \mu_3^{/} = \frac{6(\phi+4)}{\phi^3(\phi+1)},\ \mu_4^{/} = \frac{24(\phi+5)}{\phi^3(\phi+1)} \qquad (1.5)$$

Ghitany et al (2008) have discussed various properties of this distribution and showed that in many ways (1.4) provides a better model for some applications than the exponential distribution. Mazucheli and Achcar (2011), Ghitany et al (2009, 2011) and Bakouchi et al (2012) are some among others who discussed its various applications. Zakerzadah and Dolati (2009), Shanker and Mishra (2013a,2013b) obtained generalized Lindley distributions and discussed their various properties and applications. Sankaran (1970) obtained a Lindley mixture of Poisson distribution.Sah B.K. (2015a) obtained a two-parameter Quasi-Lindley distribution and discussed their various properties.

II. MISHRA DISTRIBUTION

A one-parameter Mishra distribution (MD) with parameters ϕ is defined by its probability density function (pdf)

$$f(x;\phi) = \frac{\phi^3(1+x+x^2)e^{-\phi x}}{(\phi^2+\phi+2)};\phi > 0, x > 0 \qquad \ldots \qquad (2.1)$$

III. MOMENTS

The r^th^moment about origin of the Mishra distribution has been obtained as

$$\mu_r^{/} = \frac{\phi^3}{(\phi^2+\phi+2)}\int_0^\infty x^r(1+x+x^2)e^{-\phi x}dx \qquad \dots \qquad \dots \qquad \dots \qquad (3.1)$$

$$= \frac{r!}{\phi^r}\frac{\{\phi^2+(r+1)\phi+(1+r)(2+r)\}}{(\phi^2+\phi+2)} \qquad .. \qquad \dots \qquad \dots \qquad (3.2)$$

Putting the value r=1, 2, 3 and 4 in the expression (3.2), the first four moments about origin of the MD are obtained as

$$\mu_1^{/} = \frac{1!}{\phi}\frac{(\phi^2+2\phi+6)}{(\phi^2+\phi+2)} \qquad \mu_2^{/} = \frac{2!}{\phi^2}\frac{(\phi^2+3\phi+12)}{(\phi^2+\phi+2)}$$

$$\mu_3^{/} = \frac{3!}{\phi^3}\frac{(\phi^2+4\phi+20)}{(\phi^2+\phi+2)} \qquad \mu_4^{/} = \frac{4!}{\phi^4}\frac{(\phi^2+5\phi+30)}{(\phi^2+\phi+2)} \qquad \dots \qquad (3.3)$$

The variance is given by

$$\mu_2 = \mu_2^{/} - \mu_1^{/2}$$

$$= \frac{2(\phi^2+3\phi+12)(\phi^2+\phi+2)-(\phi^2+2\phi+6)^2}{(\phi^2+\phi+6)}\times 100 \qquad \dots \qquad \dots \qquad (3.4)$$

<u>Co – efficient of Variation (C.V.)</u>

$$\text{C.V.} = \frac{\sqrt{2(\phi^2+3\phi+12)(\phi^2+\phi+2)-(\phi^2+2\phi+6)^2}}{(\phi^2+\phi+6)}\times 100 \qquad \dots \qquad \dots \qquad (3.5)$$

Moment Generating Function $[M_x(t)]$

Moment generating function of Mishra distribution can be obtained as

$$[M_x(t)] = \int_0^\infty e^{tx} f(x)dx$$

$$= \frac{\phi^3}{(\phi^2+\phi+2)}\left\{\frac{(\phi-t)^2+(\phi-t)+2}{(\phi-t)^3}\right\} \qquad \dots \qquad \dots \qquad (3.6)$$

Distribution Function

Distribution function of the Mishra distribution is obtained as

$$F_x = \int_0^x \frac{\phi^3}{(\phi^2+\phi+2)}(1+x+x^2)e^{-\phi x}dx$$

$$= \ 1-e^{-\phi x} - \frac{\phi x(\phi+\phi x+2)e^{-\phi x}}{(\phi^2+\phi+2)} \qquad \dots \qquad \dots \qquad (3.7)$$

IV. ESTIMATION OF PARAMETER

Here, we have discussed two methods (a) the method of moments, and (b) the method of maximum likelihood to estimate parameter of the Mishra distribution.

(a) ***The method of moments***

Parameter of the Mishra distribution can be obtained by using the first moment about origin.

$$\mu_1^{/} = \frac{1!}{\phi}\frac{(\phi^2 + 2\phi + 6)}{(\phi^2 + \phi + 2)}$$

$$Or\ ,\ \mu_1^{/}\phi^3 + \mu_1^{/}\phi^2 + 2\mu_1^{/}\phi = \phi^2 + 2\phi + 6$$

$$Or\ ,\ \mu_1^{/}\phi^3 + (\mu_1^{/} - 1)\phi^2 + 2(\mu_1^{/} - 1)\phi - 6 = 0 \quad \ldots \quad \ldots \quad (4.1)$$

The expression (4.1) is the Polynomial in third degree equation. Replacing the corresponding population moment by sample moment and solving the expression (4.1) by using Regula-Falsi method, we get an estimate of ϕ.

(b) ***The method of maximum likelihood***

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n from a single parameter Mishra distribution and let f_x be the observed frequency in the sample corresponding to $X = x\ (x = 1, 2, \ldots, k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function, L of the Mishra distribution (2.1) is given by

$$L = \left(\frac{\phi^3}{\phi^2 + \phi + 2}\right)^n \prod_{x=1}^{k} (1 + x + x^2)^{f_x} e^{-n\phi\bar{x}} \quad \ldots \quad (4.2)$$

and so the log likelihood function is obtained as

$$\log L = 3n \log \phi - n \log(\phi^2 + \phi + 2) + \sum_{x=1}^{k} f_x (1 + x + x^2) - n\phi\bar{x} \quad (4.3)$$

The log likelihood equation is thus obtained as

$$\frac{\partial \log L}{\partial \phi} = \frac{3n}{\phi} - \frac{n(2\phi + 1)}{(\phi^2 + \phi + 2)} - n\bar{x} = 0 \quad \ldots \quad \ldots \quad \ldots \quad (4.4)$$

Or, $$\bar{x} = \frac{1!}{\phi}\frac{(\phi^2 + 2\phi + 6)}{(\phi^2 + \phi + 2)}$$

Or, $$\bar{x}\phi^3 + (\bar{x} - 1)\phi^2 + 2(\bar{x} - 1)\phi - 6 = 0 \quad \ldots \quad \ldots \quad (4.5)$$

Solving the expression (4.5) by using Regula-Falsi method, we get an estimate of ϕ .

V. GOODNESS OF FIT

The Mishra distribution has been fitted to a number of data- sets to which earlier the exponential distribution and the Lindley distribution have been fitted by others and to almost all these data-sets the Mishra distribution provides closer fits than the Exponential distribution. This distribution gives better fit to the second data set than Lindley distribution.

The fittings of the Mishra distribution to two such data-sets have been presented in the following tables. The data sets given in tables-I and II are the data sets reported by Ghitany et al (2008) and Bzerkedal (1960) respectively. The expected frequencies according to the exponential distribution and the Lindley distribution have also been given for ready comparison with those obtained by the Mishra distribution. The estimate of the parameter has been obtained by the method of maximum likelihood.

Table-I
Waiting times (in minutes) of 100 bank customers

Waiting Time (In minutes)	Observed frequency	Expected frequency		
		Exponential	Lindley	Mishra
0-5	30	40.3	30.4	22.9
5-10	32	24.1	30.7	36.6
10-15	19	14.4	19.2	23.4
15-20	10	8.6	10.3	10.8
20-25	5	5.1	5.1	4.2
25-30	1	3.1	2.4	1.5
30-35	2	1.8	1.1	0.5
35-40	1	2.6	0.8	0.1
Total	100	100.0	100.0	100.0
Estimates of parameters χ^2 (df) P(χ^2)		$\hat{\phi}$ = 0.103093 12.698(4) <0.025	$\hat{\phi}$ =.187 0.088(4) >0.995	$\hat{\phi}$ =0.2894276 4.82(4) 0.75

Table-II
Survival times (in days) of 72 guinea pigs infected with virulent tuberclebacilli

Survival Time (in days)	Observed frequency	Expected frequency		
		Exponential	Lindley	Mishra
0 – 80	8	14.3	16.1	10.8
80 – 160	30	20.6	21.9	24.8
160 – 240	18	13.3	15.4	19.0
240 – 320	8	8.5	9.0	10.1
320 – 400	4	5.5	5.5	4.5
400 – 480	3	3.5	1.8	1.8
480 -560	1	6.3	2.3	1.0
Total	72	72.0	72.0	72.0
Estimates of parameters χ^2 (df) P(χ^2)		$\hat{\phi}$ =0.00552 12.5970(4) 0.021	$\hat{\phi}$ =0.011 7.7712(4) 0.051	$\hat{\phi}$ =0.016517701 2.33(4) 0.46

VI. CONCLUSION

In this paper, we propose, a single parameter continuous distribution, Mishra distribution (MD). Several properties such as moments, moment generating function, distribution function have been obtained .The methods of estimation of parameter have been discussed. Finally, the proposed distribution has been fitted to a number of data-sets to test its goodness of fit and it has been observed that the MD gives better fit to all the data-sets than the Exponential distribution. In some cases, it gives better fit than the Lindley distribution.

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