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Inventory Model with Different Deterioration Rates under Exponential Demand, Shortages, Inflation and Permissible Delay in Payments

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ABSTRACT: *An inventory model with different deterioration rates under exponential demand with inflation and permissible delay in payments is developed. Holding cost is taken as linear function of time. Shortages are allowed. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for parameters.*

KEY WORDS: *Inventory model, Varying Deterioration, Exponential demand, Time varying holding cost, Shortages, Inflation, Permissible delay in payments*

I. INTRODUCTION

Inventory problems for deterioration items have been studied extensively by many researchers from time to time. Research in this area started with the work of Whitin [21], who considered fashion goods deteriorating at the end of prescribed storage period. Ghar and Schrader [5] developed an inventory model with a constant rate of deterioration. Shah and Jaiswal [14] considered an order level inventory model for items deteriorating at a constant rate. Alfares [2] developed an inventory model with a stock level demand rate and a variable holding cost with the assumption that holding cost increases with time spent in storage. The related work are found in (Nahmias [11], Raffat [13], Goyal and Giri [7]).

Buzacott [3] developed the first economic order quantity model by considering inflationary effects into account. Su et al. [17] developed model under inflation for stock dependent consumption rate and exponential decay. Moon et al. [10] developed models for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effects of inflation and time value of money. An inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions was developed by Liao et al. [8]. Singh [16] developed an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account.

In classical inventory model, it is assumed that a supplier must be paid for items as and when the customer receives the items. But in actual practice, many times the supplier allows credit for some fixed time period in settling the payment for the product and is not charged any interest from the customer for that specified period. But if he pays beyond that specified period, then the interest will be charged. Goyal [6] developed an EOQ model under the conditions of permissible delay in payments. Mandal and Phaujdar [9] extended this issue by considering the interest earned from the sales revenue. Aggarwal and Jaggi [1] extended Goyal's model to the case of deterioration. Chang [4] developed mathematical model to determine an optimal ordering policy under permissible delay of payment and/ or cash discount for the customer. Shah [15] derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. Tripathy and Misra [19] extended Jaggi and Aggrwal's model to allow for a time dependent demand rate. Teng et al. [18] developed EOQ model with trade credit financing for non-decreasing demand. Tripathy [20] developed an EOQ model for items with an exponential demand rate under permissible delay in payment and shortages. Patel and Patel [12] developed an eoq model with shortages, linear demand, time varying holding cost under permissible delay in payments.

Inventory models for non-instantaneous deteriorating items have been an object of study for a long time. Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model with different deterioration rates for the cycle time and exponential demand under time varying holding cost and permissible delay in payment. Shortages are allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. NOTATIONS AND ASSUMPTIONS

The following notations are used for the development of the model:

NOTATIONS:

D(t) : Demand is an exponential function of time t (ae^{bt}, a>0, 0<b<1)
A : Replenishment cost per order
c : Purchasing cost per unit
p : Selling price per unit
h(t) : x+yt, Inventory variable holding cost per unit excluding interest charges
c_2 : Shortage cost per unit
M : Permissible period of delay in settling the accounts with the supplier
T : Length of inventory cycle
I_e : Interest earned per year
I_p : Interest paid in stocks per year
R : Inflation rate
I(t) : Inventory level at any instant of time t, $0 \leq t \leq T$
Q : Order quantity
Q_1 : Order quantity initially
Q_2 : Quantity of shortages
θ : Deterioration rate during $\mu_1 \leq t \leq \mu_2$, $0< \theta<1$
θt : Deterioration rate during , $\mu_2 \leq t \leq t_0$, $0< \theta<1$
π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as an exponential function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- Deteriorated units neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \leq t \leq T$) as shown in figure.

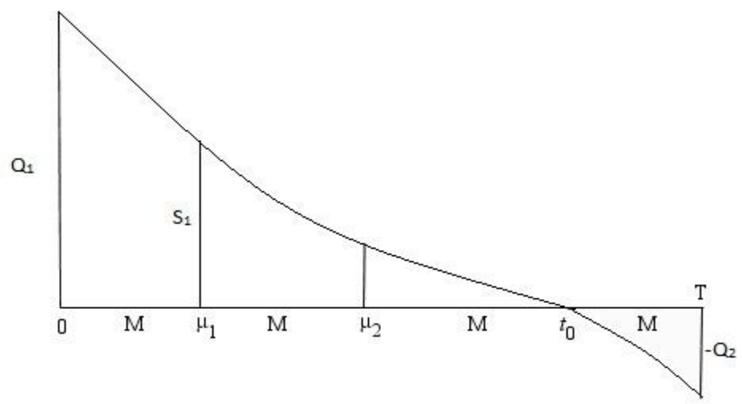


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) are given by :

$$\frac{dI(t)}{dt} = -ae^{bt}, \qquad 0 \leq t \leq \mu_1$$

(1)

$$\frac{dI(t)}{dt} + \theta I(t) = -ae^{bt}, \qquad \mu_1 \le t \le \mu_2$$
(2)

$$\frac{dI(t)}{dt} + \theta t I(t) = -ae^{bt}, \qquad \mu_2 \le t \le t_0$$
(3)

$$\frac{dI(t)}{dt} = -ae^{bt}, \qquad t_0 \le t \le T$$
(4)

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$, $I(T) = -Q_2$.
Solutions of these equations are given by:

$$I(t) = Q_1 - a(t + \frac{1}{2}bt^2),$$
(5)

$$I(t) = a\left(\mu_1 - t\right) + S_1\left[1 + \theta\left(\mu_1 - t\right)\right]$$
(6)

$$I(t) = a\left[\left(t_0 - t\right) + \frac{1}{2}b\left(t_0^2 - t^2\right) + \frac{1}{6}\theta\left(t_0^3 - t^3\right) - \frac{1}{2}\theta t^2\left(t_0 - t\right) - \frac{1}{4}b\theta t^2\left(t_0^2 - t^2\right)\right].$$
(7)

$$I(t) = a\left[\left(t_0 - t\right) + \frac{1}{2}b\left(t_0^2 - t^2\right)\right].$$
(8)

(by neglecting higher powers of θ)

From equation (5), putting $t = \mu_1$, we have

$$Q_1 = S_1 + a\left(\mu_1 + \frac{1}{2}b\mu_1^2\right).$$
(9)

From equations (6) and (7), putting $t = \mu_2$, we have

$$I(\mu_2) = a\left(\mu_1 - \mu_2\right) + S_1\left[1 + \theta\left(\mu_1 - \mu_2\right)\right] \qquad (10)$$

$$I(\mu_2) = a\left[\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{2}\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right)\right].$$
(11)

So from equations (10) and (11), we get

$$S_1 = \frac{a}{\left[1 + \theta\left(\mu_1 - \mu_2\right)\right]}\left[\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{2}\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) - \left(\mu_1 - \mu_2\right)\right]. \qquad (12)$$

Putting value of S_1 from equation (12) into equation (9), we have

$$Q_1 = \frac{a}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{2}\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) - \left(\mu_1 - \mu_2\right)\right] + \left(\mu_1 + \frac{1}{2}b\mu_1^2\right). \qquad (13)$$

Putting $t = T$ in equation (8), we have

$$Q_2 = a\left[\left(T - t_0\right) + \frac{1}{2}b\left(T^2 - t_0^2\right)\right]. \qquad (14)$$

Using (13) in (5), we have

$$I(t) = \frac{a}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\begin{bmatrix}\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}\theta\left(t_0^3 - \mu_2^3\right) \\ - \frac{1}{2}\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) - \left(\mu_1 - \mu_2\right)\end{bmatrix} + a\left[\left(\mu_1 - t\right) + \frac{1}{2}b\left(\mu_1^2 - t^2\right)\right] \qquad (15)$$

Using (12) in (6), we have

$$I(t)=\frac{a\left[1+\theta\left(\mu_1-t\right)\right]}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left[\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\left(\mu_1-\mu_2\right)\right]+a\left(\mu_1-t\right)$$

(16)

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A

(17)

(ii) Holding cost (HC) is given by

$$HC=\int_0^{t_0}(x+yt)I(t)e^{-Rt}dt$$

$$=\int_0^{\mu_1}(x+yt)I(t)e^{-Rt}dt+\int_{\mu_1}^{\mu_2}(x+yt)I(t)e^{-Rt}dt+\int_{\mu_2}^{t_0}(x+yt)I(t)e^{-Rt}dt$$

$$=\frac{1}{10}yRab\mu_1^5+\frac{1}{4}\left(-\frac{1}{2}(y-Rx)ab+yRa\right)\mu_1^4$$

$$+\frac{1}{3}\left(-\frac{1}{2}xab-(y-Rx)a-yR\left(\begin{array}{l}\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)\\+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\end{array}\right)\right)\mu_1^3$$

$$+\frac{1}{2}\left(-xa+(y-Rx)\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\right)\right)\mu_1^2$$

$$+x\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\right)\mu_1$$

$$-\frac{1}{4}yR\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\mu_2^4$$

$$+\frac{1}{3}\left(\begin{array}{l}(y-Rx)\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\\-yR\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_2^3$$

$$+\frac{1}{2}\left(\begin{array}{l}x\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\\+(y-Rx)\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_2^2$$

$$+\frac{1}{2}\left(\begin{array}{l}x\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a(1+\theta\mu_1)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\\+(y-Rx)\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_2^2$$

$$+x\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\mu_2$$

$$+\frac{1}{4}yR\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\mu_1^4$$

$$-\frac{1}{3}\left(\begin{array}{l}(y-Rx)\left(-\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\\-yR\left(\frac{1}{1+\theta\left(\mu_1-\mu_2\right)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_1^3$$

$$-\frac{1}{2}\left(\begin{array}{l} x\left(-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right) \\ +\left(y-Rx\right)\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_1^2$$

$$-x\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)a\mu_1$$

$$+a\left(\begin{array}{l} -\frac{1}{28}yRb\theta t_0^7+\frac{1}{6}\left(\frac{1}{4}\left(y-Rx\right)b\theta-\frac{1}{3}yR\theta\right)t_0^6+\frac{1}{5}\left(\frac{1}{4}xb\theta+\frac{1}{3}\left(y-Rx\right)\theta-yR\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)t_0^5 \\ +\frac{1}{4}\left(\frac{1}{3}x\theta+\left(y-Rx\right)\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)+yR\right)t_0^4+\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)+Rx-y \\ -yR\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\end{array}\right)t_0^3 \\ +\frac{1}{2}\left(-x+\left(y-Rx\right)\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)t_0^2+x\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)t_0\end{array}\right)$$

$$-a\left(\begin{array}{l} -\frac{1}{28}yRb\theta\mu_2^7+\frac{1}{6}\left(\frac{1}{4}\left(y-Rx\right)b\theta-\frac{1}{3}yR\theta\right)\mu_2^6+\frac{1}{5}\left(\frac{1}{4}xb\theta+\frac{1}{3}\left(y-Rx\right)\theta-yR\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)\mu_2^5 \\ +\frac{1}{4}\left(\frac{1}{3}x\theta+\left(y-Rx\right)\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)+yR\right)\mu_2^4+\frac{1}{3}\left(\begin{array}{l} x\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)+Rx-y \\ -yR\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\end{array}\right)\mu_2^3 \\ +\frac{1}{2}\left(-x+\left(y-Rx\right)\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)\mu_2^2+x\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\mu_2\end{array}\right) \qquad (18)$$

(by neglecting higher powers of θ and R)

(iii) Deterioration cost (DC) is given by

$$DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)e^{-Rt}dt+\int_{\mu_2}^{t_0}\theta tI(t)e^{-Rt}dt\right)$$

$$= c\theta\left(\begin{array}{l} -\frac{1}{3}R\left(-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\mu_2^3 \\ +\frac{1}{2}\left(\begin{array}{l} -\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a \\ -R\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_2^2 \\ +\frac{1}{1+\theta.(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\mu_2\right)+a\mu_1\mu_2\end{array}\right)$$

$$-c\theta\left(\begin{array}{l}-\frac{1}{3}R\left(-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\right)\mu_1^3\\ +\frac{1}{2}\left(\begin{array}{l}-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)-a\\ -R\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\mu_1\right)\end{array}\right)\mu_1^2\\ +\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(1+\theta\mu_1\right)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\mu_1\right)+a\mu_1^2\end{array}\right)$$

$$+c\theta a\left(\begin{array}{l}-\frac{1}{28}Rb\theta t_0^7+\frac{1}{6}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)t_0^6+\frac{1}{5}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)t_0^5\\ +\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2+R\right)t_0^4+\frac{1}{3}\left(-1-R\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)t_0^3+\frac{1}{2}\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)t_0^2\end{array}\right)$$

$$-c\theta a\left(\begin{array}{l}-\frac{1}{28}Rb\theta\mu_2^7+\frac{1}{6}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)\mu_2^6+\frac{1}{5}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)\mu_2^5\\ +\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2+R\right)\mu_2^4+\frac{1}{3}\left(-1-R\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)\mu_2^3+\frac{1}{2}\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\mu_2^2\end{array}\right) \quad (19)$$

(iv) $SC=-c_2\int_{t_0}^{T}I(t)e^{-Rt}dt$

$$=-c_2\left(\frac{1}{8}ab\left(T^4-t_0^4\right)+\frac{1}{3}\left(aR-\frac{1}{2}ab\right)\left(T^3-t_0^3\right)+\frac{1}{2}\left(-a\left(t_0+\frac{1}{2}bt_0^2\right)R-a\right)\left(T^2-t_0^2\right)+a\left(t_0+\frac{1}{2}bt_0^2\right)\left(T-t_0\right)\right) \quad (20)$$

(iv) $SR=p\left(\int_0^T ae^{bt}e^{-Rt}dt\right)=p\left(-\frac{1}{3}abRT^3+\frac{1}{2}(-aR+ab)T^2+aT\right)$ (21)

To determine the interest earned, there will be two cases i.e.
case I: ($0\le M\le T$) and case II: ($0\le T\le M$).

Case I: (0≤M≤T): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

(v) Interest earned per cycle:

$$IE_1=pI_e\int_0^M ae^{bt}te^{-Rt}dt$$

$$=pI_e\left[-\frac{1}{4}abRM^4+\frac{1}{3}\left(-aR+ab\right)M^3+\frac{1}{2}aM^2\right]$$

(22)

Case II: (0 ≤T ≤ M):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vi) Interest earned up to the permissible delay period is:

$$IE_2=pI_e\left[\int_0^{t_0}ae^{bt}te^{-Rt}dt+a\left(1+bt_0\right)t_0\left(M-t_0\right)\right]$$

$$=pI_e\left[-\frac{1}{4}abRt_0^4+\frac{1}{3}\left(-aR+ab\right)t_0^3+\frac{1}{2}at_0^2+a\left(1+bt_0\right)t_0\left(M-t_0\right)\right]$$

(23)

To determine the interest payable, there will be four cases i.e.

Case I: (0≤M≤μ_1):

(vii) Interest payable per cycle for the inventory not sold after the due period M is

$$IP_1 = cI_p \int_M^T I(t)e^{-Rt}dt$$

$$= cI_p \left(\int_M^{\mu_1} I(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2} I(t)e^{-Rt}dt + \int_{\mu_2}^{T} I(t)e^{-Rt}dt \right)$$

$$= cI_p \left\{ \begin{array}{l} \frac{1}{8}Rab\mu_1^4 + \frac{1}{3}\left(-\frac{1}{2}ab + Ra\right)\mu_1^3 \\ + \frac{1}{2}\left(-a-R\left(\frac{1}{1+\theta.(\mu_1-\mu_2)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\right)\right)\mu_1^2 \\ + \frac{1}{1+\theta.(\mu_1-\mu_2)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\mu_1\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\mu_1 \end{array} \right\}$$

$$- cI_p \left\{ \begin{array}{l} \frac{1}{8}RabM^4 + \frac{1}{3}\left(-\frac{1}{2}ab + Ra\right)M^3 \\ + \frac{1}{2}\left(-a-R\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)\right)\right)M^2 \\ + \frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)M\right)+a\left(\mu_1+\frac{1}{2}b\mu_1^2\right)M \end{array} \right\}$$

$$+ cI_p \left\{ \begin{array}{l} -\frac{1}{3}R\left(-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)\right)\mu_2^3 \\ + \frac{1}{2}\left\{ \begin{array}{l} -a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right) \\ -R\left(a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)\right) \end{array} \right\}\mu_2^2 \\ +a\mu_1\mu_2+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\mu_2\right) \end{array} \right\}$$

$$- cI_p \left\{ \begin{array}{l} -\frac{1}{3}R\left(-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)\right)\mu_1^3 \\ + \frac{1}{2}\left\{ \begin{array}{l} -a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right) \\ -R\left(a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right)\right) \end{array} \right\}\mu_1^2 \\ +a\mu_1^2+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\mu_1\left(t_0+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{2}\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-\mu_1\right)\right) \end{array} \right\}$$

$$+ cI_p a \left\{ \begin{array}{l} -\frac{1}{24}Rb\theta t_0^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)t_0^5 + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta t_0 - \frac{1}{4}b\theta t_0^2\right)\right)t_0^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta t_0 - \frac{1}{4}b\theta t_0^2 + R\right)t_0^3 + \frac{1}{2}\left(-1-R\left(t_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}\theta t_0^3\right)\right)t_0^2 + t_0^2 + \frac{1}{2}bt_0^3 + \frac{1}{6}\theta t_0^4 \end{array} \right\}$$

$$- cI_p a \left\{ \begin{array}{l} -\frac{1}{24}Rb\theta\mu_2^6 + \frac{1}{5}\left(\frac{1}{4}b\theta - \frac{1}{3}R\theta\right)\mu_2^5 + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(-\frac{1}{2}b - \frac{1}{2}\theta t_0 - \frac{1}{4}b\theta t_0^2\right)\right)\mu_2^4 \\ + \frac{1}{3}\left(-\frac{1}{2}b - \frac{1}{2}\theta t_0 - \frac{1}{4}b\theta t_0^2 + R\right)\mu_2^3 + \frac{1}{2}\left(-1-R\left(t_0 + \frac{1}{2}bt_0^2 + \frac{1}{6}\theta t_0^3\right)\right)\mu_2^2 + t_0\mu_2 + \frac{1}{2}bt_0^2\mu_2 + \frac{1}{6}\theta t_0^3\mu_2 \end{array} \right\} \qquad (24)$$

Case II: ($\mu_1 \le M \le \mu_2$):

(viii) $IP_2 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt$

$$= cI_p\left(\int_M^{\mu_2} I(t)e^{-Rt}dt + \int_{\mu_2}^{t_0} I(t)e^{-Rt}dt\right)$$

$$= cI_p\left(\begin{array}{l} -\frac{1}{3}R\left(-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\right)\mu_2^3 \\ +\frac{1}{2}\left(\begin{array}{l} -a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right) \\ -R\left(a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\right)\end{array}\right)\mu_2^2 \\ a\mu_1\mu_2+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\mu_2\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\end{array}\right)$$

$$-cI_p\left(-\frac{1}{3}R\left(-a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\right)M^3\right)$$

$$-cI_p\left(\begin{array}{l}\frac{1}{2}\left(\begin{array}{l} -a-\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a\theta\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right) \\ -R\left(a\mu_1+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\right)\end{array}\right)M^2 \\ +a\mu_1M+\frac{1}{1+\theta(\mu_1-\mu_2)}\left(a(1+\theta\mu_1)M\left(t_0+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}\theta(t_0^3-\mu_2^3)-\frac{1}{2}\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2)-\mu_1\right)\right)\end{array}\right)$$

$$+cI_pa\left(\begin{array}{l}-\frac{1}{24}Rb\theta t_0^6+\frac{1}{5}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)t_0^5+\frac{1}{4}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)t_0^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2+R\right)t_0^3+\frac{1}{2}\left(-1-R\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)t_0^2+t_0^2+\frac{1}{2}bt_0^3+\frac{1}{6}\theta t_0^4\end{array}\right)$$

$$-cI_pa\left(\begin{array}{l}-\frac{1}{24}Rb\theta\mu_2^6+\frac{1}{5}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)\mu_2^5+\frac{1}{4}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)\mu_2^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2+R\right)\mu_2^3+\frac{1}{2}\left(-1-R\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)\mu_2^2+t_0\mu_2+\frac{1}{2}bt_0^2\mu_2+\frac{1}{6}\theta t_0^3\mu_2\end{array}\right)$$

(25)

Case III: ($\mu_2 \le M \le t_0$):

(ix) $IP_3 = cI_p \int_M^{t_0} I(t)e^{-Rt}dt$

$$+cI_pa\left(\begin{array}{l}-\frac{1}{24}Rb\theta t_0^6+\frac{1}{5}\left(\frac{1}{4}b\theta-\frac{1}{3}R\theta\right)t_0^5+\frac{1}{4}\left(\frac{1}{3}\theta-R\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2\right)\right)t_0^4 \\ +\frac{1}{3}\left(-\frac{1}{2}b-\frac{1}{2}\theta t_0-\frac{1}{4}b\theta t_0^2+R\right)t_0^3+\frac{1}{2}\left(-1-R\left(t_0+\frac{1}{2}bt_0^2+\frac{1}{6}\theta t_0^3\right)\right)t_0^2+t_0^2+\frac{1}{2}bt_0^3+\frac{1}{6}\theta t_0^4\end{array}\right)$$

$$- cI_p a \left(\begin{array}{l} - \frac{1}{24} Rb\theta M^6 + \frac{1}{5}\left(\frac{1}{4} b\theta - \frac{1}{3} R\theta \right) M^5 + \frac{1}{4}\left(\frac{1}{3}\theta - R\left(- \frac{1}{2} b - \frac{1}{2}\theta t_0 - \frac{1}{4} b\theta t_0^2 \right) \right) M^4 \\ + \frac{1}{3}\left(- \frac{1}{2} b - \frac{1}{2}\theta t_0 - \frac{1}{4} b\theta t_0^2 + R \right) M^3 + \frac{1}{2}\left(-1 - R\left(t_0 + \frac{1}{2} bt_0^2 + \frac{1}{6}\theta t_0^3 \right) \right) M^2 + t_0 M + \frac{1}{2} bt_0^2 M + \frac{1}{6}\theta t_0^3 M \end{array} \right)$$

(26)

Case IV: ($t_0 \leq M \leq T$):

(x) $IP_4 = 0$

(27)

The total profit (π_i), i=1,2,3 and 4 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}[SR - OC - HC - DC - SC - IP_i + IE_i] \quad (28)$$

Substituting values from equations (17) to (27) in equation (28), we get total profit per unit. Putting $\mu_1 = v_1 t_0$ and $\mu_2 = v_2 t_0$ in equation (28), we get profit in terms of t_0 and T for the four cases will be as under:

$$\pi_1 = \frac{1}{T}[SR - OC - HC - DC - SC - IP_1 + IE_1]$$

(29)

$$\pi_2 = \frac{1}{T}[SR - OC - HC - DC - SC - IP_2 + IE_1]$$

(30)

$$\pi_3 = \frac{1}{T}[SR - OC - HC - DC - SC - IP_3 + IE_1]$$

(31)

$$\pi_4 = \frac{1}{T}[SR - OC - HC - DC - SC - IP_4 + IE_2]$$

(32)

The optimal value of t_0* and T* (say), which maximizes π_i can be obtained by solving equation (29), (30), (31) and (32) by differentiating it with respect to t_0 and T and equate it to zero, we have

i.e. $\frac{\partial \pi_i(t_0, T)}{\partial t_0} = 0, \quad \frac{\partial \pi_i(t_0, T)}{\partial T} = 0, \quad i=1,2,3,4$

(33)

provided it satisfies the condition

$$\frac{\partial^2 \pi_i(t_0, T)}{\partial t_0^2} < 0, \quad \frac{\partial^2 \pi_i(t_0, T)}{\partial T^2} < 0, \quad \text{and} \quad \left[\frac{\partial^2 \pi_i(t_0, T)}{\partial t_0^2} \right]\left[\frac{\partial^2 \pi_i(t_0, T)}{\partial T^2} \right] - \left[\frac{\partial^2 \pi_i(t_0, T)}{\partial t_0 \partial T} \right]^2 > 0, \quad i=1,2,3,4.$$

(34)

IV. NUMERICAL EXAMPLES

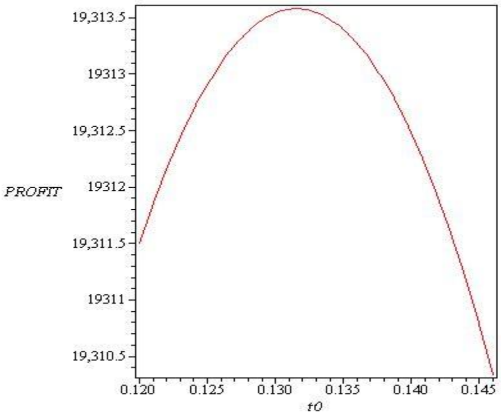
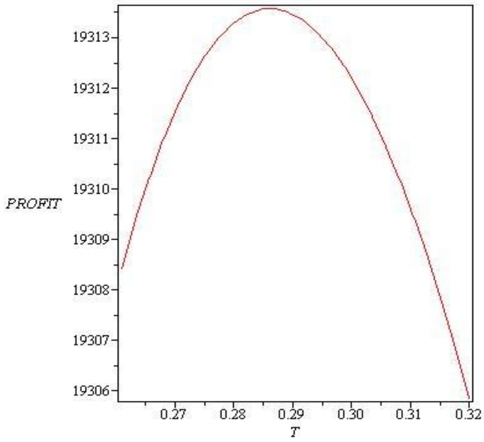
Case I: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, c_2 = Rs. 8, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.01 in appropriate units. The optimal value of t_0* = 0.1316, T* =0.2860, Profit*= Rs. 19313.5824 and optimum order quantity Q* = 144.0534.

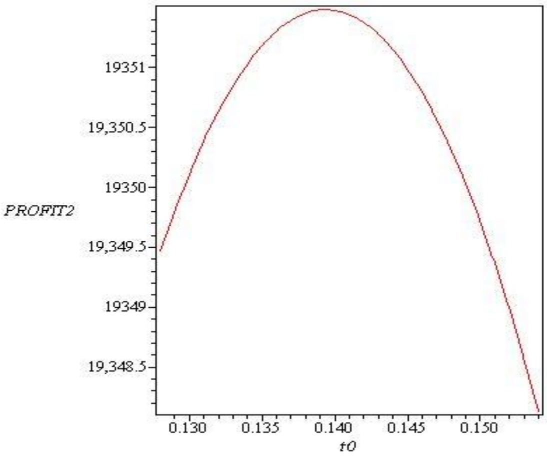
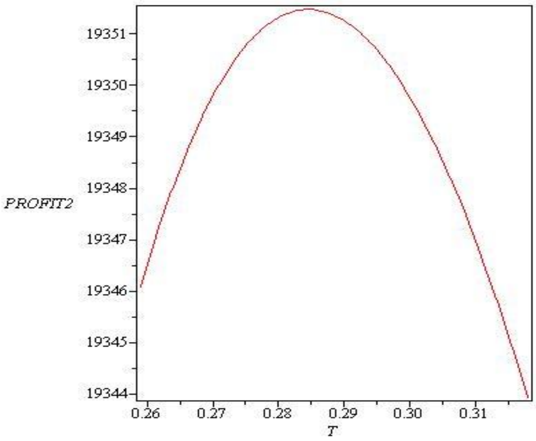
Case II: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, c_2 = Rs. 8, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.07 in appropriate units. The optimal value of t_0* = 0.1394, T* = 0.2845, Profit*= Rs. 19351.4866 and optimum order quantity Q* = 143.2969.

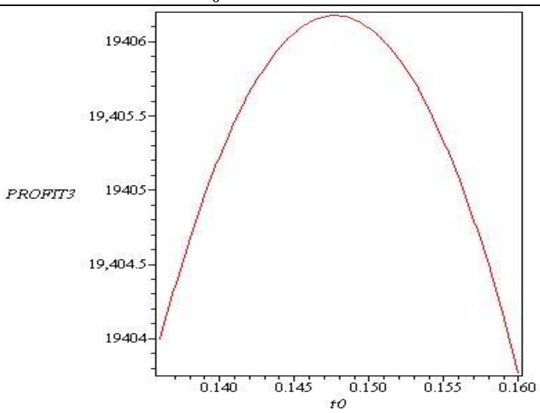
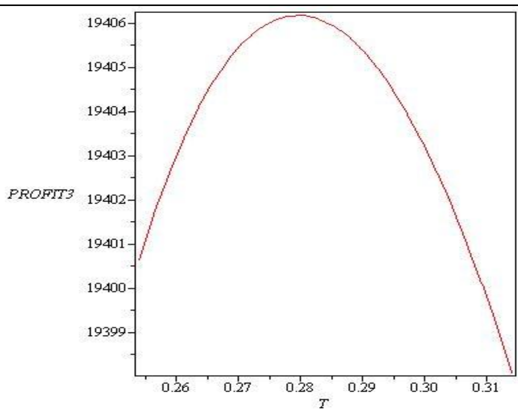
Case III: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, c_2 = Rs. 8, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.10 in appropriate units. The optimal value of t_0* = 0.1477, T* =0.2796, Profit*= Rs. 19406.1798 and optimum order quantity Q* = 140.8170.

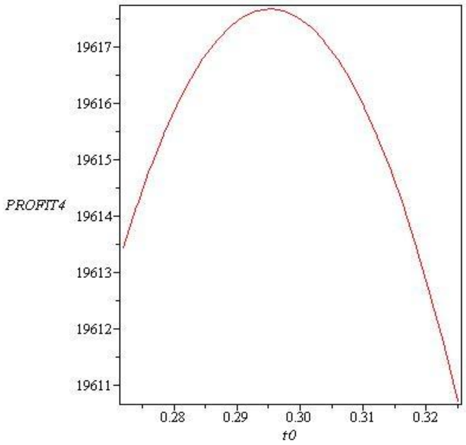
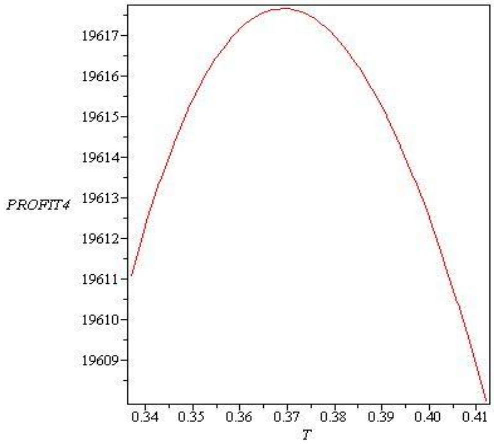
Case IV: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, θ=0.05, x = Rs. 5, y=0.05, c_2 = Rs. 8, v_1=0.30, v_2=0.50, R = 0.06, Ie = 0.12, Ip = 0.15, M = 0.37 in appropriate units. The optimal value of t_0* = 0.2952, T* =0.3691, Profit*= Rs. 19617.6820 and optimum order quantity Q* = 186.4403.

The second order conditions given in equation (34) are also satisfied.

<table>
<tr><td colspan="2">Case I</td></tr>
<tr><td>t_0 and Profit</td><td>T and Profit</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Graph 1</td><td>Graph 2</td></tr>
</table>

<table>
<tr><td colspan="2">Case II</td></tr>
<tr><td>t_0 and Profit</td><td>T and Profit</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Graph 3</td><td>Graph 4</td></tr>
</table>

<table>
<tr><td colspan="2">Case III</td></tr>
<tr><td>t_0 and Profit</td><td>T and Profit</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Graph 5</td><td>Graph 6</td></tr>
</table>

<table>
<tr><td colspan="2">Case IV</td></tr>
<tr><td>t_0 and Profit</td><td>T and Profit</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Graph 7</td><td>Graph 8</td></tr>
</table>

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis
Case I: ($0 \leq t \leq \mu_1$)

Parameter	%	t_0	T	Profit	Q
a	+20%	0.1204	0.2614	23249.3671	157.8956
	+10%	0.1256	0.2729	21280.7234	151.1498
	-10%	0.1385	0.3013	17348.1761	136.6374
	-20%	0.1467	0.3194	15384.8028	128.8115
x	+20%	0.1222	0.2805	19299.3605	141.2599
	+10%	0.1267	0.2832	19306.2432	142.6311
	-10%	0.1368	0.2891	19321.4263	145.6284
	-20%	0.1425	0.2925	19329.8299	147.3563
θ	+20%	0.1296	0.2849	19310.7420	143.5066
	+10%	0.1306	0.2854	19312.1531	143.7551
	-10%	0.1325	0.2866	19315.0303	144.3515
	-20%	0.1336	0.2872	19316.4970	144.6495
A	+20%	0.1438	0.3130	19246.8152	157.7622
	+10%	0.1378	0.2998	19279.4475	151.0577
	-10%	0.1250	0.2715	19349.4520	136.6991
	-20%	0.1180	0.2561	19387.3547	128.8943
M	+20%	0.1320	0.2860	19315.3561	144.0536
	+10%	0.1317	0.2860	19314.4677	144.0535
	-10%	0.1314	0.2860	19312.7004	144.0533
	-20%	0.1312	0.2861	19311.8216	144.0532
R	+20%	0.1257	0.2729	19279.8224	137.4090
	+10%	0.1285	0.2792	19296.5026	140.6038
	-10%	0.1349	0.2934	19331.0918	147.8088
	-20%	0.1384	0.3013	19349.0646	151.8193

Table 2
Sensitivity Analysis
Case II: ($\mu_1 \leq t \leq \mu_2$)

Parameter	%	t_0	T	Profit	Q
a	+20%	0.1281	0.2596	23295.2841	156.8059
	+10%	0.1334	0.2712	21322.6202	150.2064
	-10%	0.1463	0.2999	17382.1165	136.0019
	-20%	0.1545	0.3180	15414.8097	128.2464
x	+20%	0.1296	0.2791	19335.4253	140.5537
	+10%	0.1343	0.2817	19343.2031	141.8743
	-10%	0.1448	0.2876	19360.3281	144.8721
	-20%	0.1508	0.2909	19369.7873	146.5494
θ	+20%	0.1373	0.2834	19348.2818	142.7523
	+10%	0.1383	0.2839	19349.8742	142.9990
	-10%	0.1404	0.2851	19353.1196	143.5942
	-20%	0.1414	0.2857	19354.7734	143.8913
A	+20%	0.1516	0.3116	19284.3927	157.0558
	+10%	0.1456	0.2984	19317.1770	150.3516
	-10%	0.1327	0.2699	19387.5599	135.3516
	-20%	0.1257	0.2544	19425.7044	128.0370
M	+20%	0.1411	0.2838	19361.7668	142.9428
	+10%	0.1403	0.2842	19356.5862	143.1452
	-10%	0.1384	0.2848	19346.4677	143.4485
	-20%	0.1375	0.2851	19341.5292	143.6001
R	+20%	0.1335	0.2714	19317.9093	136.6527
	+10%	0.1363	0.2778	19334.4994	139.8981
	-10%	0.1426	0.2918	19368.9008	147.0012
	-20%	0.1461	0.2996	19386.7758	150.9609

Table 3
Sensitivity Analysis
Case III: ($\mu_2 \leq t \leq t_0$)

Parameter	%	t_0	T	Profit	Q
a	+20%	0.1362	0.2542	23362.3415	153.5293
	+10%	0.1416	0.2661	21383.4491	147.3686
	-10%	0.1548	0.2952	17430.7703	133.8601
	-20%	0.1631	0.3137	15457.5251	126.5036
x	+20%	0.1376	0.2744	19387.7938	138.1753
	+10%	0.1424	0.2769	19396.7039	139.4452
	-10%	0.1533	0.2825	19416.2784	142.2908
	-20%	0.1595	0.2858	19427.0649	143.9681
θ	+20%	0.1456	0.2785	19402.5089	140.2743
	+10%	0.1466	0.2790	19404.3331	140.5204
	-10%	0.1488	0.2801	19408.0494	141.0623
	-20%	0.1498	0.2807	19409.9423	141.3588
A	+20%	0.1602	0.3072	19338.0128	154.8272
	+10%	0.1541	0.2937	19371.2956	147.9719
	-10%	0.1409	0.2647	19442.9248	133.2618
	-20%	0.1338	0.2488	19481.8687	125.2059
M	+20%	0.1506	0.2766	19430.4202	139.2979
	+10%	0.1492	0.2782	19418.1264	140.1081
	-10%	0.1461	0.2808	19394.5749	141.4245
	-20%	0.1445	0.2820	19383.3072	142.0320
R	+20%	0.1419	0.2667	19373.1558	134.2756
	+10%	0.1447	0.2729	19389.4730	137.4190
	-10%	0.1509	0.2867	19423.3056	144.4192
	-20%	0.1544	0.2945	19440.8837	148.3780

Table 4
Sensitivity Analysis
Case IV: ($t_0 \leq t \leq T$)

Parameter	%	t_0	T	Profit	Q
a	+20%	0.2727	0.3369	23597.8677	204.0299
	+10%	0.2832	0.3519	21607.1862	195.4350
	-10%	0.3091	0.3890	17629.5307	176.9398
	-20%	0.3254	0.4122	15642.9576	166.7668
x	+20%	0.2552	0.3438	19564.5728	173.5118
	+10%	0.2736	0.3553	19589.6616	179.3853
	-10%	0.3210	0.3860	19649.2316	195.0897
	-20%	0.3524	0.4073	19685.1019	206.0062
θ	+20%	0.2857	0.3628	19606.2251	183.2867
	+10%	0.2904	0.3659	19611.8824	184.8389
	-10%	0.3003	0.3725	19623.6303	188.1417
	-20%	0.3055	0.3760	19629.7349	189.8919
A	+20%	0.3197	0.4040	19565.9480	204.2654
	+10%	0.3078	0.3870	19591.2313	195.5784
	-10%	0.2820	0.3501	19645.4859	176.7508
	-20%	0.2679	0.3301	19674.8842	166.5621
M	+20%	0.3001	0.3666	19648.0676	185.1745
	+10%	0.2977	0.3679	19632.7856	185.8329
	-10%	0.2927	0.3702	19602.7523	186.9968
	-20%	0.2901	0.3711	19587.9958	187.4514
R	+20%	0.2757	0.3414	19574.3552	172.3170
	+10%	0.2849	0.3545	19595.5905	178.9935
	-10%	0.3069	0.3857	19640.7401	194.9144
	-20%	0.3203	0.4046	19664.9006	204.5724

From the table we observe that as parameter a increases/ decreases, average total profit increases/ decreases for case I, case II, case III and case IV respectively.
From the table we observe that with increase/ decrease in parameter θ, there is almost no change in total profit for all the four cases.
Also, we observe that with increase and decrease in the value of x, A and R, there is corresponding decrease/ increase in total profit for case I, case II, case III and case IV respectively.
From the table we observe that as parameters M increases/ decreases, average total profit increases/ decreases for case I and case II, case III and case IV respectively.

VI. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with different deterioration rates, exponential demand and shortages. We show that with the increase/ decrease in the parameter values there will be corresponding increase/ decrease in the value of profit.

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