Inventory Model with Different Deterioration Rates under Exponential Demand, Shortages, Inflation and Permissible Delay in Payments

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**ABSTRACT:** An inventory model with different deterioration rates under exponential demand with inflation and permissible delay in payments is developed. Holding cost is taken as linear function of time. Shortages are allowed. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.

**KEY WORDS:** Inventory model, Varying Deterioration, Exponential demand, Time varying holding cost, Shortages, Inflation, Permissible delay in payments

**I. INTRODUCTION**

Inventory problems for deterioration items have been studied extensively by many researchers from time to time. Research in this area started with the work of Whitin [21], who considered fashion goods deteriorating at the end of prescribed storage period. Ghar and Schrader [5] developed an inventory model with a constant rate of deterioration. Shah and Jaiswal [14] considered an order level inventory model for items deteriorating at a constant rate. Alfares [2] developed an inventory model with a stock level demand rate and a variable holding cost with the assumption that holding cost increases with time spent in storage. The related work are found in (Nahmias [11], Raffat [13], Goyal and Giri [7]).

Buzacott [3] developed the first economic order quantity model by considering inflationary effects into account. Su et al. [17] developed model under inflation for stock dependent consumption rate and exponential decay. Moon et al. [10] developed models for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [8]. Singh [16] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account.

In classical inventory model, it is assumed that a supplier must be paid for items as and when the customer receives the items. But in actual practice, many times the supplier allows credit for some fixed time period in settling the payment for the product and is not charged any interest from the customer for that specified period. But if he pays beyond that specified period, then the interest will be charged. Goyal [6] developed an EOQ model under the conditions of permissible delay in payments. Mandal and Phaujdar [9] extended this issue by considering the interest earned from the sales revenue. Aggarwal and Jaggi [1] extended Goyal’s model to the case of deterioration. Chang [4] developed mathematical model to determine an optimal ordering policy under permissible delay of payment and/ or cash discount for the customer. Shah [15] derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. Tripathy and Misra [19] extended Jaggi and Aggrwal’s model to allow for a time dependent demand rate. Teng et al. [18] developed EOQ model with trade credit financing for non-decreasing demand. Tripathy [20] developed an EOQ model for items with an exponential demand rate under permissible delay in payment and shortages. Patel and Patel [12] developed an EOQ model with shortages, linear demand, time varying holding cost under permissible delay in payments.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with different deterioration rates for the cycle time and exponential demand under time varying holding cost and permissible delay in payment. Shortages are allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.
II. NOTATIONS AND ASSUMPTIONS

The following notations are used for the development of the model:

**NOTATIONS:**
- \( D(t) \): Demand is an exponential function of time \( t \) \( (ae^{bt}, a>0, \ 0<b<1) \)
- \( A \): Replenishment cost per order
- \( c \): Purchasing cost per unit
- \( p \): Selling price per unit
- \( h(t) \): Inventory variable holding cost per unit excluding interest charges
- \( c_2 \): Shortage cost per unit
- \( M \): Permissible period of delay in settling the accounts with the supplier
- \( T \): Length of inventory cycle
- \( I_e \): Interest earned per year
- \( I_p \): Interest paid in stocks per year
- \( R \): Inflation rate
- \( I(t) \): Inventory level at any instant of time \( t \), \( 0 \leq t \leq T \)
- \( Q \): Order quantity
- \( Q_1 \): Order quantity initially
- \( Q_2 \): Quantity of shortages
- \( \theta \): Deterioration rate during \( \mu_1 \leq t \leq \mu_2 \), \( 0<\theta<1 \)
- \( \theta t \): Deterioration rate during \( \mu_2 \leq t \leq t_0 \), \( 0<\theta<1 \)
- \( \pi \): Total relevant profit per unit time.

**ASSUMPTIONS:**
- The following assumptions are considered for the development of the model.
  - The demand of the product is declining as an exponential function of time.
  - Replenishment rate is infinite and instantaneous.
  - Lead time is zero.
  - Shortages are allowed and completely backlogged.
  - Deteriorated units neither be repaired nor replaced during the cycle time.
  - During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let \( I(t) \) be the inventory at time \( t \) \( (0 \leq t \leq T) \) as shown in figure.

The differential equations which describes the instantaneous states of \( I(t) \) over the period \( (0, T) \) are given by:

\[
\frac{dI(t)}{dt} = -ae^{bt}, \quad 0 \leq t \leq \mu_1
\]

(1)
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\[ I(t) = \frac{a}{1 + \theta(\mu, t)} \left[ \left( t_{1} - \mu_{1} \right) + \frac{1}{2} b \left( t_{0} - \mu_{0} \right) + \frac{1}{6} \left( t_{1} - \mu_{1} \right) - \frac{1}{2} \theta \mu_{2} \left( t_{1} - \mu_{1} \right) - \frac{1}{4} b \theta \mu_{2} \left( t_{0} - \mu_{0} \right) \right] + a(\mu, t) \]

(16)

Based on the assumptions and descriptions of the model, the total annual relevant profit (\( \pi \)), include the following elements:

(i) Ordering cost (OC) = A

(ii) Holding cost (HC) is given by

\[ HC = \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt \]

\[ = \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt + \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt + \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt \]

\[ = \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt + \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt + \int_{0}^{T} (x+y)I(t)e^{-\lambda t} dt \]

Based on the given mathematical expressions and the assumptions of the model, the total annual relevant profit (\( \pi \)) is derived. The solution involves integrating the inventory model over time, considering the elements of ordering cost and holding cost. The model is particularly useful for systems with different deterioration rates, providing a comprehensive approach to optimizing inventory management in such scenarios.
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\[\begin{align*}
&x = \frac{1}{1 + \theta(t) \mu(t) + \mu_j} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \times \mu_j^2 \\
&y = \frac{1}{1 + \theta(t) \mu(t) + \mu_j} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \times \mu_j^2 \\
&z = \frac{1}{1 + \theta(t) \mu(t) + \mu_j} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \times \mu_j^2
\end{align*}\]

(iii) Deterioration cost (DC) is given by

\[DC = c \int_{t_i}^{t_f} \theta(t) e^{-\mu_j t} dt + \int_{t_i}^{t_f} \theta(t) e^{-\mu_j t} dt\]

\[\begin{align*}
&= c \left[ \left( \frac{-R}{3} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \right) \right] \times \mu_j^2 \\
&+ \left( \frac{-R}{2} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \right) \times \mu_j^2 \\
&+ \left( \frac{-R}{1 + \theta(t) \mu(t) + \mu_j} \left[ a \left( t_0 + \frac{1}{2} b \left( t_0^2 \mu_j^2 \right) - \frac{1}{6} \theta \left( t_0 \mu_j^3 \right) - \frac{1}{2} \theta\mu_j \left( t_0 \mu_j^2 \right) - \frac{1}{4} b \theta\mu_j \left( t_0 \mu_j^2 \right) - \mu_j \right) \right] \right) \times \mu_j^2
\end{align*}\]

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\[
\begin{align*}
&\left(1 - \frac{1}{3} \frac{1}{1 + \theta (\mu_i - \mu_j)} \left(a \left(t_i + \frac{1}{2} b \left(t_i + \mu_i - \mu_j\right) + \frac{1}{6} \left(t_i - \mu_i\right) \left(1 - \mu_i\right) \frac{1}{4} \left(\mu_i - \mu_j\right) - \mu_j\right) - \mu_j\right) - \mu_j\right) - a \mu_j \\
&\left(1 - \frac{1}{2} \frac{1}{1 + \theta (\mu_i - \mu_j)} \left(a \left(t_i + \frac{1}{2} b \left(t_i + \mu_i - \mu_j\right) + \frac{1}{6} \left(t_i - \mu_i\right) \left(1 - \mu_i\right) \frac{1}{4} \left(\mu_i - \mu_j\right) - \mu_j\right) - \mu_j\right) - a \mu_j \\
&\left(1 - \frac{1}{2} \frac{1}{1 + \theta (\mu_i - \mu_j)} \left(a \left(t_i + \frac{1}{2} b \left(t_i + \mu_i - \mu_j\right) + \frac{1}{6} \left(t_i - \mu_i\right) \left(1 - \mu_i\right) \frac{1}{4} \left(\mu_i - \mu_j\right) - \mu_j\right) - \mu_j\right) - a \mu_j \\
&\left(1 + \frac{1}{2} \frac{1}{1 + \theta (\mu_i - \mu_j)} \left(a \left(t_i + \frac{1}{2} b \left(t_i + \mu_i - \mu_j\right) + \frac{1}{6} \left(t_i - \mu_i\right) \left(1 - \mu_i\right) \frac{1}{4} \left(\mu_i - \mu_j\right) - \mu_j\right) - \mu_j\right) - a \mu_j \\
&\sum_{i=1}^{n}
\end{align*}
\]

\(v\) Interest earned per cycle:

\[
IE_i = p \int_0^M ae^{-at} dt = p \left(1 + \frac{1}{4} \left(\frac{a R - \frac{1}{2} \left(T - t_i\right)}{T - t_i}\right) + \frac{1}{3} \left(\frac{a R - \frac{1}{2} \left(T - t_i\right)}{\left(T - t_i\right) - \frac{1}{2} \left(T - t_i\right)}\right) + \frac{1}{2} \left(\frac{a R - \frac{1}{2} \left(T - t_i\right)}{\left(T - t_i\right) - \frac{1}{2} \left(T - t_i\right)}\right)\right)
\]

\(22\)

Case II: \(0 \leq T \leq M\):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

\[
IE_2 = p \int_0^M ae^{-at} dt + a \left(1 + bt_i\right) t_i \left(M - t_i\right)
\]

\(23\)

To determine the interest payable, there will be four cases i.e.

Case I: \(0 \leq M \leq T\) and case II: \(0 \leq T \leq M\).

**Case I: \(0 \leq M \leq T\):** In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

\[
(19)
\]

\[
(20)
\]

\[
(21)
\]

To determine the interest earned, there will be two cases i.e. case I: \(0 \leq M \leq T\) and case II: \(0 \leq T \leq M\).
\[ IP_1 = c_I \int_I^r t(t)e^{-b_1 t} dt \]
\[ = c_I \int_I^r t(t)e^{-b_1 t} dt + \int_I^r t(t)e^{-b_1 t} dt + \int_I^r t(t)e^{-b_1 t} dt \]
\[ = c_I \left( \frac{1}{8Rab} \mu_1^2 + \frac{1}{3} \left( \frac{1}{2ab + Ra} \right) \mu_1^2 \right) \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{1 + 0 (\mu_1 - \mu_2)} \left( \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \right) \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{2} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
\[ + \frac{1}{3} \left( -a - R \right) \left( \frac{1}{1 + 0 (\mu_1 - \mu_2)} \right) a \left( t_1 + \frac{1}{2} b (t_1 \cdot \mu_1) + \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \frac{1}{2} b (t_1 \cdot \mu_1) \cdot \mu_1 \right) + a \left( \mu_1 + \frac{1}{2} b \mu_1 \right) \mu_1 \]
Case II: ($\mu_1\leq M\leq \mu_2$):

(viii) $IP_2 = c_1 \left[ \int_{0}^{t} I(t)e^{-at} \, dt + \int_{0}^{t} I(t)e^{-bt} \, dt \right]$

\[
\begin{align*}
\frac{1}{3} R & \left\{ -a \cdot \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0 + \frac{1}{2} b \left( t_0^2 - \mu_2 \right) + \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) - \frac{1}{4} b^2 \left( t_0^3 \mu_2 - \mu_2 \right) - \mu_2 \right) \right] \right\} M^0 \\
- \frac{1}{3} R & \left\{ -a \cdot \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right] \right\} M^1 \\
+ a_1 M & \left\{ a_1 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right\} M^2
\end{align*}
\]

\[
\frac{1}{2} R \left\{ \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) - \frac{1}{4} b^2 \left( t_0^3 \mu_2 - \mu_2 \right) - \mu_2 \right] \right\} M^0 \\
- \frac{1}{2} R & \left\{ \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right] \right\} M^1 \\
+ a_1 M & \left\{ a_1 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right\} M^2
\end{align*}
\]

(25)

Case III: ($\mu_1 \leq M \leq \mu_0$):

(ix) $IP_3 = c_1 \left[ \int_{0}^{t} I(t)e^{-at} \, dt + \int_{0}^{t} I(t)e^{-bt} \, dt \right]$

\[
\begin{align*}
\frac{1}{24} R b \left\{ \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) - \frac{1}{4} b^2 \left( t_0^3 \mu_2 - \mu_2 \right) - \mu_2 \right] \right\} M^0 \\
+ c_1 a & \left\{ a_1 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right\} M^1 \\
- \frac{1}{24} R b & \left\{ \frac{1}{1+0(\mu_1 - \mu_2)} \left[ a_0 \left( t_0^3 - \mu_2 \right) - \frac{1}{6} \left( t_0^3 - \mu_2 \right) - \frac{1}{2} b \left( t_0^2 \mu_2 - \mu_2 \right) \right] \right\} M^2
\end{align*}
\]

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(26)

Case IV: \( (t_0 \leq M \leq T) \):

\[(x) IP_4 = 0 \] (27)

The total profit \( (\pi_i) \), \( i=1,2,3 \) and 4 during a cycle consisted of the following:

\[ \pi_i = \frac{1}{T} [SR - OC - HC - DC - SC - IP_i + IE] \] (28)

Substituting values from equations (17) to (27) in equation (28), we get total profit per unit. Putting \( \mu_1 = v_1 t_0 \) and \( \mu_2 = v_2 \) in equation (28), we get profit in terms of \( t_0 \) and \( T \) for the four cases will be as under:

\[ \pi_1 = \frac{1}{T} [SR - OC - HC - DC - SC - IP_1 + IE_1] \] (29)

\[ \pi_2 = \frac{1}{T} [SR - OC - HC - DC - SC - IP_2 + IE_1] \] (30)

\[ \pi_3 = \frac{1}{T} [SR - OC - HC - DC - SC - IP_3 + IE_1] \] (31)

\[ \pi_4 = \frac{1}{T} [SR - OC - HC - DC - SC - IP_4 + IE_1] \] (32)

The optimal value of \( t_0^* \) and \( T^* \) (say), which maximizes \( \pi_i \) can be obtained by solving equation (29), (30), (31) and (32) by differentiating it with respect to \( t_0 \) and \( T \) and equate it to zero, we have

\[ \frac{\partial \pi_i (t_0, T)}{\partial t_0} = 0, \quad \frac{\partial \pi_i (t_0, T)}{\partial T} = 0. \quad i=1,2,3,4 \] (33)

provided it satisfies the condition

\[ \frac{\partial^2 \pi_i (t_0, T)}{\partial t_0^2} < 0, \quad \frac{\partial^2 \pi_i (t_0, T)}{\partial T^2} < 0, \quad \text{and} \quad \left[ \frac{\partial^2 \pi_i (t_0, T)}{\partial t_0^2} \right]^2 + \left[ \frac{\partial^2 \pi_i (t_0, T)}{\partial t_0 \partial T} \right] < 0, \quad i=1,2,3,4. \] (34)

IV. NUMERICAL EXAMPLES

Case I: Considering \( A=Rs.100, \quad a=500, \quad b=0.05, \quad c=Rs.25, \quad p=Rs.40, \quad \theta=0.05, \quad x=Rs.5, \quad y=0.05, \quad c_2 = Rs.8, \quad v_1=0.30, \quad v_2=0.50, \quad R=0.06, \quad Ie=0.12, \quad Ip=0.15, \quad M=0.01 \) in appropriate units. The optimal value of \( t_0^* = 0.1316, \quad T^* =0.2860, \quad \text{Profit}^* = Rs. 19313.5824 \) and optimum order quantity \( Q^* = 144.0534 \).

Case II: Considering \( A=Rs.100, \quad a=500, \quad b=0.05, \quad c=Rs.25, \quad p=Rs.40, \quad \theta=0.05, \quad x=Rs.5, \quad y=0.05, \quad c_2 = Rs.8, \quad v_1=0.30, \quad v_2=0.50, \quad R=0.06, \quad Ie=0.12, \quad Ip=0.15, \quad M=0.07 \) in appropriate units. The optimal value of \( t_0^* = 0.1394, \quad T^* =0.2845, \quad \text{Profit}^* = Rs. 19351.4866 \) and optimum order quantity \( Q^* = 144.0534 \).

Case III: Considering \( A=Rs.100, \quad a=500, \quad b=0.05, \quad c=Rs.25, \quad p=Rs.40, \quad \theta=0.05, \quad x=Rs.5, \quad y=0.05, \quad c_2 = Rs.8, \quad v_1=0.30, \quad v_2=0.50, \quad R=0.06, \quad Ie=0.12, \quad Ip=0.15, \quad M=0.10 \) in appropriate units. The optimal value of \( t_0^* = 0.1477, \quad T^* =0.2845, \quad \text{Profit}^* = Rs. 19406.1798 \) and optimum order quantity \( Q^* = 144.0534 \).

Case IV: Considering \( A=Rs.100, \quad a=500, \quad b=0.05, \quad c=Rs.25, \quad p=Rs.40, \quad \theta=0.05, \quad x=Rs.5, \quad y=0.05, \quad c_2 = Rs.8, \quad v_1=0.30, \quad v_2=0.50, \quad R=0.06, \quad Ie=0.12, \quad Ip=0.15, \quad M=0.37 \) in appropriate units. The optimal value of \( t_0^* = 0.2952, \quad T^* =0.3691, \quad \text{Profit}^* = Rs. 19617.6820 \) and optimum order quantity \( Q^* = 186.4403 \).

The second order conditions given in equation (34) are also satisfied.
<table>
<thead>
<tr>
<th>Case</th>
<th>$t_0$ and Profit</th>
<th>$T$ and Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case I</strong></td>
<td><img src="image1" alt="Graph 1" /></td>
<td><img src="image2" alt="Graph 2" /></td>
</tr>
<tr>
<td><strong>Case II</strong></td>
<td><img src="image3" alt="Graph 3" /></td>
<td><img src="image4" alt="Graph 4" /></td>
</tr>
<tr>
<td><strong>Case III</strong></td>
<td><img src="image5" alt="Graph 5" /></td>
<td><img src="image6" alt="Graph 6" /></td>
</tr>
</tbody>
</table>
V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

### Table 1
Sensitivity Analysis
Case IV: \(0 \leq t \leq \mu_1\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>(t_0)</th>
<th>(T)</th>
<th>Profit</th>
<th>(Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>+20%</td>
<td>0.1204</td>
<td>0.2614</td>
<td>23249.3671</td>
<td>157.8956</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1256</td>
<td>0.2729</td>
<td>21280.7234</td>
<td>151.1498</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1385</td>
<td>0.3013</td>
<td>17348.1761</td>
<td>136.6374</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1467</td>
<td>0.3194</td>
<td>15384.8028</td>
<td>128.8115</td>
</tr>
<tr>
<td>(x)</td>
<td>+20%</td>
<td>0.1222</td>
<td>0.2805</td>
<td>19299.3605</td>
<td>141.2599</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1267</td>
<td>0.2832</td>
<td>19306.2432</td>
<td>142.6311</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1368</td>
<td>0.2891</td>
<td>19321.4263</td>
<td>145.6284</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1425</td>
<td>0.2925</td>
<td>19329.8299</td>
<td>147.3563</td>
</tr>
<tr>
<td>(\theta)</td>
<td>+20%</td>
<td>0.1296</td>
<td>0.2849</td>
<td>19310.7420</td>
<td>143.5066</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1306</td>
<td>0.2854</td>
<td>19312.1531</td>
<td>143.7551</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1325</td>
<td>0.2866</td>
<td>19315.0303</td>
<td>144.3515</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1336</td>
<td>0.2872</td>
<td>19316.4970</td>
<td>144.6495</td>
</tr>
<tr>
<td>(A)</td>
<td>+20%</td>
<td>0.1438</td>
<td>0.3130</td>
<td>19246.8152</td>
<td>157.7622</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1378</td>
<td>0.2998</td>
<td>19279.4475</td>
<td>151.0577</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1250</td>
<td>0.2715</td>
<td>19349.4520</td>
<td>136.6991</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1180</td>
<td>0.2561</td>
<td>19387.3547</td>
<td>128.8943</td>
</tr>
<tr>
<td>(M)</td>
<td>+20%</td>
<td>0.1320</td>
<td>0.2860</td>
<td>19315.3561</td>
<td>144.0536</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1317</td>
<td>0.2860</td>
<td>19314.4677</td>
<td>144.0535</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1314</td>
<td>0.2860</td>
<td>19312.7004</td>
<td>144.0533</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1312</td>
<td>0.2861</td>
<td>19311.8216</td>
<td>144.0532</td>
</tr>
<tr>
<td>(R)</td>
<td>+20%</td>
<td>0.1257</td>
<td>0.2729</td>
<td>19279.8224</td>
<td>137.4090</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1285</td>
<td>0.2792</td>
<td>19296.5026</td>
<td>140.6038</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.1349</td>
<td>0.2934</td>
<td>19331.0918</td>
<td>147.8088</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.1384</td>
<td>0.3013</td>
<td>19349.0646</td>
<td>151.8193</td>
</tr>
</tbody>
</table>
### Table 2
Sensitivity Analysis
Case II: ($\mu_1 \leq t \leq \mu_2$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>$t_0$</th>
<th>$T$</th>
<th>Profit</th>
<th>$Q$</th>
</tr>
</thead>
</table>
| a         | +20%| 0.1281| 0.2596    | 23295.2841 | 156.8059  
|           | +10%| 0.1334| 0.2712    | 21322.6202 | 150.2064  
|           | -10%| 0.1463| 0.2999    | 17382.1165 | 136.0019  
|           | -20%| 0.1545| 0.3180    | 15414.8097 | 128.2464  
| x         | +20%| 0.1296| 0.2791    | 19335.4253 | 140.5537  
|           | +10%| 0.1343| 0.2817    | 19343.2031 | 141.8743  
|           | -10%| 0.1448| 0.2876    | 19360.3281 | 144.8721  
|           | -20%| 0.1508| 0.2909    | 19369.7873 | 146.5494  
| $\theta$  | +20%| 0.1373| 0.2834    | 19348.2818 | 142.7523  
|           | +10%| 0.1383| 0.2839    | 19349.8742 | 142.9990  
|           | -10%| 0.1404| 0.2851    | 19353.1196 | 143.5942  
|           | -20%| 0.1414| 0.2857    | 19354.7734 | 143.8913  
| A         | +20%| 0.1516| 0.3116    | 19284.3927 | 157.0558  
|           | +10%| 0.1456| 0.2984    | 19317.1770 | 150.3516  
|           | -10%| 0.1327| 0.2699    | 19387.5599 | 135.3516  
|           | -20%| 0.1257| 0.2544    | 19425.7044 | 128.0370  
| M         | +20%| 0.1411| 0.2838    | 19361.7668 | 142.9428  
|           | +10%| 0.1403| 0.2842    | 19356.5862 | 143.1452  
|           | -10%| 0.1384| 0.2848    | 19346.4677 | 143.4485  
|           | -20%| 0.1375| 0.2851    | 19341.5292 | 143.6001  
| R         | +20%| 0.1335| 0.2714    | 19317.9093 | 136.6527  
|           | +10%| 0.1363| 0.2778    | 19334.4994 | 139.8981  
|           | -10%| 0.1426| 0.2918    | 19368.9008 | 147.0012  
|           | -20%| 0.1461| 0.2996    | 19386.7758 | 150.9609  

### Table 3
Sensitivity Analysis
Case III: ($\mu_2 < t \leq t_0$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>$t_0$</th>
<th>$T$</th>
<th>Profit</th>
<th>$Q$</th>
</tr>
</thead>
</table>
| a         | +20%| 0.1362| 0.2542    | 23362.3415 | 153.5293  
|           | +10%| 0.1416| 0.2661    | 21383.4491 | 147.3686  
|           | -10%| 0.1548| 0.2952    | 17430.7703 | 133.8601  
|           | -20%| 0.1631| 0.3137    | 15457.5251 | 126.5036  
| x         | +20%| 0.1376| 0.2744    | 19387.7938 | 138.1753  
|           | +10%| 0.1424| 0.2769    | 19396.7039 | 139.4452  
|           | -10%| 0.1533| 0.2825    | 19416.2784 | 142.2908  
|           | -20%| 0.1595| 0.2858    | 19427.0649 | 143.9681  
| $\theta$  | +20%| 0.1456| 0.2785    | 19402.5089 | 140.2743  
|           | +10%| 0.1466| 0.2790    | 19404.3331 | 140.5204  
|           | -10%| 0.1488| 0.2801    | 19408.0494 | 141.0623  
|           | -20%| 0.1498| 0.2807    | 19409.9423 | 141.3588  
| A         | +20%| 0.1602| 0.3072    | 19338.0128 | 154.8272  
|           | +10%| 0.1541| 0.2937    | 19371.2956 | 147.9719  
|           | -10%| 0.1409| 0.2647    | 19442.9248 | 133.2618  
|           | -20%| 0.1338| 0.2488    | 19481.8687 | 125.2059  
| M         | +20%| 0.1506| 0.2766    | 19430.4202 | 139.2979  
|           | +10%| 0.1492| 0.2782    | 19418.1264 | 140.1081  
|           | -10%| 0.1461| 0.2808    | 19394.5749 | 141.4245  
|           | -20%| 0.1445| 0.2820    | 19383.3072 | 142.0320  
| R         | +20%| 0.1419| 0.2667    | 19373.1558 | 134.2756  
|           | +10%| 0.1447| 0.2729    | 19389.4730 | 137.4190  
|           | -10%| 0.1509| 0.2867    | 19423.3056 | 144.4192  
|           | -20%| 0.1544| 0.2945    | 19440.8837 | 148.3780  

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Table 4
Sensitivity Analysis
Case IV: (t₀ ≤ t ≤ T)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>t₀</th>
<th>T</th>
<th>Profit</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>+20%</td>
<td>0.2727</td>
<td>0.3369</td>
<td>23597.8677</td>
<td>204.0299</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.2832</td>
<td>0.3519</td>
<td>21607.1862</td>
<td>195.4350</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.3091</td>
<td>0.3890</td>
<td>17629.5307</td>
<td>176.9398</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.3254</td>
<td>0.4122</td>
<td>15642.9576</td>
<td>166.7668</td>
</tr>
<tr>
<td>x</td>
<td>+20%</td>
<td>0.2552</td>
<td>0.3438</td>
<td>19564.5728</td>
<td>173.5118</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.2736</td>
<td>0.3553</td>
<td>19589.6616</td>
<td>179.3853</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.3210</td>
<td>0.3860</td>
<td>19649.2316</td>
<td>195.0897</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.3524</td>
<td>0.4073</td>
<td>19685.1019</td>
<td>206.0062</td>
</tr>
<tr>
<td>θ</td>
<td>+20%</td>
<td>0.2857</td>
<td>0.3628</td>
<td>19606.2251</td>
<td>183.2867</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.2904</td>
<td>0.3659</td>
<td>19611.8824</td>
<td>184.8389</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.3003</td>
<td>0.3725</td>
<td>19623.6303</td>
<td>188.1417</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.3055</td>
<td>0.3760</td>
<td>19629.7349</td>
<td>189.8919</td>
</tr>
<tr>
<td>A</td>
<td>+20%</td>
<td>0.3197</td>
<td>0.4040</td>
<td>19565.9480</td>
<td>172.3170</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.3078</td>
<td>0.3870</td>
<td>19591.2313</td>
<td>195.5784</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.2820</td>
<td>0.3501</td>
<td>19645.4859</td>
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</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.2679</td>
<td>0.3301</td>
<td>19674.8842</td>
<td>166.5621</td>
</tr>
<tr>
<td>M</td>
<td>+20%</td>
<td>0.3001</td>
<td>0.3666</td>
<td>19648.0676</td>
<td>185.1745</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.2977</td>
<td>0.3679</td>
<td>19632.7856</td>
<td>185.8329</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.2927</td>
<td>0.3702</td>
<td>19602.7523</td>
<td>186.9968</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.2901</td>
<td>0.3711</td>
<td>19587.9958</td>
<td>187.4514</td>
</tr>
<tr>
<td>R</td>
<td>+20%</td>
<td>0.2757</td>
<td>0.3414</td>
<td>19574.3552</td>
<td>172.3170</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.2849</td>
<td>0.3545</td>
<td>19595.5905</td>
<td>178.9935</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.3069</td>
<td>0.3857</td>
<td>19640.7401</td>
<td>194.9144</td>
</tr>
<tr>
<td></td>
<td>-20%</td>
<td>0.3203</td>
<td>0.4046</td>
<td>19664.9006</td>
<td>204.5724</td>
</tr>
</tbody>
</table>

From the table we observe that as parameter a increases/ decreases, average total profit increases/ decreases for case I, case II, case III and case IV respectively.

From the table we observe that with increase/ decrease in parameter θ, there is almost no change in total profit for all the four cases.

Also, we observe that with increase and decrease in the value of x, A and R, there is corresponding decrease/increase in total profit for case I, case II, case III and case IV respectively.

From the table we observe that as parameters M increases/ decreases, average total profit increases/ decreases for case I and case II, case III and case IV respectively.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with different deterioration rates, exponential demand and shortages. We show that with the increase/ decrease in the parameter values there will be corresponding increase/decrease in the value of profit.

REFERENCES