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Even Harmonious Labeling of the Graph H (2n, 2t+1)

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ABSTRACT: *A function f is said to be an even harmonious labeling of a graph G with q edges if f is an injection from the vertices of G to the integers from 0 to 2q and the induced function f* from the edges ofG to {0, 2………2(q-1)} defined by f* (uv) = f (u) +f (v) (mod 2q) is bijective. The graph G is said to have an even harmonious labeling. In this paper the even harmonious labeling of a class of graph namely H (2n, 2t+1) is established.*

KEYWORDS: *Even harmonious, Harmonious and odd harmonious labelling Subject classification code: 05C78*

I. INTRODUCTION

Graph labling is an active area of research in graph theory which has mainly evolved through its many applications in coding theory, communication networksand mobile telecommunication system.

Lots of research work have been carried out in labeling of graphs in recent and past and was first initiated by A.Rosa in 1967[1].Also graph labeling methods can be traced back to Graham and Sloane[2].

If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling.

Most of the graph labeling problem will have the following three common characteristics

1. A set of numbers from which vertex labels are chosen.
2. A rule that assigns a value to each edge.
3. A condition that those values must satisfy.

II. Preliminaries

Definition2.1

A graph G with q edges is said to be harmonious if there is an injection f from the vertices of G to the group of integers modulo q such that when each edge xy is assigned the label f(x)+f(y) (mod q), the resulting edge labels are distinct. Recently two variants of harmonious labelings have been defined.

Definition2.2

A function f is said to be an odd harmonious labeling of a graph G with q edges if f is an injection from the vertices of G to the integers from 0 to 2q – 1 such that the induced mapping f* (uv) = f(u)+f(v) (mod 2q) from the edges of G to the odd integers between 1 to 2q -1 is bijection.

Definition2.3

A function f is said to be an even harmonious labeling of a graph G with q edges if f is an injection from the vertices of G to the integers from 0 to 2q and the induced function f* from the edges of G to {0,2……….2(q-1)} defined by f* (uv) = f(u)+f(v)(mod 2q) is bijective.

Even harmonious labeling was introduced by Sarasija and Binthiya[3] in 2011. The following results have been proved by Lori Ann Schoenhard in 2013

Theorem 2.4

A tree cannot have an even harmonious labeling.

Definition 2.5

The wheel, W_n is the graph obtained by joining every vertex of a cycle C_n to exactly one isolated vertex called the Center. The edges incident to the center are called SPOKES.

Theorem 2.6

The wheel, w_n is even harmonious when n is odd

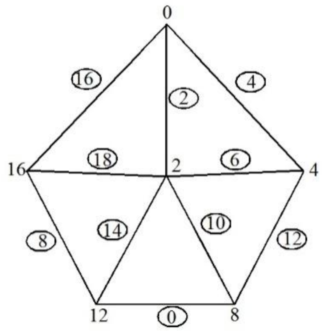


Fig 2.1

Definition 2.7

The helm H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

Theorem 2.8

The helm H_n is even harmonious when n is odd.

Theorem 2.9

The graph, C_{2n} is not even harmonious when n is odd.

Theorem 2.10

K_n is even harmonious if and only if $n<4$

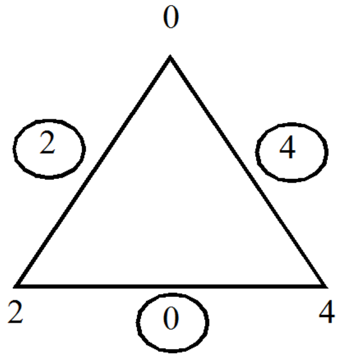


Fig 2.2

III. EVEN HARMONIOUS LABELING OF H(2n,2t+1)

In his survey article, "A dynamic survey of graph labeling – Electronic journal of Combinatorics" – 2015[5] J.A.Gallian mentioned as a conjecture that C_n with K consecutive chords has even harmonious labeling.

Definition 3.1

H(2n,2t+1)(n=2,3,4,…t=0,1,2,3,…) denote the graph obtained from a cycle C_{2n} by adding 2t+1 consecutive diagonals including the central diagonal and 2t diagonals symmetric to the central diagonal. If n=m,a positive integer then t will take only the values 0,1,2,..,m-2.

Theorem: 3.2

H(2n,2t+1) has even harmonious labeling for n=2,3,4,5,6

Even harmonious labeling of H (4,1) (V=4,q=5,2q=10)

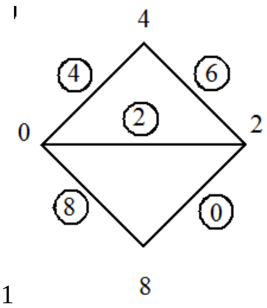


Fig 3.1

Even harmonious labeling of H (6,1) (V=6,q=7,2q=14)

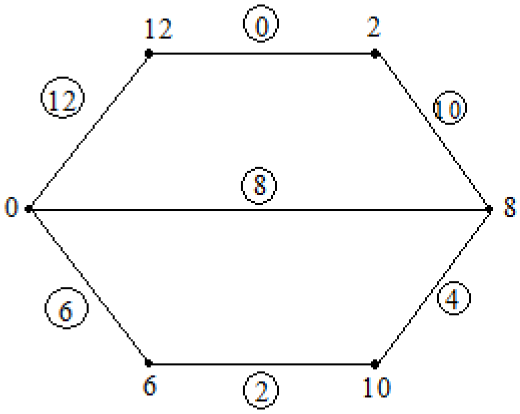


Fig 3.2

H (6,3) (V=6,q=9,2q=18)

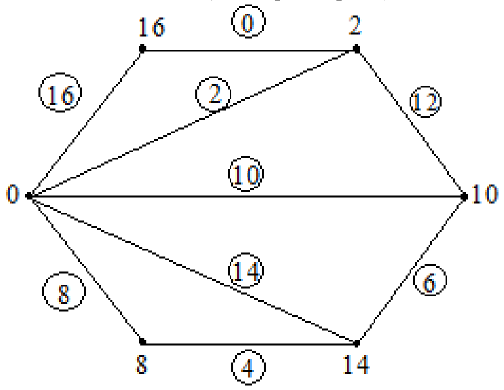


Fig 3.3

Even harmonious labeling of H (8,1) (V=8,q=9,2q=18)

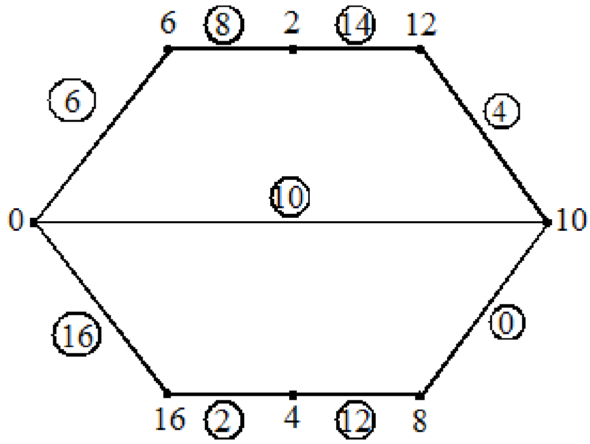


Fig 3.4

H (8,3) (V=8,q=11,2q=22)

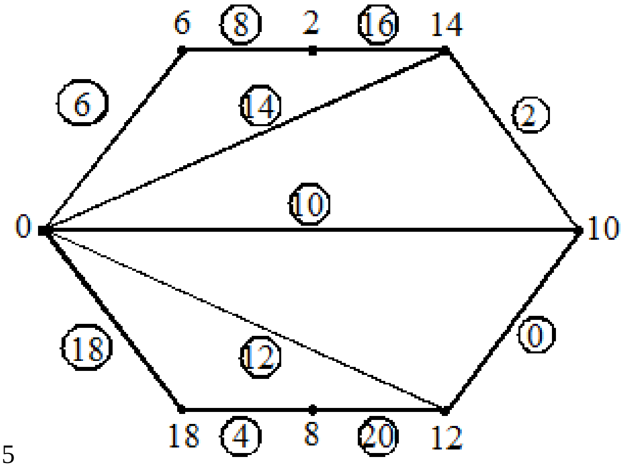


Fig 3.5

H (8,5) (V=8,q=13,2q=26)

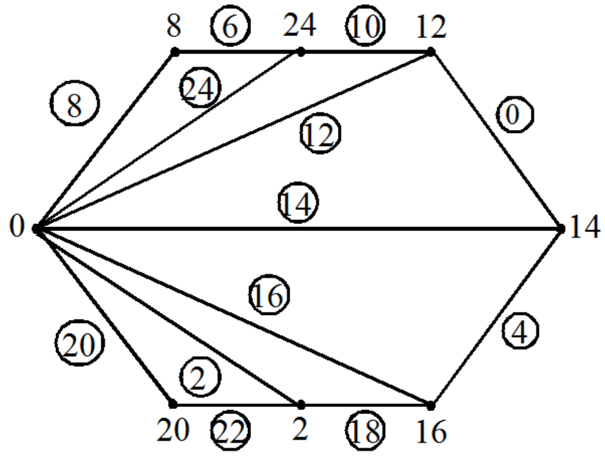


Fig 3.6

Even harmonious labeling of H (10,1) (V=10,q=11,2q=22)

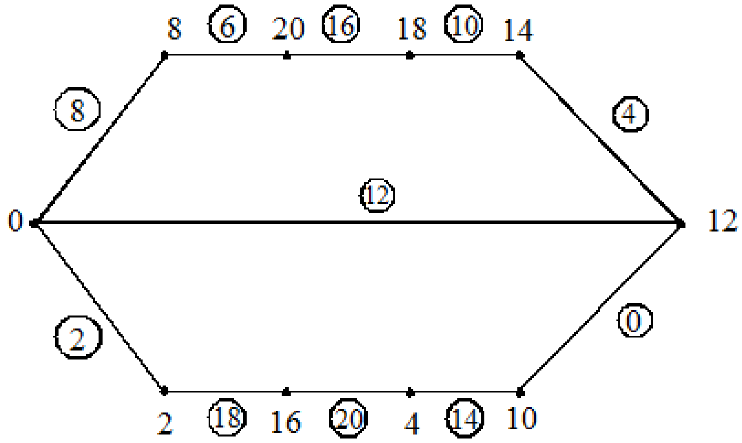
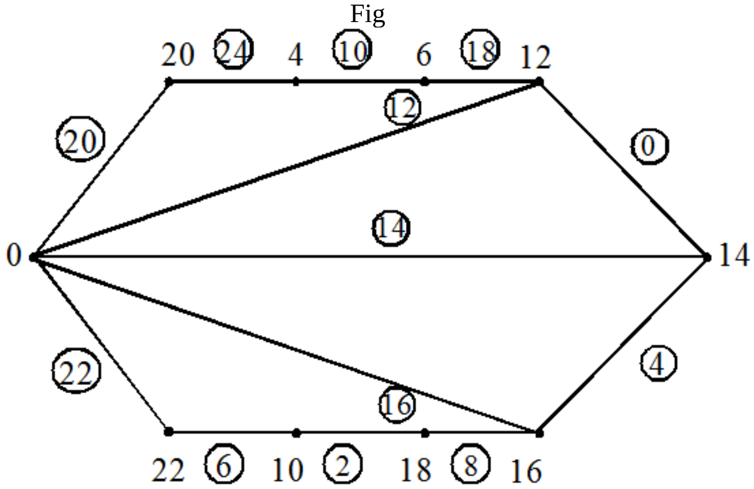


Fig 3.7

H (10,3) (V=10,q=13,2q=26)

Fig



3.8

H (10,5) (V=10,q=15,2q=30)

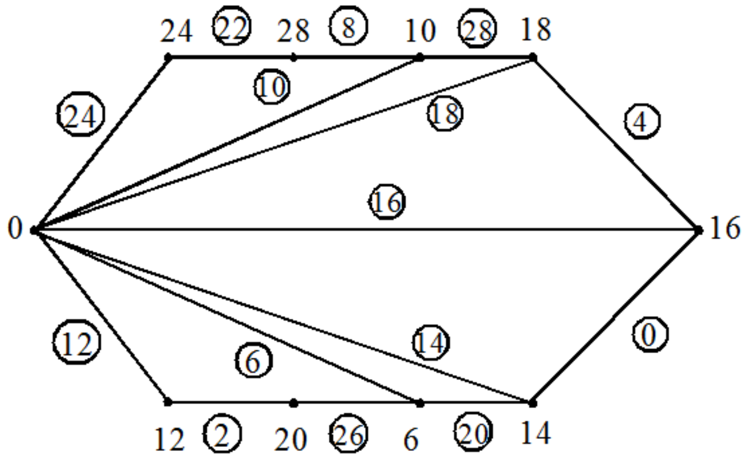


Fig 3.9

H (10,7) (V=10,q=17,2q=34)

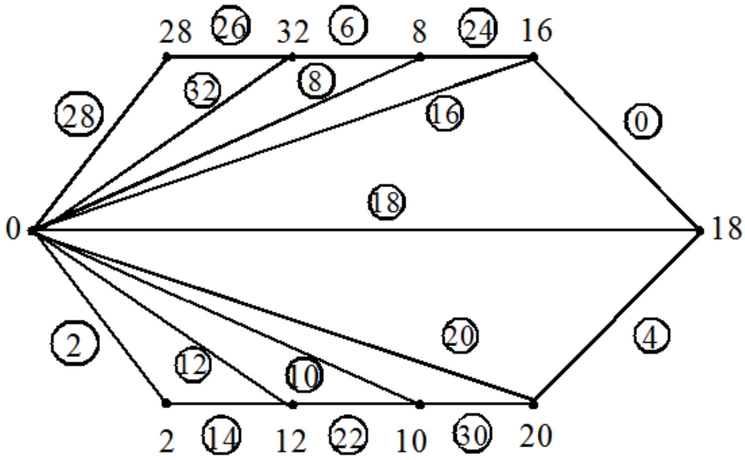


Fig 3.10

Even harmonious labeling of H (12,1) (V=12,q=13,2q=26)

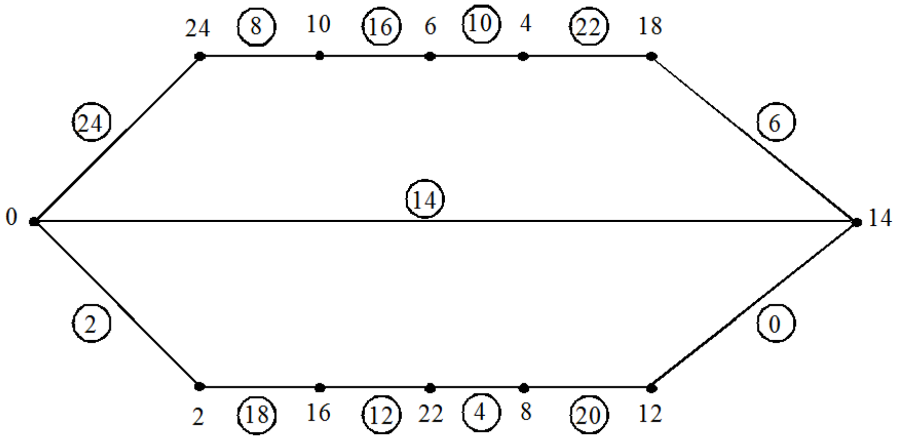


Fig 3.11

H (12,3) (V=12,q=15,2q=30)

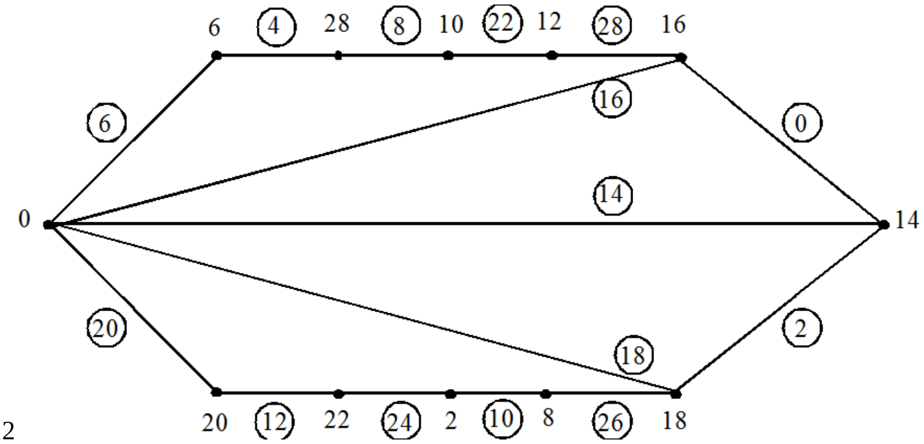


Fig 3.12

H (12,5) (V=12,q=17,2q=34)

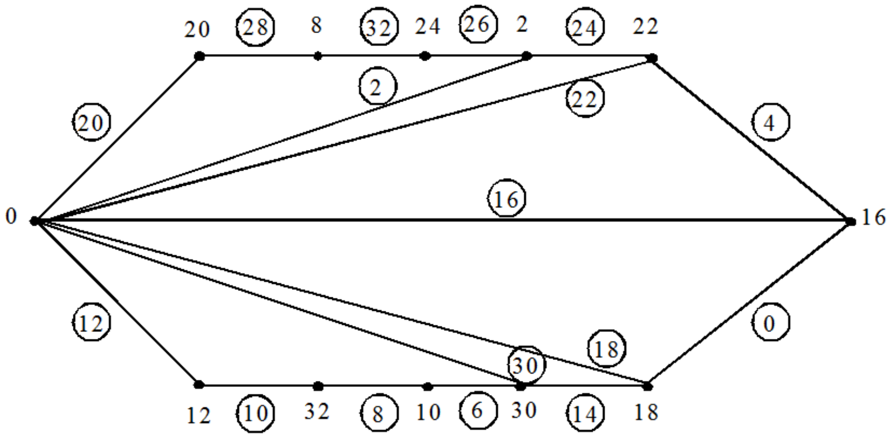


Fig 3.13

H (12,7) (V=12,q=19,2q=38)

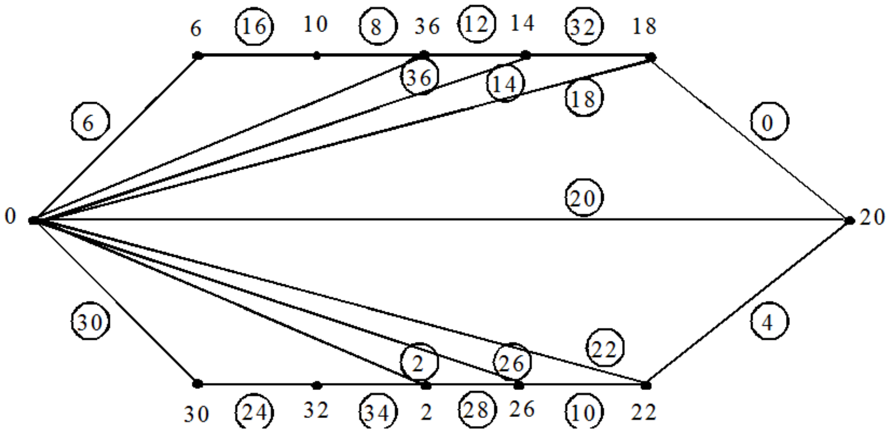


Fig 3.14

H (12,9) (V=12,q=21,2q=42)

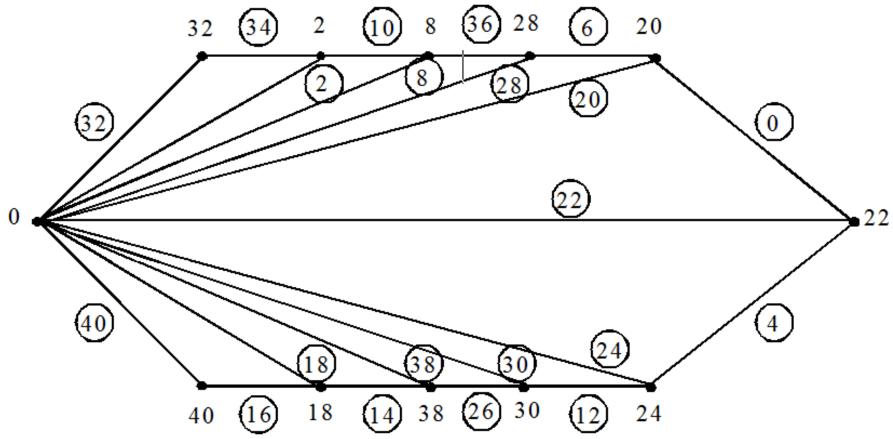


Fig 3.15

Observation3.3

H(4,1) has even harmonious labeling. Moreover, H(4,1) has even harmonious labeling only in the following 8 different ways.

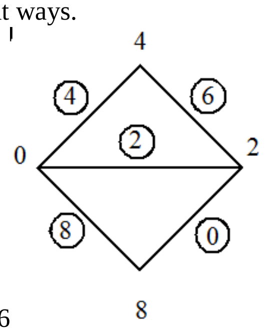


Fig3.16

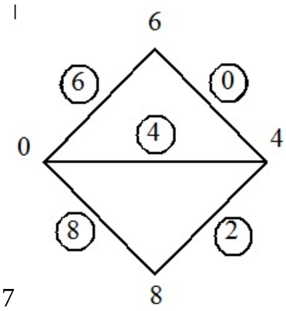


Fig 3.17

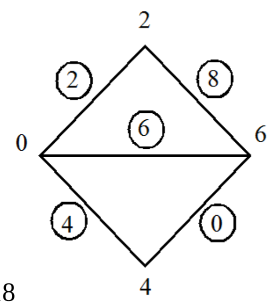


Fig 3.18

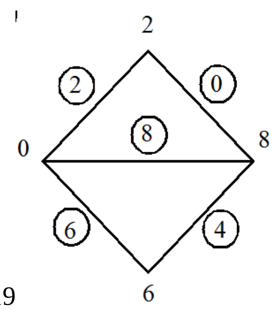


Fig 3.19

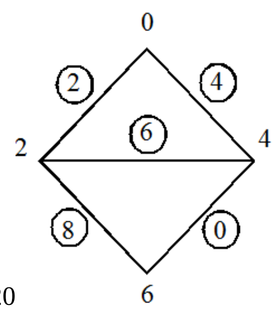


Fig 3.20

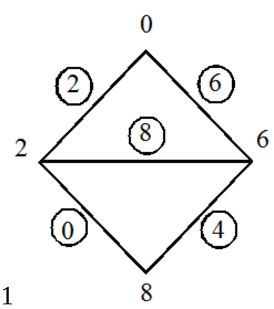
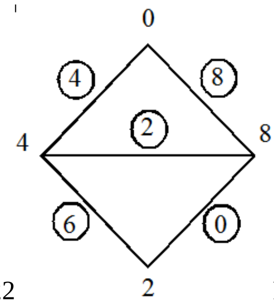


Fig 3.21



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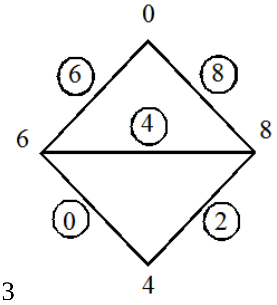


Fig 3.23

Observation3.4
H(6,1) has even harmonious labeling only in 12 different ways as given below.

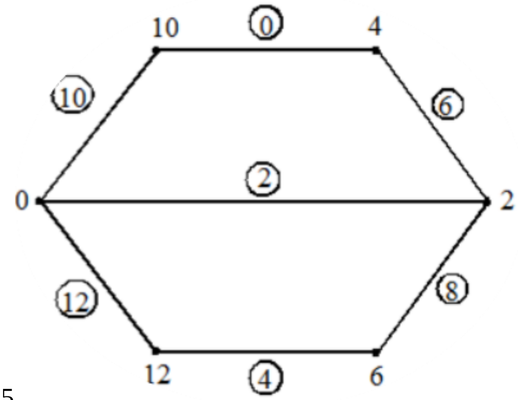


Fig3.25

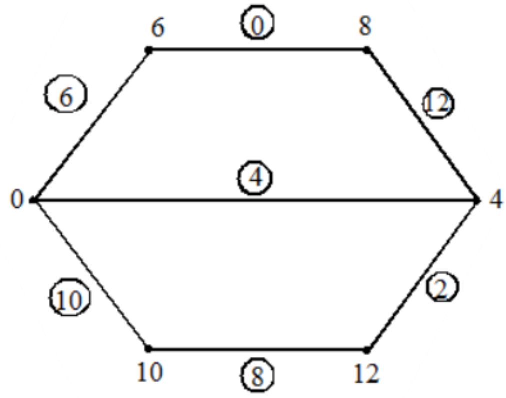


Fig 3.26

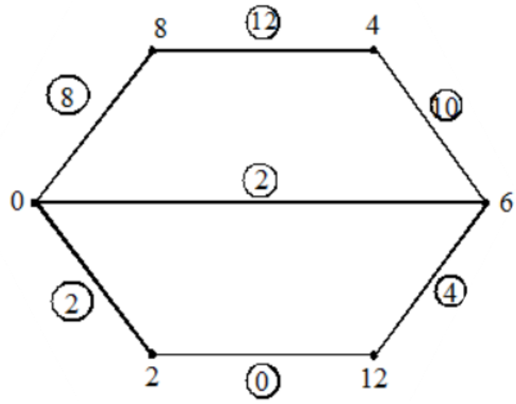


Fig 3.27

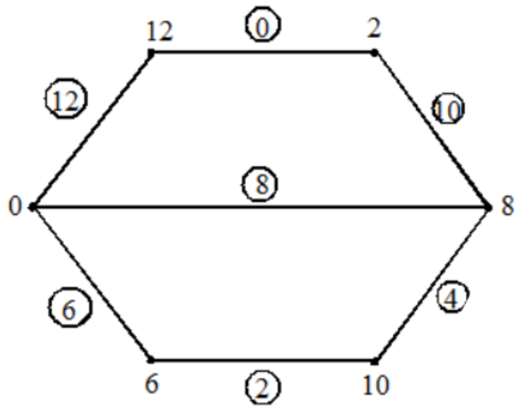


Fig 3.28

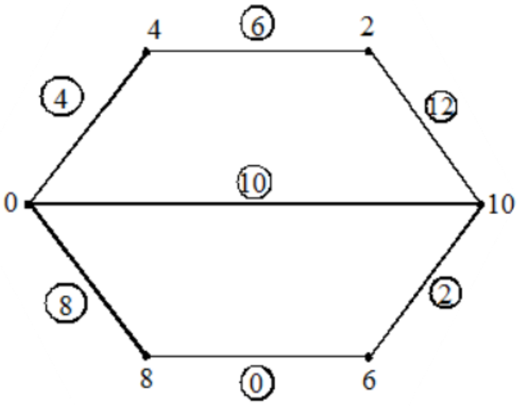


Fig 3.29

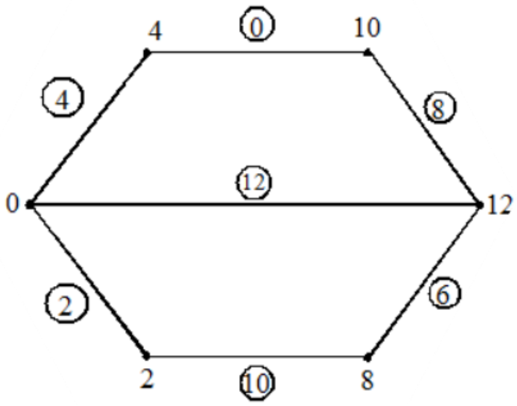


Fig 3.30

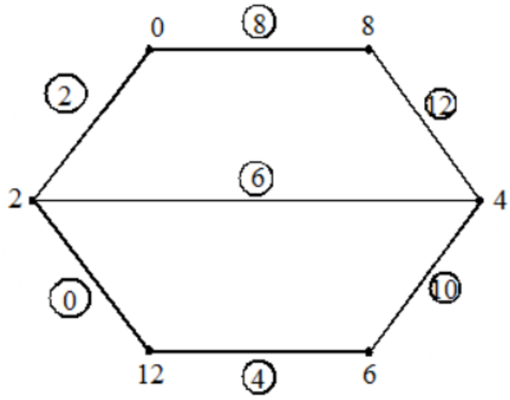
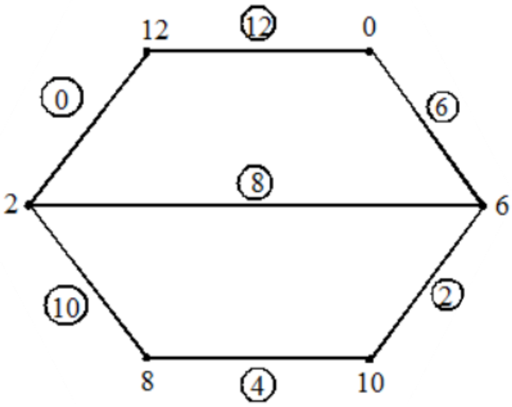


Fig 3.31



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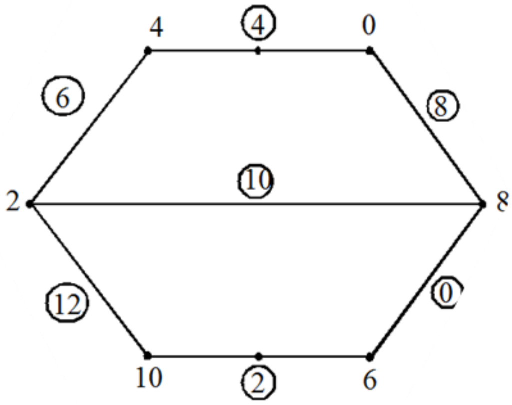

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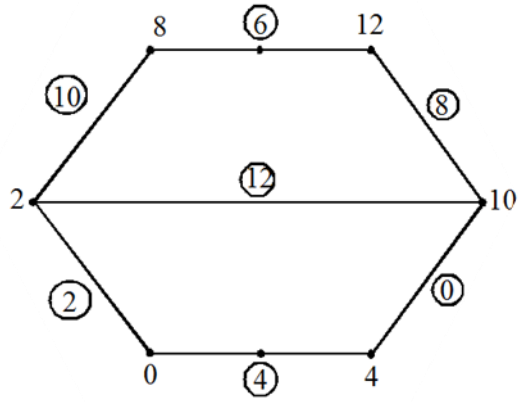

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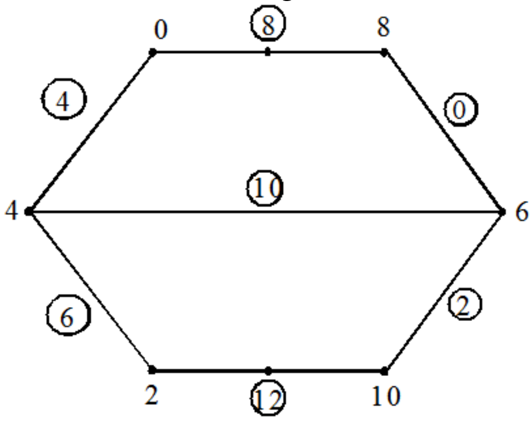
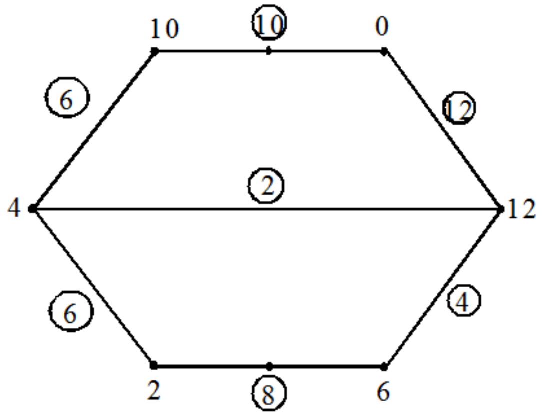


Fig 3.35

Fig 3.36



IV. Conclusion

We proceed this work to find the number of ways in which an even harmonious labeling can be assigned for the graphs H (2n, 2t+1) for n=4,5,6,...

Reference

[1] A.Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris (1967)349-355.
[2] R.L.Graham and J.J.A.Sloane, On additive bases and harmonious graphs, SIAM J. Alg. Discrete Methods.1 (1980) 382-404.
[3] P.B. Sarasija and R. Binthiya, Even harmonious graphs with application, International Journal of Computer Science and Information Security,(2011).
[4] Lori Ann Schoenhard, Even harmonious graphs (2013).
[5] J.A.Gallian, A dynamic survey of graphlabeling, Electronic Journal of Combinatorics,(2012).