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# Stochastic Analysis of a Cold Standby System with Server Failure

Dhiraj Yadav* and M.S.Barak
**Department of Mathematics, Govt. college for women, Rewari*
*Department of Mathematics, I. G. Univeristy, Meerpur (Rewari)*

***ABSTRACT:*** *This paper throws light on the stochastic analysis of a cold standby system having two identical units. The operative unit may fail directly from normal mode and the cold standby unit can fail owing to remain unused for a longer period of time. There is a single server, who may also follow as precedent unit happens to be in the line of failure .After having treated, the operant unit for functional purposes, the server may eventually perform the better service efficiently. The time to take treatment and repair activity follows negative exponential distribution whereas the distributions of unit and server failure are taken as arbitrary with different probability density functions. The expressions of various stochastic measures are analyzed in steady state using semi-Markov process and regenerative point technique. The graphs are sketched for arbitrary values of the parameters to delineate the behavior of some important performance measures to check the efficacy of the system model under such situations.*

***KEYWORDS:*** *Stochastic Analysis, Cold-standby, regenerative point, steady state and semi-Markov process*

## I. INTRODUCTION

Redundancy is a common approach to enrich the reliability and availability of a system. In literature, the stochastic behavior of cold standby system has been widely discussed by many researchers [1-3]. Some of them have generally imagined the server to be always in good condition and it never fails while working. But this imagination seems to be quite impractical when a server has to work in varying environmental conditions. We may observe many cases where the server fails during his performance. In a standby redundant system some additional paths are created for proper functioning of the system. Standby unit is the assistance to increase the reliability of the system.

In a cold standby redundant system as the operating unit fails, the standby unit takes its place and the failed unit goes under treatment .But it may be the possibility that the standby unit is already damaged owing to remain unused for a longer period of time or erosion etc. we may face a situation when the operating unit fails but the standby unit is already damaged. So far, the cold standby systems with the possibility of server failure have been discussed much[4-5] but the standby failure have not been researched so much .Though a two unit redundant system with standby failure has been discussed by[6-8] while the concept of standby failure needs more attention due to its significant contribution during study. So keeping this aspect in view we developed a stochastic model of redundant system with standby and server failure. The model consists of two identical units; one unit is in operative mode and other in cold standby mode. The cold standby unit becomes operative after failure of the previous unit. The failure of the server during any service activity can produce undesirable results in terms of safety as well as economic losses. There is single server for repair activity who may go for treatment to increase his efficiency whenever required. The server works as afresh after taking treatment with full efficiency. The time to take treatment and repair activity follows negative exponential distribution whereas the distributions of unit and server failure are taken as arbitrary with different probability density functions. The expression for various reliability measures such as transition probabilities, mean sojourn times, mean time to system failure, steady state availability are deduced by using semi-Markov process and regenerative point technique. The graphs are delineated for arbitrary values of the parameters to highlight the behavior of some important performance measures to check the efficacy of the system model under such situations.

**NOTATIONS**

E : Set of regenerative states.
O / Cs : The unit is operative /cold standby
F$U_r$/ F$U_R$ : The failed unit is under repair/ under repair continuously from previous state.
SFU$_t$ / SFU$_T$ : The server has failed and is under treatment /under treatment continuously from previous state.
F$W_r$ / $FW_R$ : The unit is failed and waiting for repair/waiting for repair continuously from previous state.
$\lambda$ / $\mu$ : Constant failure rate of unit /failure of server.
f(t) / F(t): p.d.f. / c.d.f. of repair rate of the failed unit.

g(t) / G(t) : pdf / cdf of treatment time of the server.

$q_{ij}(t)$ / $Q_{ij}(t)$ : pdf / cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state.

$q_{i,jk}(t)$ / $Q_{i,jk}(t)$: pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].

$q_{i,jkrs}(t)$ / $Q_{i,jkrs}(t)$ : pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k , r and s once in (0,t].

$M_i(t)$ : Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state .

$W_i(t)$ : Probability that the system is busy in state Si up at time t without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

$m_{ij}$ : Contribution to mean sojourn time ($u_i$) in state $S_i$ when system transit directly to state j so that $\mu_i = \sum_j m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*'}(0)$

Ⓢ/© : symbol for stieltjes convolution/ laplace convolution

**Transition Probabilities**

Simple probabilities considerations yield the following expressions for the non-zero elements

$$P_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)\,dt \quad (1)$$

$P_{01} = a$ , $P_{03} = b$ , $P_{10} = \frac{\phi}{\phi+\lambda+\mu}$ , $P_{12} = \frac{\mu}{\phi+\lambda+\mu}$ $P_{14} = \frac{\lambda}{\phi+\lambda+\mu}$

$P_{21} = \frac{\theta}{\lambda+\theta}$ , $P_{27} = \frac{\lambda}{\lambda+\theta}$ , $P_{31} = P_{41} = P_{61} = P_{81} = \frac{\phi}{\phi+\mu}$

$P_{35} = P_{49} = P_{65} = P_{89} = \frac{\mu}{\phi+\mu}$ , $P_{56} = P_{78} = P_{98} = 1$ , $a + b = 1$ (2)

For these transition probabilities, it can be verified that:

$$P_{01} + P_{03} = P_{10} + P_{12} + P_{14} = P_{21} + P_{27} = P_{31} + P_{35} = P_{41} + P_{49} = 1 \quad (3)$$

$P_{01,356}^{(n)} = b\frac{\mu}{\phi+\mu}$, $P_{11,498}^{(n)} = \frac{\lambda}{\phi+\lambda+\mu}\cdot\frac{\mu}{\phi+\mu}$, $P_{21,789}^{(n)} = \frac{\lambda}{\phi+\lambda}$

It can be verified that, $P_{01} + P_{01,3} + P_{01,356}^{(n)} = P_{01} + P_{11,4} + P_{11,498}^{(n)} + P_{12} = P_{21} + P_{21,789}^{(n)} = 1$ (4)

**Mean Sojourn Time**

Let T denotes the time to system failure then the mean sojourn times ($\mu_i$) in the state $S_i$ are given by $\mu_i = E(t) = \int_0^\infty P(T > t)\,dt$

Hence, $\mu_0 = \frac{1}{\lambda}$ $\mu_1 = \frac{1}{\phi+\lambda+\mu}$ $\mu_2 = \frac{1}{\theta+\lambda}$

$\mu_0' = \frac{\theta\phi+b\lambda(\theta+\mu)}{\theta\phi\lambda}$ $\mu_1' = \frac{\theta\phi+\lambda(\theta+\mu)}{\theta\phi(\phi+\lambda+\mu)}$ $\mu_2' = \frac{\theta\phi+\lambda(\phi+\mu)}{\theta\phi(\theta+\lambda)}$ (5)

**Mean Time to System Failure**

Let $\phi_i(t)$ be the c.d.f. of the passage time from regenerative state $S_i$ to a failed state .Regarding the failed state as absorbing state ,we have the following recursive relations for $\phi_i(t)$

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_{01}(t) + Q_{03}(t)$

$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_{12}(t)Ⓢ\phi_2(t) + Q_{14}(t)$

$\phi_2(t) = Q_{21}(t)Ⓢ\phi_1(t) + Q_{27}(t)$ (6)

Taking L.S.T. of relation (6) and solving for $\phi_0^*(s)$, $\phi_0^* = \frac{N}{D}$

$$\text{MTSF} = \lim_{s\to 0} R^*(s) = \lim_{s\to 0} \frac{1-\phi_0^*(s)}{s} \quad (7)$$

$$= \lim_{s\to 0} \frac{1-N/D}{s}$$

$$= \frac{\mu_0(1-P_{12}P_{21})+\mu_1P_{01}+P_{12}P_{01}\mu_2}{1-P_{12}P_{21}-P_{01}P_{10}} \quad (8)$$

**Steady State Availability**

Let $A_i(t)$ be the probability that the system is in up-state at instant t ,given that the system entered the regenerative state $S_i$ at t = 0.The recursive relations for Ai(t) are as follows :

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{01,3}(t)©A_1(t) + q_{01,356}^{n}(t)©A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t) + q_{11,4}(t)©A_1(t) + q_{11,498}^{n}(t)©A_1(t)$

$A_2(t) = M_2(t) + q_{21}(t)©A_1(t) + q_{21,789}{}^n(t) © A_1(t)$ (9)

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ at time t without visiting to any other regenerative state where

$M_0(t) = e^{-\lambda t}$ , $M_1(t)=e^{-(\lambda+\mu)t}\bar{G}(t)$ , $M_2(t) = e^{-\lambda t}\bar{F}(t)$

Taking Laplace transform of equation (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(s) = \lim_{s\to 0} sA_0^*(s) = \frac{\mu_0 P_{10}+\mu_1+\mu_2 P_{12}}{\mu'_{10}P_{10}+\mu'_1+\mu'_2 P_{12}} \quad (10)$$

**Busy period of the server**

Let $B_i(t)$ be the probability that the server is busy at instant t ,given that the system entered the regenerative state i at t = 0 .The recursive relations for $B_i(t)$ are as follows :

$B_0^R(t) = q_{01}(t) © B_1^R(t) + q_{01,3}(t)© B_1^R(t)+ q_{01,356}{}^n(t) © B_1^R(t)$

$B_1^R(t) = W_1(t) + q_{10}(t)©B_0^R(t) + q_{12}(t)©B_2^R(t) + q_{11,4}(t)© B_1^R(t) + q_{11,498}{}^n(t) © B_1^R(t)$

$B_2^R(t) = q_{21}(t) © B_1^R(t) + q_{21,789}{}^n(t) © B_1^R(t)$ (11)

Where Wi(t) is the probability that the server is busy in state Si due to repairing of unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states so

$W_1(t) = e^{-(\lambda+\mu)t}\bar{G}(t) + \lambda e^{-(\lambda+\mu)t}\bar{G}(t) © g(t)e^{-\mu t} + \lambda e^{-(\lambda+\mu)t}\bar{G}(t) ©\mu e^{-\mu t}\bar{G}(t)$+-------up to n

Taking L.T. of relation (11) and solving for $B_0^*(s)$, the time for which server is busy is given as

$$B_0^R = \lim_{s\to 0} s\, B_0^*(s) = \frac{w_1^*(0)}{\mu'_0 P_{10}+\mu'_1+\mu'_2 P_{12}} \quad (12)$$

**Expected Number of visits by the server**

Let $R_i(t)$ be the expected number of visits by the server in(0,t],given that the system entered the regenerative state $S_i$ at t=0 .The recursive relations for $R_i(t)$ are as follows :

$R_o(t) = Q_{01}(t)©R_1(t) + Q_{01,3}(t)©R_1(t) + Q_{01,356}{}^n(t)©R_1(t)$

$R_1(t) = Q_{10}(t)©[ 1+R_0(t) ] + Q_{12}(t)©[R_2(t) +1] + Q_{11,4}(t)©[ R_1(t) +1 ] + Q_{11,498}{}^n(t)©[ R_1(t) + 1]$

$R_2(t) = Q_{21}(t)©R_1(t) + Q_{21,789}{}^n(t)© R_1(t)$ (13)

Taking Laplace transform of the above relation and solving for $R_0^*(s)$, the expected number of visits by the server are given by

$$R_0 = \lim_{s\to 0} s\, R_0^*(s) = \frac{1}{\mu'_0 P_{10}+\mu'_1+\mu'_2 P_{12}} \quad (14)$$

**Expected Number of treatments given to server**

Let $T_i(t)$ be the expected number of treatments given to server in (0,t] such that the system entered the regenerative state at t = 0 .The recursive relations for $T_i(t)$ are as follows:

$T_0(t) = Q_{01}(t) ©T_1(t) + Q_{01,3}(t) ©T_1(t) + Q_{01,356}{}^n(t) ©T_1(t)$

$T_1(t) = Q_{10}(t) ©T_0(t) + Q_{12}(t) ©[1+T_2(t)] + Q_{11,4}(t) ©T_1(t) + Q_{11,498}{}^n(t) ©[1+T_1(t)]$

$T_2(t) = Q_{21}(t) ©T_1(t) + Q_{01,3}(t) ©T_1(t) + Q_{21,789}{}^n(t) ©[1+T_1(t)]$ (15)

Taking Laplace transform of the above relation and solving for $T_0^*(s)$ , we get

$$T_0 = \lim_{s\to 0} T_0^*(s) = \frac{P_{12}\left(1+P_{21,789}{}^n\right)+P_{10}P_{01,356}{}^n+P_{11,498}{}^n}{\mu'_0 P_{10}+\mu'_1+\mu'_2 P_{12}} \quad (16)$$

**Cost -Benefit Analysis:**

The profit occurred in the system model in steady state can be calculated as

$P_0 = K_0A_0 - K_1B_0^R - K_2R_0 - K_3T_0$ (17)

Where $K_0$ = (5000); Revenue per unit up- time of the system

$K_1$ = (650); Cost per unit time for which server is busy

$K_2$ = (450 ); Cost per unit visits by the server

$K_3$ = (350); Cost per unit time treatment given to server

**Particular Case:**

Let us take $f(t) = \theta e^{-\theta t}$ , $f(t) = \phi e^{-\phi t}$

$$\textbf{MTSF} = \frac{\theta\phi+\lambda(\theta+\phi+\lambda+\mu+\alpha\mu+\alpha\theta+\alpha\lambda)}{\lambda[\theta\phi+\lambda(\theta+\phi+\lambda+\mu-\alpha\phi)-\alpha\theta\phi]} \quad (18)$$

**Availability = $A_0$**

$$=\frac{\theta\phi[(\phi+\lambda)(\theta+\lambda)+\lambda\mu]}{[\theta^2\phi^2+\lambda\theta\phi(\theta+\phi+\lambda+\mu)+b\lambda\phi(\theta+\lambda)(\theta+\mu)+\lambda^2(\theta^2+\mu\theta+\mu^2+\mu\varphi)+\lambda^3(\theta+\mu)]} \tag{19}$$

**Busy Period of the server due to Repair =$B_0^R$**

$$=\frac{\lambda\theta\phi(1+\lambda)(\theta+\lambda)}{[(\theta+\lambda)\phi(\theta\phi+b\lambda\theta+b\lambda\mu)+\lambda(\theta+\lambda)(\theta\phi+\lambda\theta+\lambda\mu)+\mu\lambda(\theta\phi+\lambda\phi+\lambda\mu)]} \tag{20}$$

**Expected number of visits due to Repair = $R_0$**

$$\frac{\lambda\theta\phi(\theta+\lambda)(\phi+\lambda+\mu)}{[(\theta+\lambda)\phi(\theta\phi+b\lambda\theta+b\lambda\mu)+\lambda(\theta+\lambda)(\theta\phi+\lambda\theta+\lambda\mu)+\mu\lambda(\theta\phi+\lambda\phi+\lambda\mu)]} \tag{21}$$

**Expected Number of Treatments given to Server =$T_0$**

$$=\frac{\lambda\mu[(\theta+2\lambda)(\phi+\mu)+(b\phi+\lambda)(\theta+\lambda)]}{(\varphi+\mu)[(\theta+\lambda)\phi(\theta\phi+b\lambda\theta+b\lambda\mu)+\lambda(\theta+\lambda)(\theta\phi+\lambda\theta+\lambda\mu)+\mu\lambda(\theta\phi+\lambda\phi+\lambda\mu)]} \tag{22}$$

## II. CONCLUSION

In this exploration, the effect of various parameters on system model is visualized. Graphs are penned down by assigning particular values to various parameters and costs shown in figure 2-4.

Fig.2. The pictorial behavior of MTSF w.r.t repair rate explains that the increase rate in treatment causes increment in MTSF also. It also shows that as the rate of server failure decreases, MTSF increases

Fig.3 expose Availability Vs Repair rate having increasing pattern. The curve indicates that as the rate of treatment decreases, graph declines having still increasing pattern .But as the rate of server failure decrease and treatment rate is increased, slop rises rapidly.

Fig.4 demonstrate Profit Vs Repair rate having same nature as the case of availability. Hence the system performance can be increased by providing service facility of lower failure rate and by increasing treatment rate. It is analyzed that mean time to system failure(MTSF) , Availability , profit function go on increasing w.r.t. Repair rate .This study winds up that the system performance can be enriched by decreasing server failure rate , increasing repair rate and treatment rate .The applied approach of this undertaken model has its utility in variegated industrial and other setups.

**About the Authors:**

M.S.Barak is presently working as a Assistant Professor, Department of Mathematics, Indira Gandhi Univeristy, Meerpur, Rewari, India. He has been actively engaged in the research area of Reliability as well as theoretical seismology for many years. He has been visited many academic institutions working in India and abroad to present his research work at the conferences/ symposium of international repute. He is a member of various academic/professional bodies

Dhiraj is presently working as a Assistant professor in Mathematics, Govt. college for women Rewari and research scholar in Department of Mathematics, Indira Gandhi University, Meerpur, Rewari. She has the research interest in the area of Reliability modeling of repairable systems.

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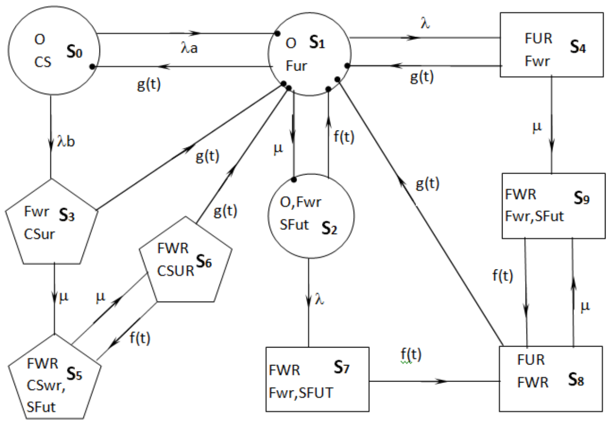
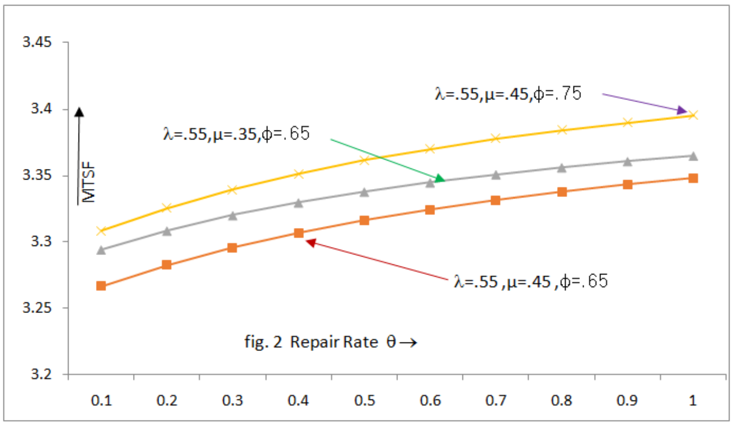
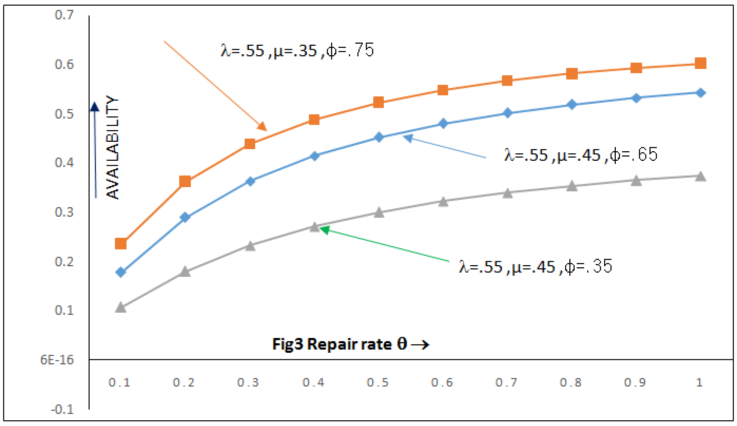
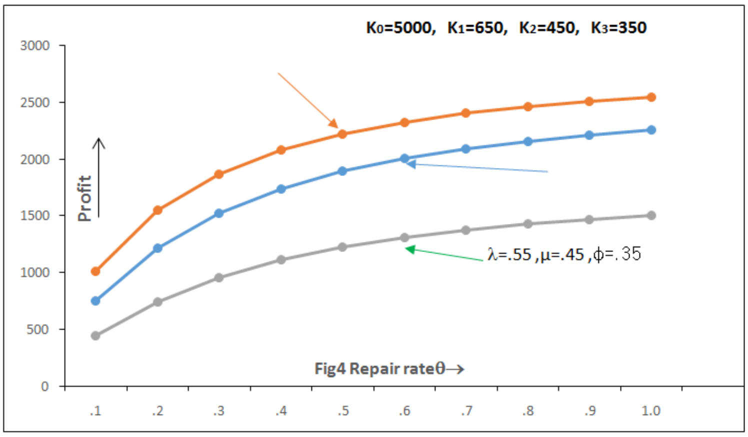



State Transition Diagram Fig.1





K0=5000, K1=650, K2=450, K3=350
3000
2500
2000
1500
1000
500
0
Profit
λ=.55 ,μ=.45 ,φ=.35
Fig4 Repair rateθ→
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0