International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 4 Issue 10 || December. 2016 || PP-22-28

k– Hermitian Circulant, s-Hermitian Circulant and s – k Hermitian Circulant Matrices

Dr. N. Elumalai[1], Mr. K. Rajesh kannan[2],
[1]Associate professor, [2]Assistant Professor
[1]*Department of Mathematics, A.V.C. College (Autonomous), Mannampandal, TamilNadu*
[2]Department of Mathematics, Annai College of Engineering and technology, Kovilacheri, TamilNadu.

ABSTRACT: *The basic concepts and theorems of k-Hermitian Circulant s- Hermitian Circulant and s-k Hermitian Circulant matrices are introduced with examples.*

Keywords: *k- Hermitian Circulant matrix, s- Hermitian Circulant matrix and s-k- Hermitian Circulant matrix.*
AMS Classifications: *15B57, 15B05, 47B15*

I. INTRODUCTION

The concept of s- Hermitian matrices, k- Hermitian matrices and of s-k Hermitian matrices was introduced in [1], [2] and [3] Some properties of Hermitian matrices given in [5],[6] .In this paper, our intention is to define s- Hermitian circulant matrices, k- Hermitian circulant matrices and of s-k Hermitian circulant matrices and prove some results on Hermitian circulant matrices

II. PRELIMINARIES AND NOTATIONS

Z^T is called Transpose of Z, Z^S is called secondary transpose of Z.Let k be a fixed product of disjoint transposition in S_n and 'K'be the permutation matrix associated with k, V is a permutation matrix with units in the secondary diagonal. Clearly K and V are satisfies the following properties. $K^2 = I$, $K^T = K$,$V^2 = I$, $V^T = V$

III. DEFINITIONS AND THEOREMS

Definition: 1 *For any given $z_0, z_1, z_2, z_3, \ldots z_{n-1} \in C^{nxn}$ the Circulant matrix $Z = [Z_{ij}]$ is defined by*

$$Z=\begin{bmatrix} z0 & z1 & z2 & \ldots & zn-1 \\ zn-1 & z0 & z1 & \ldots & zn-2 \\ zn-2 & zn-1 & z0 & \ldots & zn-3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ zn-1 & zn-2 & zn-3 & \ldots & z0 \end{bmatrix}$$

Definition: 2 *A matrix $Z \in C^{nxn}$ is said to be Hermitian Circulant matrix if $Z = Z^*$*

Example: 1

$$Z = \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix} \quad \overline{Z} = \begin{bmatrix} 2 & 1-i & 1+i \\ 1+i & 2 & 1-i \\ 1-i & 1+i & 2 \end{bmatrix} \quad Z^* = \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix}$$

Result:

(i) All Hermitian is not a Circulant matrix , $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$

(ii) All Circulant matrix is not a Hermitian matrix, $\begin{bmatrix} 1 & 1+i \\ 1+i & 1 \end{bmatrix}$

Definition: 3 *A matrix $Z \in C^{nxn}$ is said to be k-Hermitian Circulant matrix if $\overline{z} = K Z^* K$*

Example: 2

$$Z= \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix}, \overline{Z} = \begin{bmatrix} 2 & 1-i & 1+i \\ 1+i & 2 & 1-i \\ 1-i & 1+i & 2 \end{bmatrix}, Z^*= \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix}, K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now, $KZ^*K=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}2&1+i&1-i\\1-i&2&1+i\\1+i&1-i&2\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}=\begin{bmatrix}2&1-i&1+i\\1+i&2&1-i\\1-i&1+i&2\end{bmatrix}=\overline{Z}$

Result: *(i)* $KZ=\overline{ZK}$ *(ii)* $ZK=\overline{KZ}$

(i) $KZ=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}2&1+i&1-i\\1-i&2&1+i\\1+i&1-i&2\end{bmatrix}=\begin{bmatrix}2&1+i&1-i\\1+i&1-i&2\\1-i&2&1+i\end{bmatrix}$

$ZK=\begin{bmatrix}2&1+i&1-i\\1-i&2&1+i\\1+i&1-i&2\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}=\begin{bmatrix}2&1-i&1+i\\1-i&1+i&2\\1+i&2&1-i\end{bmatrix}$ $\overline{ZK}=\begin{bmatrix}2&1+i&1-i\\1+i&1-i&2\\1-i&2&1+i\end{bmatrix}$

$KZ=\overline{ZK}$ *Similarly* $ZK=\overline{KZ}$

Theorem: 1 *Let* $Z\in C^{nxn}$ *is k-Hermitian Circulant matrix if* $\overline{Z}=KZ^*K$

Proof:

$KZ^*K = KZK$ *where* $Z^* = Z$

$= \overline{ZK}\,K$ *where* $KZ=\overline{ZK}$

$= \overline{Z}\,\overline{K}\,K$

$= \overline{Z}\,K^2 = \overline{Z}$ *where* $K^2 = I$

Theorem: 2 *Let* $Z\in C^{nxn}$ *is k-Hermitian Circulant matrix if* $Z^*= K\,\overline{Z}\,K$

Proof: $K\,\overline{Z}\,K = K\,\overline{Z}\,\overline{K}$ *where* $\overline{K}=K$

$= K\,\overline{ZK}$

$= K\,KZ$ *where* $\overline{ZK}=KZ$

$= Z = Z^*$ *where* $Z^* = Z$

Theorem: 3 *Let* Z_1 *and* Z_2 *are k-Hermitian Circulant matrices then* Z_1Z_2 *is also k-Hermitian Circulant matrix*

Proof: *Let* Z_1 *and* Z_2 *are k-Hermitian Circulant matrices then* $Z_1^*= K\,\overline{Z_1}\,K$, $Z_2^*= K\,\overline{Z_2}\,K$ *and* $\overline{Z_1}=KZ_1^*K$, $\overline{Z_2}=KZ_2^*K$. *To prove* Z_1Z_2 *is k-Hermitian Circulant matrix*

We will show that $Z_1Z_2=\overline{Z_2Z_1}=K(Z_1Z_2)^*K$

Now $K(Z_1Z_2)^*K = KZ_2^*Z_1^*K$

$= K[(K\,\overline{Z_2}\,K)(K\,\overline{Z_1}\,K)]K$ *where* $Z_1^*= K\,\overline{Z_1}\,K$, $Z_2^*= K\,\overline{Z_2}\,K$

$= K^2\,\overline{Z_2}\,K^2\,\overline{Z_1}\,K^2$

$= \overline{Z_2}\,\overline{Z_1}$

$= \overline{Z_2Z_1}$

Theorem: 4 *Let* Z_1 *and* Z_2 *are k-Hermitian Circulant matrices then* Z_1+Z_2 *is also k-Hermitian Circulant matrix*

Proof: *Let* Z_1 *and* Z_2 *are k-Hermitian Circulant matrices then* $Z_1^*= K\,\overline{Z_1}\,K$, $Z_2^*= K\,\overline{Z_2}\,K$ *and* $\overline{Z_1}=KZ_1^*K$, $\overline{Z_2}=KZ_2^*K$. *To prove* Z_1+Z_2 *is k-Hermitian Circulant matrix*

We will show that $(\overline{Z_1+Z_2})=K(Z_1+Z_2)^*K$

Now $K(Z_1+Z_2)^*K = K(Z_1^*+Z_2^*)K$

$= KZ_1^*K + KZ_2^*K$

$= \overline{Z_1}+\overline{Z_2}$

$= \overline{Z_1+Z_2}$

Theorem: 5 *Let* $Z\in C^{nxn}$ *is k-Hermitian Circulant matrix and K is the Permutation matrix , k=(1)(2 3) then KZ is also k-Hermitian Circulant matrix*

Proof: *Let* $Z\in C^{nxn}$ *is k-Hermitian Circulant matrix* $\overline{Z}=KZ^*K$, $Z^*=K\overline{Z}K$

To Prove KZ is k-Hermitian Circulant matrix

We will show that, $KZ = \overline{ZK} = K(KZ)^*K$

Now $K(KZ)^*K = K[K(\overline{ZK})K]K$

$= K^2\,\overline{ZK}\,K^2 = \overline{ZK}$

Theorem: 5 *Let* $Z \in C^{nxn}$ *is k-Hermitian Circulant matrix then* ZZ^T *is also k-Hermitian Circulant matrix*

Proof: *Let* $Z \in C^{nxn}$ *is k-Hermitian Circulant matrix* $\overline{Z} = KZ^*K$, $Z^* = K\overline{Z}K$

To Prove ZZ^T *is k-Hermitian Circulant matrix .We will show that,* $ZZ^T = \overline{Z^T Z} = K(ZZ^T)^*K$

Now $K(ZZ^T)^*K = K[K(\overline{Z^T Z})K]K$

$= K^2(\overline{Z^T Z})K^2$

$= \overline{Z^T Z}$

Result: *Let* Z_1 *and* Z_2 *are k-Hermitian Circulant matrices for the following conditions are holds*

(i) $Z_1 Z_2 = Z_2 Z_1$ *and also k-Hermitian Circulant matrix*

(ii) $Z_1^T Z_2 Z_1 = Z_2^T Z_1 Z_2$ *are also k-Hermitian Circulant matrices*

Theorem: 7 *Let Z is k-Hermitian Circulant matrix then the following conditions are equal*

1) $KZ^* = KZ$
2) $Z^*K = ZK$
3) $(KZ)^* = Z^*K$
4) $(Z^*K) = KZ$

Proof: *(i)* $KZ^* = KZ$

Now, $KZ^* = K(K\overline{Z}K)$ *where* $Z^* = K\overline{Z}K$

$= K(K\overline{Z}\,\overline{K})$

$= K(K\overline{ZK})$

$= K(KKZ) = K^2(KZ)$

$= KZ$

(ii) $Z^*K = ZK$

Now, $Z^*K = (K\overline{Z}K)K$ *where* $Z^* = K\overline{Z}K$

$= (K\overline{Z}\,\overline{K})K$

$= (K\overline{ZK})K$

$= (KKZ)K = K^2(ZK)$

$= ZK$

(iii) $(KZ)^* = Z^*K$

Now, $(KZ)^* = K(\overline{ZK})K$ *where* $Z^* = K\overline{Z}K$

$= K(\overline{Z}\,\overline{K})K$

$= K(\overline{Z}K)K$

$= (K\overline{Z}K)K$

$= Z^*K$

(iv) $(Z^*K)^* = KZ$

Now, $(Z^*K)^* = (ZK)^*$ *where (ii)*

$= K(\overline{KZ})K$

$= K(\overline{K}\,\overline{Z})K$

$= K(K\overline{Z}K)$

$= KZ^* = KZ$

Definition: 4 *A matrix $Z \in C^{nxn}$ is said to be S-Hermitian Circulant matrix if $\overline{Z} = VZ^*V$ where V is Exchange matrix*

Example: 3

$$VZ^*V = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1-i & 1+i \\ 1+i & 2 & 1-i \\ 1-i & 1+i & 2 \end{bmatrix} = \overline{Z}$$

Result: *(i) $VZ = \overline{ZV}$ (ii) $ZV = \overline{VZ}$*

$$(i)\ VZ = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix} = \begin{bmatrix} 1+i & 1-i & 2 \\ 1-i & 2 & 1+i \\ 2 & 1+i & 1-i \end{bmatrix}$$

$$(ii)\ ZV = \begin{bmatrix} 2 & 1+i & 1-i \\ 1-i & 2 & 1+i \\ 1+i & 1-i & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1-i & 1+i & 2 \\ 1+i & 2 & 1-i \\ 2 & 1-i & 1+i \end{bmatrix},\ \overline{ZV} = \begin{bmatrix} 1+i & 1-i & 2 \\ 1-i & 2 & 1+i \\ 2 & 1+i & 1-i \end{bmatrix}$$

$\therefore$ *$VZ = \overline{ZV}$ Similarly $ZV = \overline{VZ}$*

Theorem: 8 *Let $Z \in C^{nxn}$ is s-Hermitian Circulant matrix if $\overline{Z} = V Z^* V$*

Proof: *$VZ^*V = V\,Z\,V$ where $Z^* = Z$*

$= \overline{ZV}\ K$ where $VZ = \overline{ZV}$

$= \overline{Z}\ \overline{V}\ V$

$= \overline{Z}\ V^2 = \overline{Z}$ where $V^2 = I$

$= Z = Z^*$ where $Z^* = Z$

Theorem:9 *Let $Z \in C^{nxn}$ is s-Hermitian Circulant matrix if $Z^* = V\overline{Z}\ V$*

Proof: *$V\ \overline{Z}\ \ V = V\ \overline{Z}\ \overline{V}$ where $\overline{V} = V$*

$= V\ \overline{ZV}$

$= V\,VZ$ where $\overline{ZV} = VZ$

$= V^2\,Z$ where $V^2 = I$

$= Z = Z^*$ where $Z^* = Z$

Theorem:10 *Let Z_1 and Z_2 are s-Hermitian Circulant matrices then $Z_1 Z_2$ is also s-Hermitian Circulant matrix*

Proof: *Let Z_1 and Z_2 are s-Hermitian Circulant matrices then $Z_1^* = V\ \overline{Z_1}\ V$, $Z_2^* = V\ \overline{Z_2}\ V$ and $\overline{Z_1} = V Z_1^* V$, $\overline{Z_2} = V Z_2^* V$. To prove $Z_1 Z_2$ is s-Hermitian Circulant matrix*

We will show that $Z_1 Z_2 = \overline{Z_2 Z_1} = V (Z_1 Z_2)^* V$

Now $V (Z_1 Z_2)^* V = V Z_2^* Z_1^* V$

$= V\,[(V\ \overline{Z_2}\ V)\ (V\ \overline{Z_1}\ V)]\ V$ where $Z_1^* = V\ \overline{Z_1}\ V$, $Z_2^* = V\ \overline{Z_2}\ V$

$= V^2\ \overline{Z_2}\ V^2\ \overline{Z_1}\ V^2$

$= \overline{Z_1}\ \ \overline{Z_2}$

$= \overline{Z_2 Z_1}$

$= Z_1 Z_2$ where $\overline{Z_2 Z_1} = Z_1 Z_2$

Theorem:11 *Let Z_1 and Z_2 are s-Hermitian Circulant matrices then $Z_1 + Z_2$ is also s-Hermitian Circulant matrix*

Proof: Let Z_1 and Z_2 are s-Hermitian Circulant matrices then $Z_1^* = V\ \overline{Z_1}\ V$, $Z_2^* = V\ \overline{Z_2}\ V$ and $\overline{Z_1} = V Z_1^* V$, $\overline{Z_2} = V Z_2^* V$. To prove $Z_1 + Z_2$ is s-Hermitian Circulant matrix

We will show that $(\overline{Z_1 + Z_2}) = V (Z_1 + Z_2)^* V$

Now $V (Z_1 + Z_2)^* V = V (Z_1^* + Z_2^*) V$

$= V Z_1^* V + V Z_2^* V$

$= \overline{Z_1} + \overline{Z_2}$

$= \overline{Z_1 + Z_2}$

Theorem: 12 *Let $Z \in C^{nxn}$ is s-Hermitian Circulant matrix and V is the Exchange matrix then VZ is also s-Hermitian Circulant matrix*

***Proof**: Let Z is s-Hermitian Circulant matrix then $\overline{Z} = V Z^* V$, $Z^* = V \overline{Z} V$*
To Prove VZ is s-Hermitian Circulant matrix

We will show that, $VZ = \overline{ZV} = V (VZ)^* V$

Now $V (VZ)^* V = V [V (\overline{ZV}) V] V$

$= V^2 \overline{ZV} V^2$

$= \overline{ZV} = VZ$

Theorem: 13 *Let $Z \in C^{nxn}$ is s-Hermitian Circulant matrix then ZZ^T is also s-Hermitian Circulant matrix*

***Proof:** Let $Z \in C^{nxn}$ is s-Hermitian Circulant matrix $\overline{Z} = V Z^* V$, $Z^* = V \overline{Z} V$*

To Prove ZZ^T is s-Hermitian Circulant matrix .We will show that, $ZZ^T = \overline{Z^T Z} = V (ZZ^T)^* V$

Now $V (ZZ^T)^* V = V [V (\overline{Z^T Z}) V] V$

$= V^2 (\overline{Z^T Z}) V^2$

$= \overline{Z^T Z} = Z Z^T$

***Result:** Let Z_1 and Z_2 are s-Hermitian Circulant matrices for the following conditions are holds*
(i) $Z_1 Z_2 = Z_2 Z_1$ and also s-Hermitian Circulant matrix
(ii) $Z_1^T Z_2 Z_1$ and $Z_2^T Z_1 Z_2$ are also s-Hermitian Circulant matrices

Theorem: 14
Let Z is s-Hermitian Circulant matrix then the following conditions are equal
1) $VZ^* = VZ$
2) $Z^*V = ZV$
3) $(VZ)^* = Z^*V$
4) $(Z^*V) = VZ$

Proof: *(i) $VZ^* = VZ$*

Now, $VZ^* = V (V \overline{Z} V)$ where $Z^* = V \overline{Z} V$

$= V (V \overline{Z} \overline{V})$

$= V (V \overline{ZV})$

$= V (VV Z) = V^2 (VZ)$

$= VZ$

(ii) $Z^*V = ZV$

Now, $Z^*V = (V \overline{Z} V) V$ where $Z^* = V \overline{Z} V$

$= (V \overline{Z} \overline{V}) V$

$= (V \overline{ZV}) V$

$= (VV Z) V = V^2 (ZV)$

$= ZV$

(iii) $(VZ)^* = Z^*V$

Now, $(VZ)^* = V (\overline{ZV}) V$ where $Z^* = V \overline{Z} V$

$= V (\overline{Z} \overline{V}) V$

$= V (\overline{Z} V) V$

$= (V \overline{Z} V) V$

$= Z^*V$

(iv) $(Z^*V)^* = VZ$

Now, $(Z^*V)^* = (ZV)^*$ where (ii)

$= V (\overline{VZ}) V$

$= V(\overline{V}\ \overline{Z})V$

$= V(V\overline{Z}V)$

$= VZ^* = VZ$

Definition: 5 *A matrix $Z \in C^{nxn}$ is said to be s-k Hermitian Circulant matrix if (i)$Z = KVZ^*VK$ (ii) $Z = VKZ^*KV$ (iii) $\overline{Z} = KV\overline{Z}VK$ (ii) $\overline{Z} = VK\overline{Z}KV$ where V is Exchange matrix and K is Permutation matrix K = (1)(2 3)*

Theorem: 15*Let Z_1 and Z_2 are s-k Hermitian Circulant matrices then $Z_1 + Z_2$ is also s-k Hermitian Circulant matrix*

Proof: *Let Z_1 and Z_2 are s-k Hermitian Circulant matrices then $Z_1^* = KV\overline{Z_1}VK$, $Z_2^* = KV\overline{Z_2}VK$*
To prove $Z_1 + Z_2$ is s-k Hermitian Circulant matrix
We will show that $(Z_1+Z_2)^* = KV(Z_1+Z_2)^*VK$
Now $KV(Z_1+Z_2)^*VK = K[V(Z_1^* + Z_2^*)V]K$

$= K(\overline{Z_1 + Z_2})K$

$= (Z_1^* + Z_2^*)$

Theorem: 16 *Let Z_1 and Z_2 are s-k Hermitian Circulant matrices then $Z_1 Z_2$ is also s-k Hermitian Circulant matrix*

Proof: *Let Z_1 and Z_2 are s-k Hermitian Circulant matrices then $Z_1^* = KV\overline{Z_1}VK$, $Z_2^* = KV\overline{Z_2}VK$*
To prove $Z_1 Z_2$ is s-k Hermitian Circulant matrix
We will show that $(Z_1 Z_2)^* = KV(Z_1 Z_2)^*VK$
Now $KV(Z_1 Z_2)^*VK = K[VZ_2^*Z_1^*V]K$

$= K(\overline{Z_1 Z_2})K$

$= (Z_1 Z_2)^*$

Theorem: 17 *Let $Z \in C^{nxn}$ is s-k Hermitian Circulant matrix and V is the Exchange matrix and K is Permutation matrix K= (1)(2 3) then VZ is also s-k Hermitian Circulant matrix*
Proof: *Let Z is s-k Hermitian Circulant matrix then $Z^* = KVZ^*VK$*
To Prove VZ is s-k Hermitian Circulant matrix
We will show that, $(VZ)^* = KV(VZ)^*VK$
Now $KV(VZ)^*VK = K[V(VZ)^*V]K$

$= K\ ZV\ K$

$= (VZ)^*$

Theorem: 18 *Let $Z \in C^{nxn}$ is s-k Hermitian Circulant matrix then ZZ^T is also s-k Hermitian Circulant matrix*
Proof: *Let $Z \in C^{nxn}$ is s-k Hermitian Circulant matrix $Z^* = KVZ^*VK$*
To Prove ZZ^T is s-k Hermitian Circulant matrix .We will show that, $(ZZ^T)^* = KV(ZZ^T)^*VK$
Now $KV(ZZ^T)^*VK = K[V(ZZ^T)^*V]K$

$= K(\overline{Z^T Z})K$

$= (\overline{Z^T Z})^*$

Theorem: 19 *Let $Z \in C^{nxn}$ is s-k Hermitian Circulant matrix then the following conditions are equal*
(i)$Z^* = KVZ^*VK$
(ii) $Z^* = VKZ^*KV$
(iii) $\overline{Z} = KV\overline{Z}VK$
(iv) $\overline{Z} = VK\overline{Z}KV$
Proof:

(i) $KVZ^*VK = K(VZ^*V)K$

$= K\overline{Z}K$

$= Z^*$

(ii) $VKZ^*KV = V(KZ^*K)V$

$= V\overline{Z}V$

$= Z^*$

(iii) $KV\overline{Z}VK = K[V\overline{Z}V]K$

$= KZ^*K$

$$= \bar{z}$$

(iv) $VK\bar{z}\,KV = V\,(K\bar{z}\,K)\,V$

$$= \bar{V}\,Z^*\,V$$

$$= \bar{z}$$

REFERENCES

[1]. Ann Lec. Secondary symmetric and skew symmetric secondary or thogonal matrices; (i) Period, Math Hungary, 7, 63-70 (1976).

[2]. Elumalai.N, Rajesh kannan.K," k-symmetric Circulant, s- symmetric Circulant and s- k symmetric Circulant Matrices", Journal of Ultra Scientist of Physical Sciences, Vol. 28(6), 322-327 (2016).

[3]. Gunasekaran. K., Mohana. N., " k-symmetric Double stochastic , s-symmetric Double stochastic , s-k- symmetric Double stochastic Matrices" International Journal of Engineering Science Invention, Vol 3, Issue 8, (2014).

[4]. Hill, R.D, and Waters, S.R., On k-real and k-hermitian matrices; Lin. Alg. Appl., 169, 17-29 (1992).

[5]. Hazewinkel, Michiel, ed.(2001), "Symmetric matrix", Encyclopedia of Mathematics, Springer, ISBN978-1- 55608-010-4

[6]. Krishnamoorthy. S, Vijayakumar. R, Ons-normal matrices, Journal of Analysis and Computation, Vol 15, No 2, (2009).

[7]. Krishnamoorthy. S, Gunasekaran. K, Mohana. N, "Characterization and theorems on doubly stochastic matrices" Antartica Journal of Mathematics, 11(5) (2014)