

Back in Action: How Spare Parts Keep Broken Factories Moving

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Abstract

Unplanned machine failure converts directly and immediately into lost production, and the size of the loss depends on two decisions that are often made separately: how many spare units to hold and how many repairers to employ. This article treats both decisions inside one stochastic model. The machine repair problem is formulated as a birth-death Markov chain with M required machines, S cold spares, and R repairers, and its steady-state distribution is obtained from the cut equations. Expected operating machines, production availability, repairer utilization, and mean time in the repair system follow from that distribution and from Little's law. The analysis then moves from whole machines to repairable components. Under a one-for-one base-stock policy, Palm's theorem implies that the number of units in the repair pipeline is Poisson distributed, which yields closed-form fill rates and expected backorders. A marginal-analysis rule extends the result to many items under a budget. Later sections cover repair systems in which failed machines abandon the queue, forecasting of intermittent demand with Croston's method and its bias correction, and a cost model that compares spares with repairers. A worked example with assumed parameters, three required machines and a failure-to-repair ratio of 0.2, shows that one spare machine raises availability from 78.4 percent to 88.5 percent with a single repairer. In that setting the spare adds more than twice the output gained from a second repairer.

Keywords: spare parts inventory, machine repair model, Markov chain, M/M/R queue, base-stock policy, availability

I. Introduction

1.1 Background

Consider a plant in which three presses must run throughout a shift. One press stops because a bearing has failed. Management then faces a set of linked questions. How long until a replacement bearing arrives? Is a technician free to fit it? Can the line meet its target on two presses in the meantime?

These questions describe a queue. Failures arrive at random, repairs take random time, and failed machines wait when the repair crew is occupied. Queueing theory has treated this situation since the early twentieth century (Palm, 1938), and the finite-source models that result are standard textbook material (Cooper, 1981; Gross, Shortle, Thompson, & Harris, 2008; Kleinrock, 1975; Taha, 2011). Their data requirements are modest: a failure rate, a repair rate, and a few lines of algebra.

Maintenance practice often relies on a rule of thumb: hold enough spares. The rule is hard to apply because "enough" is never defined. The models below define it as the point where the next spare no longer buys a reduction in expected downtime that justifies its holding cost.

1.2 Two Forms of Spares

Spares appear in two forms. A spare machine is a complete unit that takes over when an operating unit fails. A spare component, such as a motor, gearbox, or control board, is fitted into a failed machine so that the machine can restart. The machine repair literature addresses the first form (Gross et al., 2008; Taha, 2011). The inventory literature addresses the second (Feeney & Sherbrooke, 1966; Sherbrooke, 1968). Both lead to Markov models with state-dependent rates, and comparing them is instructive.

Two studies by Baghel bear directly on the repair side of the problem. Baghel (2013) compared generalist and specialist repair crews within an M/M/R framework. Baghel (2014) modeled systems in which failed machines abandon the repair queue while spares are limited. This article does not reproduce their results.

II. The Machine Repair Model with Spares

2.1 Model Formulation

The system contains M machines that must operate, together with S spare machines held in reserve, so $N = M + S$ machines exist in total. Each operating machine fails after an exponential time with rate λ . Spares are cold, meaning they do not fail while idle. Failed machines enter a repair facility with R identical repairers, each

serving at exponential rate μ . A repaired machine takes an open production position or, if none is open, joins the spare pool.

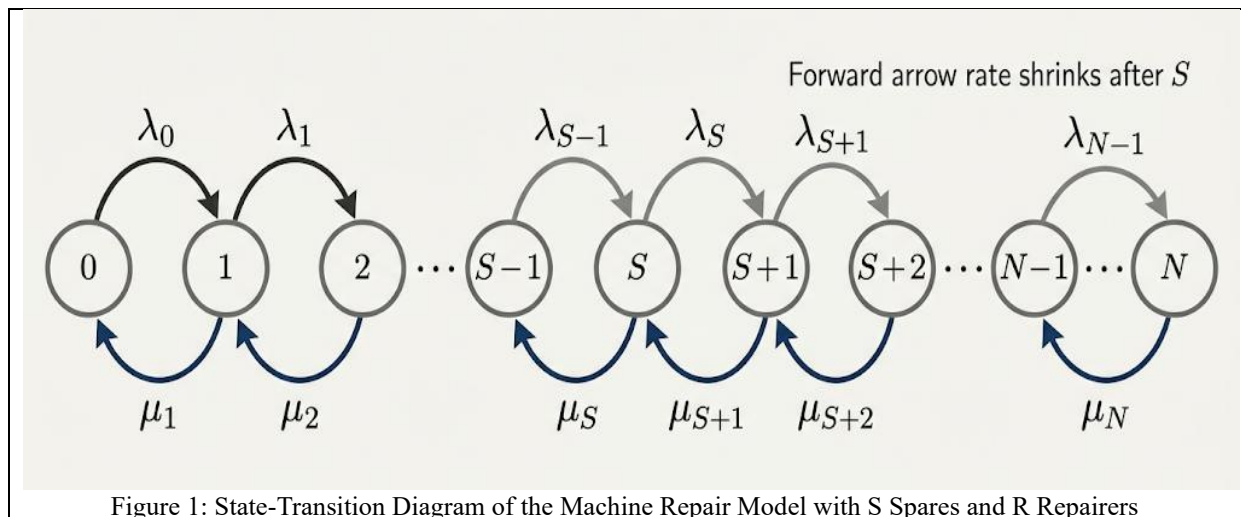
Let n denote the number of machines in the repair facility, whether in service or waiting, so that $n \in \{0, 1, \dots, N\}$. The number of operating machines is $\min(M, N - n)$. This gives the state-dependent failure rate

$$\lambda_n = M\lambda \text{ for } 0 \leq n \leq S, \text{ and } \lambda_n = (N - n)\lambda \text{ for } S \leq n \leq N. \dots (1)$$

While $n \leq S$, spares fill every vacancy and all M positions run. Beyond that point each additional failure removes one operating machine, so the failure rate falls by λ per failed unit. The repair rate is

$$\mu_n = n\mu \text{ for } 1 \leq n \leq R, \text{ and } \mu_n = R\mu \text{ for } n \geq R. \dots (2)$$

The resulting process is a birth-death chain, drawn in Figure 1. A failure moves the state up one step, and a repair moves it down one step.



States 0 through N appear as circles in a row. Forward arrows from state n to $n + 1$ carry the failure rate λ_n of Equation (1), and backward arrows from $n + 1$ to n carry the repair rate $\mu_{(n+1)}$ of Equation (2). The forward arrows stay equal up to state S , where spares absorb failures, and then shrink by λ at each step as the plant loses capacity. This is a conceptual diagram of the model in Gross et al. (2008) and Taha (2011), and it contains no empirical data.

2.2 Steady-State Solution

A birth-death chain in steady state satisfies the cut equations. The probability flow upward across the boundary between n and $n + 1$ equals the flow downward:

$$\lambda_n p_n = \mu_{(n+1)} p_{(n+1)} \dots (3)$$

Solving recursively from p_0 gives

$$p_n = p_0 \prod_{i=0}^{n-1} (\lambda_i / \mu_{(i+1)}) \dots (4)$$

and the normalization condition $\sum p_n = 1$ fixes the constant:

$$p_0 = [1 + \sum_{n=1}^N \prod_{i=0}^{n-1} (\lambda_i / \mu_{(i+1)})]^{(-1)} \dots (5)$$

A special case deserves attention. With no spares ($S = 0$) and one repairer ($R = 1$), Equation (4) reduces to

$$p_n = p_0 \cdot [M! / (M - n)!] \cdot \rho^n, \text{ where } \rho = \lambda / \mu. \dots (6)$$

This is the classical machine interference model, treated in Cooper (1981) and Kleinrock (1975).

2.3 Performance Measures

In state n the number of operating machines is $\min(M, N - n)$. The expected number of operating machines is therefore

$$E[O] = \sum_{n=0}^N \min(M, N - n) p_n \dots (7)$$

Production availability, the fraction of required positions that are actually running, is

$$A = E[O]/M \dots (8)$$

The mean number of machines in the repair facility is $L = \sum n p_n$. The long-run rate at which machines fail, which equals the rate at which they complete repair, is

$$\gamma = \sum \lambda_n p_n = \sum \mu_n p_n \dots (9)$$

Little's law (Little, 1961) connects these quantities:

$$W = L/\gamma \dots (10)$$

where W is the mean time a failed machine spends in the repair facility, waiting and in service. Equation (1) shows that $\lambda_n = \lambda \cdot \min(M, N - n)$, so $\gamma = \lambda E[O]$. Two consequences follow. First, $W = L / (\lambda E[O])$. Second, repairer utilization, defined as

$$U = (1/R) \sum_{n=0}^N \min(n, R) p_n \dots (11)$$

satisfies $U = \rho E[O]/R$, because $\sum \mu_n p_n = \mu R \cdot U$. The identity $\gamma = \lambda E[O] = \mu R U$ also serves as an arithmetic check, and Section 6 uses it for every case.

2.4 The Production-Loss Threshold

Production suffers only when $n > S$. The probability of production loss is

$$P_{loss} = \sum_{n=S+1}^N p_n \dots (12)$$

Adding a spare moves the threshold from S to $S + 1$. For $M = 3$, $R = 1$, and $\rho = 0.2$, Section 6 shows $P_{loss} = 0.470$ without a spare and 0.250 with one. The reduction is 0.220 for the first spare alone.

III. Spare Components and Base-Stock Inventory

3.1 The One-for-One Policy

A repairable component fails inside a machine. The technician fits a spare from stock, and the failed unit goes to a repair shop. On return, it rejoins the stock. This is a one-for-one, or $(S - 1, S)$, policy, and it suits expensive, slow-moving, repairable parts (Feeney & Sherbrooke, 1966; Sherbrooke, 2004).

Let s be the base-stock level: serviceable units on the shelf, plus units in repair, plus units in transit, held constant. Demand for the part arrives at rate λ_p , and the mean repair turnaround time is T . The subscript distinguishes part demand from the machine failure rate λ of Section 2. Let X denote the number of units in the repair pipeline at a random moment.

3.2 Palm's Theorem

Palm (1938) established a result that makes the theory tractable. If demands follow a Poisson process and repair times are independent with mean T , and if the repair shop has enough capacity that units never queue for service, then X is Poisson distributed in steady state with mean

$$\theta = \lambda_p T \dots (13)$$

The pipeline behaves as an $M/G/\infty$ queue, so the result depends on the repair-time distribution only through its mean. Hence

$$P(X = x) = e^{(-\theta)} \theta^x / x!, x = 0, 1, 2, \dots (14)$$

3.3 Fill Rate and Expected Backorders

A demand finds the shelf empty when all s units are in the pipeline, that is, when $X \geq s$. By PASTA (Poisson arrivals see time averages), the fill rate is

$$FR(s) = P(X \leq s - 1) = \sum_{x=0}^{s-1} e^{(-\theta)} \theta^x / x! \dots (15)$$

Expected backorders are

$$EBO(s) = \sum_{x=s+1}^{\infty} (x - s) P(X = x) \dots (16)$$

Taking the difference $EBO(s - 1) - EBO(s)$ gives the recursion

$$EBO(s) = EBO(s - 1) - P(X \geq s), \text{ with } EBO(0) = \theta \dots (17)$$

Sherbrooke (1968) adopted expected backorders as the objective of the METRIC model for multi-echelon recoverable items. If every backordered unit corresponds to one stopped machine, EBO(s) is the expected number of machines down because of that part.

3.4 A Numerical Illustration

Suppose $\lambda_p = 0.5$ demands per day and $T = 4$ days, so that $\theta = 2$. These are assumed values for illustration, not field data. Equation (14) gives $P(X = 0) = 0.1353$, $P(X = 1) = 0.2707$, $P(X = 2) = 0.2707$, $P(X = 3) = 0.1804$, $P(X = 4) = 0.0902$, and $P(X = 5) = 0.0361$. Equation (15) then gives fill rates of 0.135, 0.406, 0.677, 0.857, and 0.947 for $s = 1$ through 5. Equation (17) gives expected backorders of 2.000, 1.135, 0.541, 0.218, 0.075, and 0.022 for $s = 0$ through 5.

Figure 2 plots the expected backorder curve. The first unit of stock cuts EBO by 0.865, while the fifth cuts it by only 0.053.

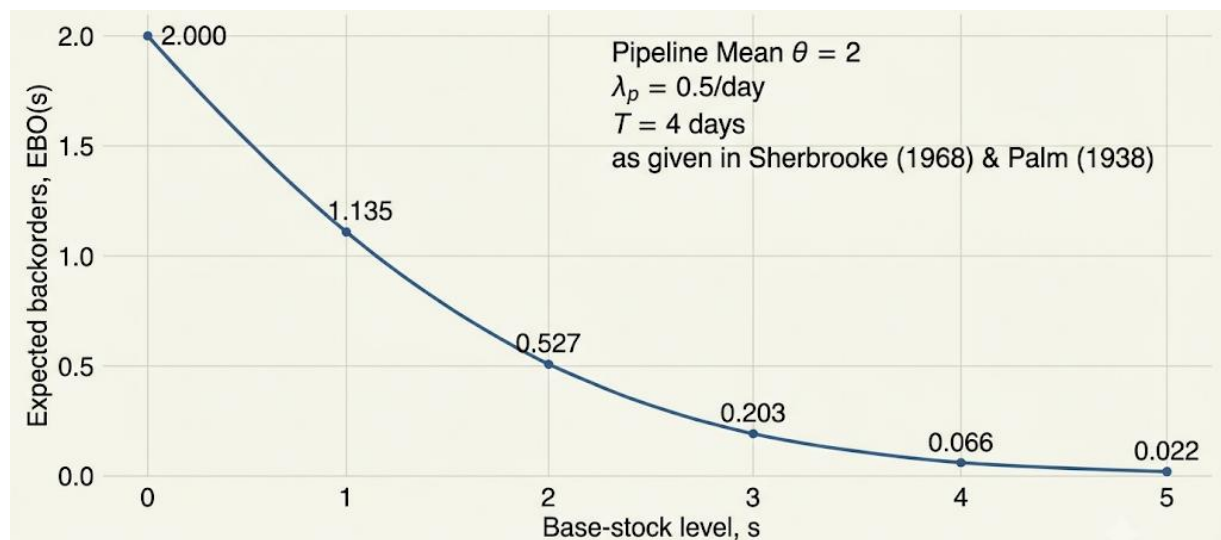


Figure 2: Expected Backorders Versus Base-Stock Level for a Repairable Part with Pipeline Mean $\theta = 2$

The horizontal axis shows the base-stock level s from 0 to 5, and the vertical axis shows expected backorders EBO(s) from 0 to 2. The curve falls from 2.000 at $s = 0$ to 0.022 at $s = 5$, with a steep early drop that flattens into a long tail. Each added spare reduces downtime by less than the one before it. Values are computed from the Poisson pipeline of Palm (1938) using Equations (14) to (17) with assumed $\lambda_p = 0.5$ per day and $T = 4$ days, following the method of Sherbrooke (1968).

The marginal reduction at level s equals $P(X \geq s)$ exactly, and that probability falls as s grows. The curve is therefore convex, a property that Section 5.2 exploits.

3.5 Finite Repair Capacity

Palm's result assumes that units never wait for the repair shop. Real shops have limited crews, and a busy crew lengthens the effective turnaround time T in Equation (13). The pipeline mean θ grows with it, and expected backorders rise at every stock level.

Section 2 gives the exact treatment for a closed population of machines, since W in Equation (10) includes waiting for a free repairer. For multi-echelon repairable systems, Graves (1985) approximated the pipeline distribution by matching its first two moments with a negative binomial distribution, which allows variance larger than the Poisson mean. Muckstadt (1973) added an indenture structure, in which failed machines contain failed modules that contain failed parts. Both extensions give up the clean Poisson result. The qualitative message is unchanged: any delay in the repair loop acts like a longer T .

IV. When Machines Leave the Queue

4.1 Reneging in Repair Systems

Until now every failed machine waits patiently. Real plants behave differently. A failed machine may be stripped for parts, sent to an outside vendor, or replaced by a rental unit. In queueing terminology the machine reneges, meaning it leaves the queue before service begins (Ancker & Gafarian, 1963; Haight, 1957). Baghel (2014) studied this behavior in a Markovian setting with limited spares. The sketch below is a generic version, not a restatement of that paper's results.

Assume each machine waiting for a free repairer abandons at rate β , and assume a machine that abandons returns to the operating pool after outside handling, which for simplicity takes no time. That assumption keeps the machine population closed at N . With R repairers and n machines in the facility, $(n - R)^+$ machines are waiting, where $(x)^+ = \max(x, 0)$. The total downward rate becomes

$$\mu_n = \min(n, R)\mu + (n - R)^+\beta \dots (18)$$

Substituting Equation (18) into Equation (4) leaves the method unchanged.

4.2 Consequences of Reneging

Higher death rates make the chain drain faster, so the mean number in the facility L falls. The rate at which machines leave through reneging is

$$\gamma_r = \beta \sum_{n>R} (n - R)p_n \dots (19)$$

Whether γ_r is a loss depends on where the machine goes. A vendor repair at higher cost makes γ_r a leak, while cannibalization makes it repair by other means.

The flow balance now has two exits:

$$\lambda E[O] = \mu \sum_n \min(n, R)p_n + \gamma_r \dots (20)$$

The first term counts repairs completed in-house and the second counts abandonments. Equation (20) is an exact identity for the chain defined by Equation (18), and it should hold to rounding error in any implementation. A failure of the identity points to an indexing error.

4.3 Reneging and Scarce Spares

The link between reneging and spares is direct. When spares run out, each machine waiting for repair means lost output, and pressure to expedite or abandon increases. In the model this pressure is visible once n exceeds S . A state-dependent form such as $\beta_n = \beta_0[1 + \kappa(n - S)^+]$, with $\kappa \geq 0$, represents it. Only the values of μ_n change. The product formula in Equation (4) still applies, so the solution method survives.

The form of β_n is a modeling assumption and should be fitted to plant records where those exist.

V. Forecasting Demand and Choosing Stock Levels

5.1 Intermittent Demand

Equation (13) requires λ_p , the demand rate for a part. For slow-moving spares, demand is intermittent, with long runs of zeros and occasional orders of one or two units. Ordinary exponential smoothing handles such series badly. Croston (1972) split the series into two components: the size of nonzero demands, z_t , and the interval between them, p_t . When a demand of size y_t occurs, and q_t periods have passed since the previous demand, the updates are

$$z_t = \alpha y_t + (1 - \alpha)z_{(t-1)} \dots (21)$$

$$p_t = \alpha q_t + (1 - \alpha)p_{(t-1)} \dots (22)$$

In periods with no demand, both estimates carry forward unchanged. The forecast of demand per period is

$$\hat{y}_t = z_t/p_t \dots (23)$$

Syntetos and Boylan (2005) proposed the corrected estimator

$$\hat{y}_t = (1 - \alpha/2)z_t/p_t \dots (24)$$

which reduces the positive bias of Croston's ratio. For $\alpha = 0.1$ the factor is 0.95, a five percent reduction. Kennedy, Patterson, and Fredendall (2002) reviewed the wider literature on spare parts inventories. An upward bias in λ_p passes straight into θ and then into every fill-rate figure derived from it.

5.2 Many Parts and One Budget

A plant holds thousands of spare parts, each with price c_i , pipeline mean θ_i , and a different cost of stock-out. Sherbrooke's (1968) approach selects stock levels s_i to minimize total expected backorders for a budget B :

$$\text{minimize } \sum_i EBO_i(s_i) \text{ subject to } \sum_i c_i s_i \leq B \dots (25)$$

Because each EBO_i is convex in s_i , as Section 3.4 showed, marginal analysis applies. At each step compute

$$\Delta_i = [EBO_i(s_i) - EBO_i(s_i + 1)]/c_i = P(X_i \geq s_i + 1)/c_i \dots (26)$$

add one unit of the part with the largest Δ_i , and repeat until the budget is exhausted. The sequence of purchases traces the efficient frontier of backorders against spending. The result is optimal at the budget levels the sequence reaches and near-optimal between them.

Cheap parts with high pipeline means are stocked deeply, and expensive parts with low demand receive one unit or none. Sherbrooke (2004) extends the approach to multi-echelon systems, where a central depot supplies local bases, and Muckstadt (1973) adds the indenture structure.

5.3 Condition Monitoring and the Pipeline Mean

A machine that warns before it fails changes the problem. Jardine, Lin, and Banjevic (2006) reviewed diagnostics and prognostics for condition-based maintenance. In the notation of this article, a reliable warning lets logistics lead time overlap with the remaining useful life of the component. The effective T in Equation (13) shrinks, so condition monitoring acts on θ through T and not through λ_p . This is a modeling interpretation, and the size of the effect depends on warning accuracy and lead time, which are not quantified here.

5.4 A Cost Model for Spares and Repairers

To choose between spares and repairers, write the total cost rate

$$C(S, R) = c_s S + c_r R + c_d (M - E[O]) \dots (27)$$

where c_s is the holding cost per spare per unit time, c_r is the cost per repairer per unit time, and c_d is the downtime cost per machine per unit time. $E[O]$ comes from Equation (7), so it depends on S and R through the chain of Section 2. The optimal pair (S^*, R^*) minimizes C . For small plants one can enumerate $S = 0, 1, 2, \dots$ and $R = 1, 2, \dots$ in a spreadsheet. Hopp and Spearman (2008) make a related point from factory physics: variability must be buffered by inventory, capacity, or time, and the cheapest mix usually involves more than one buffer.

VI. A Worked Example

6.1 Parameters

Let $M = 3$ machines be required, and let $\rho = \lambda/\mu = 0.2$, so that a machine fails on average once per five mean repair times. Equations (4) and (5) depend only on the ratio ρ , so absolute rates are not needed. These values are assumed for illustration and do not come from any plant. Four configurations are evaluated: $S \in \{0, 1\}$ and $R \in \{1, 2\}$.

6.2 One Repairer, No Spare

Equation (6) gives unnormalized weights of 1, $3\rho = 0.6$, $6\rho^2 = 0.24$, and $6\rho^3 = 0.048$. They sum to 1.888, so $p_0 = 0.5297$, $p_1 = 0.3178$, $p_2 = 0.1271$, and $p_3 = 0.0254$. Then $L = 0.3178 + 2(0.1271) + 3(0.0254) = 0.6483$, $E[O] = 3 - 0.6483 = 2.3517$, and $A = 0.7839$.

The check from Section 2.3 holds. Utilization is $U = 1 - p_0 = 0.4703$, and $\rho E[O] = 0.2 \times 2.3517 = 0.4703$. Mean time in the facility is $W = L / (\lambda E[O]) = 1.378$ mean repair times.

6.3 One Repairer, One Spare

Now $N = 4$. Equation (1) gives $\lambda_0 = 3\lambda$, $\lambda_1 = 3\lambda$, $\lambda_2 = 2\lambda$, and $\lambda_3 = \lambda$, and $\mu_n = \mu$ throughout. Equation (4) gives weights of 1, 0.6, 0.36, 0.144, and 0.0288, which sum to 2.1328. The probabilities are $p_0 = 0.4689$, $p_1 = 0.2813$, $p_2 = 0.1688$, $p_3 = 0.0675$, and $p_4 = 0.0135$.

The number of operating machines is 3 for $n \leq 1$, and then 2, 1, and 0 for $n = 2, 3, 4$. Hence $E[O] = 3(0.7502) + 2(0.1688) + 1(0.0675) = 2.6557$ and $A = 0.8852$. The check holds: $\rho E[O] = 0.5311 = 1 - p_0$. Here $L = 0.8755$ and $W = 1.648$ mean repair times. Mean time in the facility is longer than in Section 6.2 because spares let failed machines sit in the queue without stopping production.

6.4 Two Repairers

With $R = 2$ and $S = 0$, the weights are 1, 0.6, 0.12, and 0.012, which sum to 1.732. The probabilities are $p_0 = 0.5774$, $p_1 = 0.3464$, $p_2 = 0.0693$, and $p_3 = 0.0069$. Then $E[O] = 2.4942$ and $A = 0.8314$. As a check, $\sum \mu_n p_n = \mu[p_1 + 2(p_2 + p_3)] = 0.4988\mu$, which equals $\rho E[O] = 0.4988\mu$.

With $R = 2$ and $S = 1$, the weights are 1, 0.6, 0.18, 0.036, and 0.0036, which sum to 1.8196. The probabilities are $p_0 = 0.5496$, $p_1 = 0.3297$, $p_2 = 0.0989$, $p_3 = 0.0198$, and $p_4 = 0.0020$. Then $E[O] = 2.8556$ and $A = 0.9519$. The check is $\sum \mu_n p_n = \mu[p_1 + 2(p_2 + p_3 + p_4)] = 0.5711\mu$, against $\rho E[O] = 0.5711\mu$.

6.5 Comparison

Table 1 collects the four results.

Table 1: Performance of Four Spare and Repairer Configurations for $M = 3$ and $\rho = 0.2$

Spares (S)	Repairers (R)	E[O]	Availability A	Utilization U	Mean time in facility W (units of $1/\mu$)
0	1	2.352	78.4%	0.470	1.378
1	1	2.656	88.5%	0.531	1.648
0	2	2.494	83.1%	0.249	1.014
1	2	2.856	95.2%	0.286	1.042

Source: computed from Equations (1) to (11), the birth-death model of Gross et al. (2008) and Taha (2011), with assumed $\rho = 0.2$. These are model outputs from stated assumptions, not observed plant data.

Three observations follow. First, a spare added to a one-repairer plant raises $E[O]$ by 0.304, while a second repairer added to a plant without a spare raises it by 0.143. The ratio is about 2.1, so the spare is worth more than twice the repairer in this setting. The result depends on ρ . At much higher ρ a single repairer saturates, and the second repairer gains in value.

Second, the two resources complement each other. A spare adds 0.304 machines when $R = 1$ and 0.361 when $R = 2$. A repairer adds 0.143 when $S = 0$ and 0.200 when $S = 1$. Returns do not diminish across resource types here. They diminish within a type, as Figure 2 showed for stock of a single part.

Third, the spare lengthens W while it raises output, because it lets failed machines wait without stopping the line. Mean time in the facility is therefore a poor stand-in for production loss.

6.6 Reading the Cost Trade-Off

By Equation (27), adding the spare at $R = 1$ pays when $0.304 c_d > c_s$. Adding a second repairer at $S = 0$ pays when $0.143 c_d > c_r$. The spare is the cheaper route to availability whenever $c_s/0.304 < c_r/0.143$, which simplifies to $c_r > 0.47c_s$. A repairer would have to cost less than about 47 percent of holding one spare to be the better purchase per unit of availability gained. The threshold applies only at $\rho = 0.2$ and $M = 3$.

VII. Conclusion

Spare parts keep factories moving because a small set of equations governs how failures, repairs, and inventory interact. For a three-machine plant those equations can be solved by hand.

The machine repair model supplied the core result. Steady-state probabilities follow from a product of rate ratios (Equation 4), and availability, repairer load, and mean time in the facility follow from them (Equations 7 to 11). In the worked example with $\rho = 0.2$, one spare raised availability from 78.4 to 88.5 percent with a single repairer, and its contribution to expected output was about 2.1 times that of a second repairer. The two resources also complemented each other, so the best configuration used both.

On the component side, Palm's theorem gave a Poisson pipeline with mean $\theta = \lambda_p T$. The expected backorder recursion in Equation (17) showed why the first unit of stock matters more than the fifth. Reneging, forecasting, and budget allocation each added a layer without breaking the framework: change the rates, estimate λ_p carefully, and spend where Equation (26) shows the largest marginal gain per unit cost.

The results carry limits. Every numerical value here comes from assumed parameters and exponential timing. Real failures cluster, and real repair times have long tails. Palm's result tolerates any repair-time distribution, but the closed machine repair chain does not. Practitioners should estimate λ and μ from their own maintenance logs, test the exponential assumption, and rerun the calculations. Future work could test the renege extension against plant records and relax the exponential assumption through phase-type repair times. The method transfers readily to other plants, but the example values do not transfer at all.

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