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# K-th Upper Record Values from Power Lomax Distribution

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***ABSTRACT:*** *In this paper, recurrence relations between the single and the product moments of the K-th upper record values from Power Lomax distribution are obtained.*
***Keywords:*** *K-th upper record values, Power Lomax distribution, recurrence relations.*

## I. INTRODUCTION

The probability density function (*pdf*) of the Power Lomax (PL) distribution is defined by (Rady et al. [1])

$$f(x)=\alpha\beta\lambda^{-1}x^{\beta-1}\left(1+\frac{x^{\beta}}{\lambda}\right)^{-(\alpha+1)}\quad;\ x\geq 0,\ (\alpha,\beta,\lambda>0). \tag{1.1}$$

The corresponding reliability (survival) function of PL distribution is given by,

$$\bar{F}(x)=\left(1+\frac{x^{\beta}}{\lambda}\right)^{-\alpha}\quad;\ x\geq 0,\ (\alpha,\beta,\lambda>0). \tag{1.2}$$

In view of (1.1) and (1.2), we get

$$\alpha\beta\bar{F}(x)=\left(x+\lambda x^{1-\beta}\right)f(x). \tag{1.3}$$

For more details on this distribution and its application one may refer to Rady et al. [1].
Let $X_1,X_2,...,X_n$ be a sequence of independent and identically (*iid*) random variables with *pdf* $f(x)$ and distribution function (*df*) $F(x)$. For a fixed $k\geq 1$, we defined the sequence of K-th upper record times $\left\{U_n^{(k)},n\geq 1\right\}$, of the sequence $\{X_1,X_2,...,X_n\}$ as $U_1^{(k)}=1$ and

$$U_{n+1}^{(k)}=\min\left\{J>U_n^{(k)}:X_{j:j+k-1}>X_{U_n^{(k)}:U_n^{(k)}+k-1}\right\}$$

For $k=1$ and $n=1,2,\ldots$, we write $U_n^{(1)}=U_n$. Then $\left\{U_n^{(k)},n\geq 1\right\}$ is the sequence of record times of $\{X_1,X_2,...,X_n\}$. The sequence $\left\{Y_n^{(k)},\ n\geq 1\right\}$, where $Y_n^{(k)}=X_{U_n^{(k)}}$ is called the sequence of K-th upper record values of $\{X_1,X_2,...,X_n\}$ for convenience, we shall also take $Y_0^{(k)}=0$. Note that for $k=1$ we have $Y_n^{(1)}=X_{U_n},\ n\geq 0$ which are record values of $\{X_1,X_2,...,X_n\}$ (Ahsanullah, [2]).
The *pdf* of $Y_n^{(k)}$ $(n\geq 1)$ is given by Dziubdziela and Kopocinski [3] as follows:

$$f_{Y_n^{(k)}}(x)=\frac{k^n}{(n-1)!}\left\{-\ln\left[1-F(x)\right]\right\}^{n-1}\left[1-F(x)\right]^{k-1}f(x), \tag{1.4}$$

and the joint *pdf* of $Y_m^{(k)},Y_n^{(k)};\ 1\leq m<n,\ n\geq 2$ is denoted by Grudzien [4] as:

$$f_{Y_m^{(k)},Y_n^{(k)}}(x,y)=\frac{k^n}{(m-1)!(n-m-1)!}\left[H(x)\right]^{m-1}\left\{H(y)-H(x)\right\}^{n-m-1}h(x)\left[1-F(y)\right]^{k-1}f(y), \tag{1.5}$$

where $H(z)=-\ln\left[1-F(z)\right]$ and $h(z)=\dfrac{f(z)}{1-F(z)}$,

Recurrence relations for single and product moments of k-th record values from some distributions are established by many authors. From Weibull distribution is introduced by Pawlas and Szynal [5]. Saran and Singh [6], discussed recurrence relations for single and product moments of k-th record values from linear exponential distribution. Pawlas and Szynal [7], introduced recurrence relations for single and product moments of k-th record values from Pareto, generalized Pareto and Burr distributions. Amin [8], introduced K-th upper record values and their moments for uniform and Lomax distributions.
Relations for single and product moments of upper record values from Lomax distribution are introduced by Balakrishnan and Ahsanullah [9].

## II. SINGLE MOMENTS OF K-TH RECORD VALUES FROM PL DISTRIBUTION

In this section, recurrence relation for single moments of the K-th upper record values from PL distribution is obtained. The single moments of K-th upper record values for PL distribution can be obtained from (1.4) (when r is positive integer) as follows:

$$\mu_{(n):k}^{(r)} = \frac{k^{n}}{(n-1)!}\int_{0}^{\infty} x^{r}\left\{-\ln\left[1-F(x)\right]\right\}^{n-1}\left[1-F(x)\right]^{k-1} f(x)\,dx \tag{2.1}$$

**Lemma 2.1.** For PL distribution, the following recurrence relation is satisfied.

$$\mu_{(n):k}^{(r+1)} + \lambda\mu_{(n):k}^{(r+1-\beta)} = \frac{\alpha\beta k}{r+1}\left(\mu_{(n):k}^{(r+1)} - \mu_{(n-1):k}^{(r+1)}\right) \tag{2.2}$$

**Proof.** From (1.3) and (2.1), we get

$$\mu_{(n):k}^{(r+1)} + \lambda\mu_{(n):k}^{(r+1-\beta)} = \frac{k^{n}}{(n-1)!}\int_{0}^{\infty}\left(x^{r+1}+\lambda x^{r+1-\beta}\right)\left\{-\ln\left[\bar{F}(x)\right]\right\}^{n-1}\left[\bar{F}(x)\right]^{k-1} f(x)\,dx$$

$$= \frac{\alpha\beta k^{n}}{(n-1)!}\int_{0}^{\infty} x^{r}\left\{-\ln\left[\bar{F}(x)\right]\right\}^{n-1}\left[\bar{F}(x)\right]^{k} dx$$

Integrating by parts, treating $x^{r}$ for integration and the rest of the integrand for differentiation and after simplification yields (2.2). □

**Remarks**

- When $\beta = 1$, we get the recurrence relation for the single moments of K-th upper record values from Lomax distribution, established by Amin [8].
- When $\beta = 1$ and $k = 1$, we get the recurrence relation for the single moments of upper record values from Lomax distribution, established by Balakrishnan and Ahsanullah [9].

## III. PRODUCT MOMENTS OF K-TH RECORD VALUES FROM PL DISTRIBUTION

In this section, recurrence relation for product moments of the K-th upper record values from PL distribution is obtained. The product moments of K-th upper record values for PL distribution can be obtained from (1.5) (when r and s are positive integers) as follows:

$$\mu_{(m,n):k}^{(r,s)} = \frac{k^{n}}{(m-1)!(n-m-1)!}\int_{0}^{\infty}\int_{x}^{\infty} x^{r}y^{s}\left[H(x)\right]^{m-1}\left\{H(y)-H(x)\right\}^{n-m-1} h(x)\left[1-F(y)\right]^{k-1} f(y)\,dy\,dx \tag{3.1}$$

**Lemma 3.1.** Let *X* be a non-negative random variable having a *pdf* (1.1), then

$$\mu_{(m,n):k}^{(r,s+1)} = \frac{\lambda(s+1)}{\alpha\beta k-s-1}\mu_{(m,n):k}^{(r,s+1-\beta)} + \frac{\alpha\beta}{\alpha\beta k-s-1}\mu_{(m,n-1):k}^{(r,s+1)}, \qquad 1 \le m \le n-2 \text{ and } r,s = 0,1,2,\ldots \tag{3.2}$$

and

$$\mu_{(m,m+1):k}^{(r,s+1)} = \frac{\lambda(s+1)}{\alpha\beta k-s-1}\mu_{(m,m+1):k}^{(r,s+1-\beta)} + \frac{\alpha\beta}{\alpha\beta k-s-1}\mu_{(m):k}^{(r,s+1)}, \qquad m \ge 1 \text{ and } r,s = 0,1,2,\ldots \tag{3.3}$$

**Proof.**

For $1 \le m \le n-2 \text{ and } r,s = 0,1,2,\ldots$

From (1.3) and (3.1), we have

$$\mu_{(m,n):k}^{(r,s+1)} + \lambda \mu_{(m,n):k}^{(r,s+1-\beta)} = \frac{k^n}{(m-1)!(n-m-1)!}$$

$$\int_0^\infty \int_x^\infty x^r y^s \left(y + \lambda y^{1-\beta}\right)\left[H(x)\right]^{m-1}\left[H(y)-H(x)\right]^{n-m-1} h(x)\left[1-F(y)\right]^{k-1} f(y)\,dy\,dx$$

$$= \frac{\alpha\beta k^n}{(m-1)!(n-m-1)!}\int_0^\infty \int_x^\infty x^r y^s \left[H(x)\right]^{m-1}\left[H(y)-H(x)\right]^{n-m-1} h(x)\left[1-F(y)\right]^{k} dy\,dx$$

$$= \frac{\alpha\beta k^n}{(m-1)!(n-m-1)!}\int_0^\infty x^r \left[H(x)\right]^{m-1} h(x) I(x)\,dx,$$

where

$$I(x) = \int_x^\infty y^s \left\{H(y)-H(x)\right\}^{n-m-1}\left[1-F(y)\right]^k dy$$

Integrating by parts, we get

$$I(x) = \frac{1}{s+1}\int_x^\infty y^{s+1}\left[H(y)-H(x)\right]^{n-m-2}\left[1-F(y)\right]^{k-1}\left\{k\left[H(y)-H(x)\right]-(n-m-1)\right\} f(y)\,dy$$

This is implies that

$$\mu_{(m,n):k}^{(r,s+1)} + \lambda \mu_{(m,n):k}^{(r,s+1-\beta)} = \frac{\alpha\beta}{s+1}\left[k\,\mu_{(m,n):k}^{(r,s+1)} - \mu_{(m,n-1):k}^{(r,s+1)}\right],$$

and after simplification yields (3.2). Proceeding in a similar manner for the case $n = m+1$, the recurrence relation given in (3.3) can easily be established. □

**Remarks**

- When $\beta = 1$, we get the recurrence relation for the product moments of K-th upper record values from Lomax distribution, established by Amin [8].
- When $\beta = 1$ and $k = 1$, we get the recurrence relation for the product moments of upper record values from Lomax distribution, established by Balakrishnan and Ahsanullah [9].

## IV. CONCLUSION

In this paper, we introduce recurrence relations between the single and the product moments of the K-th upper record values from Power Lomax distribution. Two special cases are obtained. These special cases are recurrence relations between the single and the product moments of the K-th upper record values from Lomax distribution introduced by Amin [8] and recurrence relations between the single and the product moments of an upper record values from Lomax distribution established by Balakrishnan and Ahsanullah [9].

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