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Some Interesting Facts, Myths and History of Mathematics

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ABSTRACT : *This paper deals with primary concepts and fallacies of mathematics which many a times students and even teachers ignore. Also this paper comprises of history of mathematical symbols, notations and methods of calculating time. I have also included some ancient techniques of solving mathematical real time problems. This paper is a confluence of various traditional mathematical techniques and their implementation in modern mathematics.*

I. INTRODUCTION

I have heard my father saying that "Mathematics is the only genuine subject as it does not change with boundary of countries". It is lucrative just because of its simplicity. Galileo once said, "Mathematics is the language with which God wrote the Universe." He was precise in calling mathematics a language, because like any dialect, mathematics has its own rubrics, formulas, and nuances. In precise, the symbols used in mathematics are quite unique to its field and are profoundly engrained in history. The following will give an ephemeral history of some of the greatest well-known symbols employed by mathematics. Categorized by discipline within the subject, each section has its own interesting subculture surrounding it. Arithmetic is the most rudimentary part of mathematics and covers addition, subtraction, multiplication, and the division of numbers. One category of numbers are the integers, -n,…-3,-2,-1,0,1,2,3,…n , where we say that n is in $\mathbb{Z}$.The capital letter Z is written to represent integers and comes from the German word, Zahlen, meaning numbers. Two vital operations in mathematics, addition, +, and subtraction, -, credit the use of their symbols to fourteenth and fifteenth century mathematicians. Nicole d' Oresme, a Frenchman who lived from 1323-1382, used the + symbol to abbreviate the Latin "et", meaning "and", in his *AlgorismusProportionum.*

The fourteenth century Dutch mathematician Giel Vander Hoecke, used the plus and minus signs in his *Eensonderlingheboeck in dye edelconsteArithmetica* and the Brit Robert Recorde used the same symbols in his 1557 publication, *The Whetstone of Witte* (Washington State Mathematics Council). The division and multiplication signs have correspondingly fascinating past. The symbol for division,÷, called an obelus, was first used in 1659, by the Swiss mathematician Johann Heinrich Rahn in his work entitled *TeutscheAlgebr.* The symbol was later presented to London when the English mathematician Thomas Brancker deciphered Rahn's work (Cajori, A History of Mathematics, 140). Descartes, who lived in the primary part of the 1600's, turned the German Cossits "√" into the square root symbol that we now have, by knocking a bar over it .The symbol "∞" meaning infinity, was first presented by Oughtred's student, John Wallis, in his 1655 book *De SectionibusConicus* .It is theorized that Wallis borrowed the symbol ∞ from the Romans, which meant 1,000 (A History of Mathematical Notations, 44). Preceding this, Aristotle (384-322 BC) is noted for saying three things about infinity: i) the infinite exists in nature and can be identified only in terms of quantity, ii) if infinity exists it must be defined, and iii) infinity do not exist in realism. From these three statements Aristotle came to the conclusion that mathematicians had no use for infinity. This idea was later refuted and the German mathematician, Georg Cantor, who lived from 1845-1918, is quoted as saying; "I experience true pleasure in conceiving infinity as I have, and I throw myself into it…And when I come back down toward finiteness, I see with equal clarity and beauty the two concepts [of ordinal numbers(first, second, third etc.) and cardinal numbers (one, two, three etc.)] once more becoming one and converging in the concept of finite integer". Cantor not only acknowledged infinity, but used aleph, the first letter of the Hebrew alphabet, as its symbol. Cantor referred to it as "transfinite". Another interesting fact is that Euler, while accepting the concept of infinity did not use the familiar ∞ symbol, but instead he wrote a sideways "s".

1.1 Intersection and union

The notations ∩ and ∪ were used by Giuseppe Peano (1858-1932) for intersection and union in 1888 in *Calcologeometrico secondo l'Ausdehnungslehre di H. Grassmann* (Cajori vol. 2, page 298); the logical part of this work with this notations is ed. in: Peano, operescelte, 2, Rom 1958, p. 3-19.Peano also shaped the large notations for general intersection and union of more than two classes in 1908.

1.2 Existence (existential quantifier)
Peano used ∃ in volume II, number 1, of his *Formulaire de mathematiqués,* which was published in 1897 (Cajori vol. 2, page 300).Kevin C. Klement writes, "While Peano had the backwards E for a predicate of classes, Russell was the first one to practice the backwards E as a variable binding operator, and there are the delightful manuscripts printed in CPBR vol 4 in which Russell's makes large dots out of Peano's backwards epsilons to change over from the Peano-notation for existence to a more Fregean one."

1.3 Membership
Giuseppe Peano (1858-1932) used an epsilon for membership in *Arithmeticesprinicipia nova methodoexposita,* Turin 1889 .He specified that the notation was an abbreviation for *est*; the entire work is in Latin. Peano's notation for membership appears to be a lunate (or uncial) epsilon, and not the stylized epsilon ∈ that is now used. *This web page previously stated that the modern stylized epsilon was adopted by Bertrand Russell in Principles of Mathematicsin 1903; however, Russell stated he was using Peano's notation, and it appears also to be a lunate epsilon, and is not intended to be the modern notation.* Peano's *I Principii di geometrialogicamenteesposti,* also 1889, has the more common epsilon ε. The notation ∉ for negated membership was apparently introduced in 1939 by Bourbaki, Nicholas.

1.4 Such that
According to Julio González Cabillón, Peano introduced the backwards lower-case epsilon for "such that" in "Formulaire de Mathematiques vol. II, #2" (p. iv, 1898).Peano introduced the backwards lower-case epsilon for "such that" in his 1889 "Principles of arithmetic, presented by a new method," according van Heijenoort's *From Frege to Gödel: A Source Book in Mathematical Logic, 1879--1931* [Judy Green].

1.5 For all
According to M. J. Cresswell and Irving H. Anellis, ∀ originated in Gerhard Gentzen, "Untersuchungenueber das logischeSchliessen," *Math. Z.*, **39**, (1935), p, 178. In footnote 4 on that page, Gentzen explains how he came to use the sign. It is the "All-Zeichen," an analogy with ∃for the existential quantifier which Gentzen says that he borrowed from Russell.Cajori, however, shows that Peano used ∃before Russell and Whitehead (whose backwards E had serifs, unlike Peano's). Russell used the notation **(*x*)** for "for all *x*". See his "Mathematical Logic as Based on the Theory of Types," *American Journal of Mathematics*, **30**, (1908), 222-261. [Denis Roegel].

1.6 Braces enclosing the elements of a set
The notation {a} for a set with only one Element and {a, b} for a set with two Elements in the modern sense introduced by Ernst Zermelo 1907 in "Untersuchungenüber die Grundlagen der Mengenlehre," *Mathematische Annalen* 65 (1908), page 263. Georg Cantor used the set brackets {a, b},..., {a, b, ...} earlier in 1878 in "EinBeitragzurMannifaltigkeitslehre" in *Crelles Journal fürMathematik*, 84 (1878), p. 242-258, however in another meaning: here {a,b} not a set with two Elements, but the disjoint intersection of the sets a and b.

1.7 Negation
The tilde ~ for negation was used by Peano in 1897. See Peano, "Studii di logicamatematica," ed. in: Peano, operescelte, 2, Rom 1958, p. 211. [Kevin C. Klement] ***~p*** for "the negation of *p*" appears in 1908 in the article "Mathematical logic as based on the theory of types" by Bertrand Russell [Denis Roegel].The notationism was also used in 1910 by Alfred North Whitehead and Bertrand Russell in the first volume of *Principia mathematica* (Cajori vol. 2, page 307).
The main notation for negation wich is used today is ¬. It was introduced in 1930 by ArendHeyting in "Die formalenRegelnder intuitionisti schen Logik," *Sitzungsberichte der preu ßischen Akademie der Wissen schaften,* phys.-math. Klasse, 1930, p. 42-65. The ¬ appears on p. 43. [Wilfried Neumaier].

1.8 Disjunction
∨ for disjunction is found in Russell's manuscripts from 1902-1903 and in 1906 in Russell's paper "The Theory of Implication," in *American Journal of Mathematics* vol. 28, pp. 159-202, according to Kevin C. Klement.***p*** ∨ ***q*** for "*p* or *q*" appears in 1908 in the article (1908) "Mathematical Logic as Based on the Theory of Types," *American Journal of Mathematics*, 30, 222-261. by Bertrand Russell [Denis Roegel].The notationism was also used in 1910 by Alfred North Whitehead and Bertrand Russell in the first volume of *Principia mathematica.* (These authors used *p.q* for "*p* and *q*.") (Cajori vol. 2, page 307)

1.9 Conjunction
The notation ∧ for logical conjunction "and" was introduced in 1930 by ArendHeyting in the same source as shown for the negation notation above. [WilfriedNeumaier]

1.10 Implication
The arrow notation → for the logical implication was introduced in 1922 by David Hilbert in: Hilbert: Neubegründung der Mathematik, 1922, in: AbhandlungenausdemMathematischen Seminar der Hamburger Universität, Band I (1922), 157-177. The notation is found on p. 166. [WilfriedNeumaier]The arrow with double lines ⇒ was introduced 1954 by Nicholas Bourbaki, in: Bourbaki: Theorie des ensembles, 3. edition, Paris, 1954. The notation appears on p. 14. [WilfriedNeumaier]

1.11 Equivalence
The double arrow notation ↔ for the logical equivalence was apparently introduced in 1933 by Albrecht Becker *Die AristotelischeTheorie der Möglichkeitsschlüsse,* Berlin, 1933, page 4. [WilfriedNeumaier]
The double arrow with double line ⇔ was introduced 1954 by Nicholas Bourbaki, in: Bourbaki: Theorie des ensembles, 3. edition, Paris, 1954. The notation appears on p. 31. [WilfriedNeumaier]

1.12 The null set notation (Ø)
First appeared in N. Bourbaki *Éléments de mathématique Fasc.1: Les structures fondamentales de l'analyse; Liv.1: Theorie de ensembles.* (*Fascicule de resultants*) (1939): "certainespropriétés... ne sontvraies pour *aucun* élément de E... la partiequ'ellesdéfinissentestappelée la *partie vide* de E, et designée par la notation Ø." (p. 4.)

1.13 The "therefore" notation (∴)
It was first published in 1659 in the original German edition of *Teusche Algebra* by Johann Rahn (1622-1676) (Cajori vol. 1, page 212, and vol1., page 282).

1.14 The halmos (a box indicating the end of a proof)
In his *Measure Theory* (1950, p. 6) P. R. Halmos writes, "The notation ▮ is used throughout the entire book in place of such phrases as "Q.E.D." or "This completes the proof of the theorem" to signal the end of a proof."
On the last page of his autobiography, Paul R. Halmos writes: My most nearly immortal contributions are an abbreviation and a typographical notation. I invented "iff", for "if and only if"—but I could never believe that I was really its first inventor. I am quite prepared to believe that it existed before me, but I don't *know* that it did, and my invention (re-invention?) of it is what spread it thorugh the mathematical world. The notation is definitely not my invention—it appeared in popular magazines (not mathematical ones) before I adopted it, but, once again, I seem to have introduced it into mathematics. It is the notation that sometimes looks like □, and is used to indicate an end, usually the end of a proof. It is most frequently called the "tombstone", but at least one generous author referred to it as the "halmos". This quote is from *I Want to Be a Mathematician: An Automathography,* by Paul R. Halmos, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985, page 403.

1.15 The aleph null notation
It was conceived by Georg Cantor (1845-1918) around 1893, and became widely known after "BeiträgezurBegründung der transfinitenMengenlehre" [Contributions to the Foundation of Transfinite Set Theory] saw the light in MathematischeAnnalen [vol. 46], B. G. Teubner, Leipzig, 1895.On page 492 of this prestigious journal we find the paragraph *Die kleinste transfinite CardinalzahlAlef-null* [The minimum transfinite cardinal number Aleph null], and the following:...wirnennen die ihrzukommendeCardinalzahl, in Zeichen, ℵ ... [We call the cardinal number related to that (set); in notation, ℵ].In a letter dated April 30, 1895, Cantor wrote, "it seemed to me that for this purpose, other alphabets were [already] over-used" (translation by Martin Davis).In *Georg Cantor,* Dauben says that Cantor did not want to use Roman or Greek alphabets, because they were already broadly used, and "His new numbers deserved something unique. ... Not wishing to discover a new notation himself, he picked the aleph, the first letter of the Hebrew alphabet...the aleph could be taken to represent new beginnings...." Avinoam Mann points out that aleph is also the first letter of the Hebrew word "Einsof," which means infinity and that the Kabbalists use "einsof" for the Godhead.
Although his father was a Lutheran and his mother was a Roman Catholic, Cantor had at least some Jewish ancestry .(Julio González Cabillón. contributed to this entry.)

1.16 Set inclusion

According to Cajori (vol. 2, page 294), the notations ⊂ for "is included in" (*untergeordnet*) and ⊃ for "includes" (*übergeordnet*) were introduced by Schröder *Vorlesungenüber die Algebra der Logik vol. 1* (1890). Previously the notations < and > had been used. According W. V. Quine, *Methods of Logic,* 4th ed., Harvard University Press, 1982, page 132: "The inclusion signs ⊆ and ⊂ now present in set theory, are derived from Gergonne's use in 1816 of 'C' for containment."

II. MYTHS ABOUT SYMBOL

One mendacious belief which present day mathematicians are nourishing is that Null Set is denoted by Greek Notation Φ(Phi). Common notations for the **empty set** include "{}", "∅", and " ". The latter two **notations** were introduced by the Bourbaki group (specifically André Weil) in 1939, inspired by the letter Ø in the Norwegian and Danish alphabets (and not related in any way to the Greek letter Φ which is read as *"Phi"*). ***Phi*** is the 21st letter of the Greek alphabet and it does not denote an Empty Set. Null, void, empty and vacuous are synonymous and interchangeable. ∅and Φ are not same (∅ is Danish Symbol while Φ is a Greek Symbol).

One thing which is noticeable is that {∅} is an example of singleton set as this is a set of an element which is Null. When speaking of the sum of the elements of a finite set, one is inevitably led to the convention that the sum of the elements of the empty set is zero. The reason for this is that zero is the identity element for addition. Similary, the product of the elements of the empty set should be considered to be one (see empty product), since one is the identity element for multiplication. A disarrangement of a set is a permutation of the set that leaves no element in the same position. The empty set is a disarrangement of itself as no element can be found that retains its original position.

III. GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE OF NEGATIVE NUMBERS

The **least common multiple** (also called the **lowest common multiple** or **smallest common multiple**) of two integers *a* and *b*, usually denoted by ***LCM*(*a*, *b*)**, is the smallest positive integer that is divisible by both *a* and *b*. Since division of integers by zero is undefined, this definition has meaning only if *a* and *b* are both different from zero. However, some authors define *LCM*(*a*,0) as 0 for all *a*, which is the result of taking the *LCM* to be the least upper bound in the lattice of divisibility.

We know that *LCM*(2,3)=6. Let us find *LCM*(2,-3), *LCM*(-2,3) and *LCM*(-2,-3). A naïve mathematician will come with answer -6.But Least common multiple means that multiple must be minimum and if we look on number line -12<-6 and -18<-12. So, we will keep on moving on negative side of number line and we will never come with a solution. So question arises what should be appropriate answer.

We may define $LCM(a,b) = lcm(|a|,|b|) \forall\ a,b \in \mathbb{z}$.

The least common multiple can be defined generally over commutative rings as follows: Let *a* and *b* be elements of a commutative ring *R*. A common multiple of *a* and *b* is an element *m* of *R* such that both *a* and *b* divide *m* (i.e. there exist elements *x* and *y* of *R* such that *ax* = *m* and *by* = *m*). A least common multiple of *a* and *b* is a common multiple that is minimal in the sense that for any other common multiple *n* of *a* and *b*, *m* divides *n*. The *LCM* of more than two integers is also well-defined: it is the smallest positive integer that is divisible by each of them.

LCM (2, -3) = *LCM* (-2, 3) = *LCM* (-2, -3) = *LCM* (2,3) = 6.

In general, two elements in a commutative ring can have no least common multiple or more than one. However, any two least common multiples of the same pair of elements are associates. In a unique factorization domain, any two elements have a least common multiple. In a principal ideal domain, the least common multiple of *a* and *b* can be characterised as a generator of the intersection of the ideals **generated** by *a* and *b* (the intersection of a collection of ideals is always an ideal).The Greatest Common Factor (HCF) or Greatest Common Divisor (GCD) of two non-zero integers is the largest positive integer that divides both numbers without remainder.(The negative number sign may be ignored as divisibility is not affected).The Highest Common Factor (HCF) or Greatest Common Divisor (GCD) of two non-zero integers is the largest positive integer that divides both numbers without remainder.(The negative number sign may be ignored as divisibility is not affected)

$HCF(x,y)=HCF(x,-y)=HCF(-x,-y)=HCF(-x,y) \quad \forall x,y \in N$

More generalized definition will be

$HCF(a,b)=HCF(|a|,|b|) \ \forall a,b \in z.$

Computer Code for GCD

```
#include <stdio.h>
int main()
{
int a, b;
```

```
printf("Enter two integers: ");
scanf("%d %d",&a,&b);

// if user enters negative number, sign of the number is changed to positive
a = ( a> 0) ? a : -a;
b = ( b> 0) ? b : -b;

while(a!=b)
{
if(a > b)
a -= b;
else
b -= a;
}
printf("GCD = %d",a);

return 0;
}
```

IV. REMAINDER

Different programming languages have adopted different conventions: Pascal showed the result of the *mod* operation positive, but does not allow *d* to be negative or zero (so, $a = (a \text{ div } d)*d + a \text{ mod } d$ is not always valid). C99 chooses the remainder with the same sign as the dividend *a*. (Before C99, the C language allowed other choices) Perl, Python (only modern versions), and Common Lisp choose the remainder with the same sign as the divisor *d*. Haskell and Scheme offer two functions, *remainder* and *modulo* – PL/I has *mod* and *rem*, while Fortran has *mod* and *modulo*; in each case, the former agrees in sign with the dividend, and the latter with the divisor. When *a* and *d* are floating-point numbers, with *d* non-zero, *a* can be divided by *d* without remainder, with the quotient being another floating-point number. If the quotient is constrained to being an integer, however, the concept of remainder is still necessary. It can be proved that there exists a unique integer quotient *q* and a unique floating-point remainder *r* such that $a = qd + r$ with $0 \le r < |d|$.
Code snippet of remainder using C is stated below.

```
int mod(int a, int b)
{int r = a % b;
return r <0 ? r + b : r;}
```

V. FORGOTTEN TRIGONOMETRIC NOTATIONS

Versine	*versin(θ)=1-cos(θ)*
Vercosine	*vercosin(θ)=1+cos(θ)*
Coversine	*coversin(θ)=1-sin(θ)*
Covercosine	*covercosine(θ)=1+sin(θ)*
Haversine	*haversin(θ)=versin(θ)/2*
Havercosine	*havercosin(θ)=vercosin(θ)/2*
Hacoversine	*hacoversin(θ)=coversin(θ)/2*
Hacovercosine	*hacovercosin(θ)=covercosin(θ)/2*
Exsecant	*exsec(θ)=sec(θ)-1*
Excosecant	*excsc(θ)=csc(θ)-1*

VI. CONCEPT OF BRACKETS (SQUARE BRACKETS, CURLY BRACKETS AND SMALL BRACKETS)

Brackets are frequently used in mathematical notation such as parentheses ( ), square brackets [ ], braces { }, and angle brackets <>. The earliest use of brackets to indicate aggregation (i.e. grouping) was suggested in 1608 by Christopher Clavius and in 1629 by Albert Girard.

{ }	**Braces ("curly braces")** Braces are used to group statements and declarations. The contents of a class or interface are enclosed in braces. Method bodies and constructor bodies are enclosed in braces. Braces are used to group the statements in an **if**statement, a loop, or other control structures.
[ ]	**Brackets ("square brackets")** Brackets are used to index into an array.

( )	**Parentheses**Parentheses are used for two purposes: (1) to control the order of operations in an expression, and (2) to supply parameters to a constructor or method.

VII. CONCEPT OF ZEROTH POWER

Why $0^0 \neq 1$?

Any number raised to power zero is one except zero. We can take an example and demonstrate.

$2^0 = 2^{1-1} = 2^1 \times 2^{-1} = 2 \times \frac{1}{2} = 1$

In order to calculate 0^0 we may write

$0^0 = 0^{1-1} = 0^1 \times 0^{-1} \neq \frac{0}{0}$, (i.e. It is also an indeterminate form)

VIII. DEVELOPMENT OF NUMBER THEORY IN INDIA

During the Vedic period (1500–500 BCE), driven by geometric construction of the fire altars and astronomy, the use of a numerical system and of elementary mathematical operations developed in northern India. Hindu cosmology required the mastery of very huge numbers such as the *kalpa* (the lifetime of the universe) said to be 4,320,000,000 years and the "orbit of the heaven" said to be 18,712,069,200,000,000 *yojanas*. Numbers were expressed using a "named place-value notation", using names for the powers of 10: *dasa*, *shatha*, *sahasra*, *ayuta*, *niyuta*, *prayuta*, *arbuda*, *nyarbuda*, *samudra*, *madhya*, *anta*, *parardha* etc., the last of these being the name for a trillion. For example, the number 26432 was expressed as "2 *ayuta* 6 *sahasra* 4 *shatha* 3 *dasa* 2. In the Buddhist text Lalitavistara, the Buddha is said to have narrated a scheme of numbers up to 10^53.The form of numerals in Ashoka's inscriptions in the Brahmi script (middle of the third century BCE) involved separate signs for the numbers 1 to 9, 10 to 90, 100 and 1000. A multiple of 100 or 1000 was represented by a modification (or "enciphering" of the sign for the number using the sign for the multiplier number. Such enciphered numerals directly represented the named place-value numerals used verbally. They continued to be used in inscriptions till the end of the 9th century.

In his seminal text of 499 CE, Aryabhata devised a novel positional number system, using Sanskrit consonants for small numbers and vowels for powers of 10. Using the system, numbers up to a billion could be expressed using short phrases, e.g., *khyu-ghṛ* representing the number 4,320,000. The system did not catch on because it produced quite unpronouncable phrases, but it might driven home the principle of positional number system (called *dasa-gunottara*, exponents of 10) to later mathematicians. A more elegant *katapayadi* scheme was devised in later centuries representing a place-value system including zero.

The place value system, however, developed later. The Brahmi numerals have been found in engravings in caves and on coins in regions near Pune, Mumbai, and Uttar Pradesh. These numerals (with minor deviations) were in use over a long time span up to the fourth century.

While the numerals in texts and inscriptions used a named place-value notation, a more efficient notation might have been employed in calculations, possibly from the 1st century CE. Computations were carried out on clay tablets covered with a thin layer of sand, giving rise to the term *dhuli-karana* ("sand-work") for higher computation. Karl Menninger believes that, in such computations, they must have dispensed with the enciphered numerals and written down just sequences of digits to represent the numbers. A zero would have been represented as a "missing place," such as a dot. The single manuscript with worked examples available to us, the Bakhshali manuscript (thought to be a copy of an original written in fourth to seventh century CE), uses a place value system with a dot to denote the zero. The dot was called the *shunya-sthāna*, "empty-place." The same symbol was also used in algebraic expressions for the unknown (as in the canonical *x* in modern algebra). However, the date of the Bakhshali manuscript is subject to considerable debate.Textual references to a place-value system are seen from the 1st century CE onwards. The Buddhist philosopher Vasubandhu in the 1st century says "when [the same] clay counting-piece is in the place of units, it is denoted as one, when in hundreds, one hundred." A commentary on Patanjali's Yoga Sutras from the 5th century reads, "Just as a line in the hundreds place [means] a hundred, in the tens place ten, and one in the ones place, so one and the same woman is called mother, daughter and sister."

A system called *bhūta-sankhya* ("object numbers" or "concrete numbers") was employed for representing numerals in Sanskrit verses, by using a concept representing a digit to stand for the digit itself. The Jain text entitled the *Lokavibhaga*, dated 458 CE, mentions the objecit fiednumeral" *panchabhyahkhalushunyebhyahparamdvesaptachambaramekamtrini cha rupamcha*" meaning, "five voids, then two and seven, the sky, one and three and the form", i.e., the number 13107200000. Such objectified numbers were used extensively from the 6th century onwards, especially after Varahamihira (c. 575 CE). Zero is explicitly represented in such numbers as "the void" (*sunya*) or the "heaven-space" (*ambaraakasha*). Correspondingly, the dot used in place of zero in written numreals was referred to as a *sunya-bindu*.

In 628 CE, astronomer-mathematician Brahmagupta wrote his seminal text Brahma SphutaSiddhanta which contained the first mathematical treatment of zero. He defined zero as the result of subtracting a number from itself, postulated negative numbers and discussed their properties under arithmetical operations. His word for zero was *shunya* (void) the same term previously used for the empty spot in 9-digit place-value system.This provided a new perspective on the *shunya-bindu* as a numeral and paved the way for the eventual evolution of a zero digit. The dot continued to be used for at least a 100 years afterwards, and transmitted to Southeast Asia and Arabia. Kashmir's Sharada script has retained the dot for zero till this day.

By the end of the 7th century, decimal numbers begin to appear in inscriptions in Southeast Asia as well as in India. Some scholars hold that they appeared even earlier. A 6th century copper-plate grant at Mankani bearing the numeral 346 (corresponding to 594 CE) is often cited. But its reliability is subject to dispute. The first indisputable occurrence of 0 in an inscription occurs at Gwalior in 876 CE, containing a numeral "270" in a notation surprisingly similar to ours. Throughout the 8th and 9th centuries, both the old Brahmi numerals and the new decimal numerals were used, sometimes appearing in the same inscriptions. In some documents, a transition is seen to occur around 866 CE. During the Gupta period, the Gupta numerals developed from the Brahmi numerals and were spread over large areas by the Gupta empire as they conquered territory. Beginning around 7th century, the Gupta numerals developed into the Nagari numerals.

VIII. HISTORY OF INDIAN MATHEMATICS

Excavations at Harappa, Mohenjo-daro and other sites of the Indus Valley Civilisation have uncovered evidence of the use of "practical mathematics". The people of the IVC manufactured bricks whose dimensions were in the proportion 4:2:1, considered favourable for the stability of a brick structure. They used a standardised system of weights based on the ratios: 1/20, 1/10, 1/5, 1/2, 1, 2, 5, 10, 20, 50, 100, 200, and 500, with the unit weight equaling approximately 28 grams (and approximately equal to the English ounce or Greek uncia). They mass-produced weights in regular geometrical shapes, which included hexahedra, barrels, cones, and cylinders, thereby demonstrating knowledge of basic geometry.

The inhabitants of Indus civilisation also tried to standardise measurement of length to a high degree of accuracy. They designed a ruler—the *Mohenjo-daro ruler*—whose unit of length (approximately 1.32 inches or 3.4 centimetres) was divided into ten equal parts. Bricks manufactured in ancient Mohenjo-daro often had dimensions that were integral multiples of this unit of length.

SAMHITAS AND BRAHMANAS

The religious texts of the Vedic Period provide evidence for the use of large numbers. By the time of the *Yajurvedasaṃhitā*- (1200–900 BCE), numbers as high as 10^{12} were being included in the texts. For example, the *mantra* (sacrificial formula) at the end of the *annahoma* ("food-oblation rite") performed during the *aśvamedha*, and uttered just before-, during-, and just after sunrise, invokes powers of ten from a hundred to a trillion.

Hail to *śata* ("hundred," 10^2), hail to *sahasra* ("thousand," 10^3), hail to *ayuta* ("ten thousand," 10^4), hail to *niyuta* ("hundred thousand," 10^5), hail to *prayuta* ("million,"10^6), hail to *arbuda* ("ten million," 10^7), hail to *nyarbuda* ("hundred million," 10^8), hail to *samudra* ("billion," 10^9, literally "ocean"), hail to *madhya* ("ten billion," 10^{10}, literally "middle"), hail to *anta* ("hundred billion," 10^{11},lit., "end"), hail to *parārdha* ("one trillion," 10^{12} lit., "beyond parts"), hail to the dawn (*uṣas*), hail to the twilight (*vyuṣṭi*), hail to the one which is going to rise (*udeṣyat*), hail to the one which is rising (*udyat*), hail to the one which has just risen (*udita*), hail to *svarga* (the heaven), hail to *martya* (the world), hail to all.

The solution to partial fraction was known to the Rigvedic People as states in the purushSukta (RV 10.90.4):

With three-fourths Puruṣa went up: one-fourth of him again was here.

The SatapathaBrahmana (ca. 7th century BCE) contains rules for ritual geometric constructions that are similar to the Sulba Sutras.

ŚULBASŪTRAS

The *ŚulbaSūtras* (literally, "Aphorisms of the Chords" in Vedic Sanskrit) (c. 700–400 BCE) list rules for the construction of sacrificial fire altars.Most mathematical problems considered in the *ŚulbaSūtras* spring from "a single theological requirement," that of constructing fire altars which have different shapes but occupy the same area. The altars were required to be constructed of five layers of burnt brick, with the further condition that each layer consist of 200 bricks and that no two adjacent layers have congruent arrangements of bricks.

According to (Hayashi 2005, p. 363), the *ŚulbaSūtras* contain "the earliest extant verbal expression of the Pythagorean Theorem in the world, although it had already been known to the Old Babylonians."

The diagonal rope (*akṣṇayā-rajju*) of an oblong (rectangle) produces both which the flank (*pārśvamāni*) and the horizontal (*tiryaṇmānī*) <ropes> produce separately."
Since the statement is a *sūtra*, it is necessarily compressed and what the ropes *produce* is not elaborated on, but the context clearly implies the square areas constructed on their lengths, and would have been explained so by the teacher to the student.
They contain lists of Pythagorean triples, which are particular cases of Diophantine equations .They also contain statements (that with hindsight we know to be approximate) about squaring the circle and "circling the square."

Baudhayana (c. 8th century BCE) composed the *BaudhayanaSulba Sutra*, the best-known *Sulba Sutra*, which contains examples of simple Pythagorean triples, such as:(3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), and (12, 35, 37),as well as a statement of the Pythagorean theorem for the sides of a square: "The rope which is stretched across the diagonal of a square produces an area double the size of the original square."[1] It also contains the general statement of the Pythagorean theorem (for the sides of a rectangle): "The rope stretched along the length of the diagonal of a rectangle makes an area which the vertical and horizontal sides make together."Baudhayana gives a formula for the square root of two.The formula is accurate up to five decimal places, the true value being 1.41421356.This formula is similar in structure to the formula found on a Mesopotamian tabletfrom the Old Babylonian period (1900–1600 BCE) which expresses $\sqrt{2}$ in the sexagesimal system, and which is also accurate up to 5 decimal places (after rounding).
According to mathematician S. G. Dani, the Babylonian cuneiform tablet Plimpton 322 written ca. 1850 BCE "contains fifteen Pythagorean triples with quite large entries, including (13500, 12709, 18541) which is a primitive triple, indicating, in particular, that there was sophisticated understanding on the topic" in Mesopotamia in 1850 BCE. "Since these tablets predate the Sulbasutras period by several centuries, taking into account the contextual appearance of some of the triples, it is reasonable to expect that similar understanding would have been there in India." Dani goes on to say:As the main objective of the *Sulvasutras* was to describe the constructions of altars and the geometric principles involved in them, the subject of Pythagorean triples, even if it had been well understood may still not have featured in the *Sulvasutras*. The occurrence of the triples in the *Sulvasutras* is comparable to mathematics that one may encounter in an introductory book on architecture or another similar applied area, and would not correspond directly to the overall knowledge on the topic at that time. Since, unfortunately, no other contemporaneous sources have been found it may never be possible to settle this issue satisfactorily.
In all, three *Sulba Sutras* were composed. The remaining two, the *ManavaSulba Sutra* composed by Manava (fl. 750–650 BCE) and the *ApastambaSulba Sutra*, composed byApastamba (c. 600 BCE), contained results similar to the *BaudhayanaSulba Sutra*.

VYAKARANA

An important landmark of the Vedic period was the work of Sanskrit grammarian, Pāṇini (c. 520–460 BCE). His grammar includes early use of Boolean logic, of the null operator, and of context free grammars, and includes a precursor of the Backus–Naur form (used in the description programming languages).

PINGALA

Among the scholars of the post-Vedic period who contributed to mathematics, the most notable is Pingala (*piṅgalá*) (fl. 300–200 BCE), a musical theorist who authored theChhandas Shastra (*chandaḥ-śāstra*, also Chhandas Sutra *chhandaḥ-sūtra*), a Sanskrit treatise on prosody. There is evidence that in his work on the enumeration of syllabic combinations, Pingala stumbled upon both the Pascal triangle and Binomial coefficients, although he did not have knowledge of the Binomial theorem itself. Pingala's work also contains the basic ideas of Fibonacci numbers (called *maatraameru*). Although the *Chandah sutra* hasn't survived in its entirety, a 10th-century commentary on it by Halāyudha has. Halāyudha, who refers to the Pascal triangle as *Meru-prastāra* (literally "the staircase to Mount Meru"), has this to say:Draw a square. Beginning at half the square, draw two other similar squares below it; below these two, three other squares, and so on. The marking should be started by putting **1** in the first square. Put **1** in each of the two squares of the second line. In the third line put **1** in the two squares at the ends and, in the middle square, the sum of the digits in the two squares lying above it. In the fourth line put **1** in the two squares at the ends. In the middle ones put the sum of the digits in the two squares above each. Proceed in this way. Of these lines, the second gives the combinations with one syllable, the third the combinations with two syllables.

KATYAYANA

Katyayana (c. 3rd century BCE) is notable for being the last of the Vedic mathematicians. He wrote the *KatyayanaSulba Sutra*, which presented much geometry, including the general Pythagorean theorem and a computation of the square root of 2 correct to five decimal places.

JAIN MATHEMATICS (400 BCE – 200 CE)

Although Jainism as a religion and philosophy predates its most well-known exponent, the great Mahavira (6th century BCE), most Jain texts on mathematical topics were composed after the 6th century BCE. Jain mathematicians are significant historically as vital links between the mathematics of the Vedic period and that of the "conventional period."A significant historical contribution of Jain mathematicians lay in their freeing Indian mathematics from its religious and ritualistic constraints. In fussy, their fascination with the enumeration of very large numbers and infinities led them to classify numbers into three classes: enumerable, innumerable and infinite. Not satisfied with a simple notion of infinity, they went on to define five different types of infinity: the infinite in one direction, the infinite in two directions, the infinite in area, the infinite everywhere, and the infinite perpetually. In addition, Jain mathematicians devised notations for simple powers (and exponents) of numbers like squares and cubes, which enabled them to define simple algebraic equations (beejganitasamikaran). Jain mathematicians were apparently also the first to use the word shunya (literally void in Sanskrit) to refer to zero. More than a millennium later, their appellation became the English word "zero" after a meandering journey of translations and transliterations from India to Europe. In addition to Surya Prajnapti, important Jain works on mathematics included the Vaishali Ganit (c. 3rd century BCE); the Sthananga Sutra (fl. 300 BCE – 200 CE); theAnoyogdwar Sutra (fl. 200 BCE – 100 CE); and the Satkhandagama (c. 2nd century CE). Important Jain mathematicians included Bhadrabahu (d. 298 BCE), the author of two astronomical works, the Bhadrabahavi-Samhita and a commentary on the Surya Prajinapti; Yativrisham Acharya (c. 176 BCE), who authored a mathematical text calledTiloyapannati; and Umasvati (c. 150 BCE), who, although better known for his powerful writings on Jain philosophy and metaphysics, composed a mathematical work calledTattwarthadhigama-Sutra Bhashya.

ORAL TRADITION

Mathematicians of ancient and early medieval India were almost all Sanskrit pandits ,, who were trained in Sanskrit language and literature, and obsessed "a familiar stock of knowledge in grammar (vyākaraṇa), exegesis (mīmāṃsā) and logic (nyāya)." Memorisation of "what is heard" (śruti in Sanskrit) through recitation played a major role in the transmission of holy texts in ancient India. Memorisation and recitation was also used to broadcast philosophical and literary works, as well as treatises on ritual and grammar. Modern scholars of ancient India have noted the "truly remarkable achievements of the Indian pandits who have preserved enormously large texts orally for millennia."

STYLES OF MEMORIZATION

Prodigous energy was expended by ancient Indian culture in ensuring that these texts were transmitted from generation to generation with inordinate fidelity. For example, memorisation of the sacred Vedas included up to eleven forms of recitation of the same text. The texts were subsequently "proof-read" by comparing the different recited versions. Forms of recitation included the jaṭā-pāṭha (literally "mesh recitation") in which every two adjacent words in the text were first recited in their original order, then repeated in the reverse order, and finally repeated again in the original order. The recitation thus proceeded as:**word1word2, word2word1, word1word2; word2word3, word3word2, word2word3;** In another form of recitation, dhvaja-pāṭha(literally "flag recitation") a sequence of N words were recited (and memorised) by pairing the first two and last two words and then proceeding as:**word$_1$word$_2$, word$_{N-1}$word$_N$; word$_2$word$_3$, word$_{N-3}$word$_{N-2}$; ..; word$_{N-1}$word$_N$, word$_1$word$_2$;**The most complex form of recitation, ghana-pāṭha (literally "dense recitation"), according to (Filliozat 2004, p. 139), took the form:**word1word2, word2word1, word1word2word3, word3 word2 word1, word1word2word3; word2word3, word3word2, word2word3word4, word4 word3word2, word2 word3 word4;**

Mathematical activity in ancient India began as a part of a "methodological reflexion" on the sacred Vedas, which took the form of works called Vedāṇgas, or, "Ancillaries of the Veda" (7th–4th century BCE). The need to conserve the sound of sacred text by use of śikṣā (phonetics) and chhandas (metrics); to conserve its meaning by use of vyākaraṇa(grammar) and nirukta (etymology); and to correctly perform the rites at the correct time by the use of kalpa (ritual) and jyotiṣa (astrology), gave rise to the six disciplines of theVedāṇgas. Mathematics arose as a part of the last two disciplines, ritual and astronomy (which also included astrology). Since the Vedāṇgas immediately preceded the use of writing in ancient India, they formed the last of the exclusively oral literature. They were expressed in a highly compressed mnemonic form, the sūtra (literally, "thread"):

The knowers of the sūtra know it as having few phonemes, being devoid of ambiguity, containing the essence, facing everything, being without pause and unobjectionable.Extreme brevity was achieved through multiple means, which included using ellipsis "beyond the tolerance of natural language," using technical names instead of longer descriptive names, abridging lists by only mentioning the first and last entries, and using markers and variables. The sūtras create the impression that communication through the text was "only a part of the whole instruction. The rest of the instruction must have been transmitted by the so-called Guru-shishyaparamparai, 'uninterrupted succession from teacher (guru) to the student (śisya),' and it was not open to the general public" and perhaps even kept secret. The brevity achieved in a sūtra is demonstrated in the following example from the Baudhāyana ŚulbaSūtra (700 BCE).The domestic fire-altar in the Vedic period was required by ritual to have a square base and be constituted of five layers of bricks with 21 bricks in each layer. One method of constructing the altar was to divide one side of the square into three equal parts using a cord or rope, to next divide the transverse (or perpendicular) side into seven equal parts, and thereby sub-divide the square into 21 congruent rectangles. The bricks were then designed to be of the shape of the constituent rectangle and the layer was created. To form the next layer, the same formula was used, but the bricks were arranged transversely.The process was then repeated three more times (with alternating directions) in order to complete the construction. In the Baudhāyana ŚulbaSūtra, this procedure is described in the following words:

II.64. After dividing the quadri-lateral in seven, one divides the transverse [cord] in three.
II.65. In another layer one places the [bricks] North-pointing.

According to (Filliozat 2004, p. 144), the officiant constructing the altar has only a few tools and materials at his disposal: a cord (Sanskrit, rajju, f.), two pegs (Sanskrit, śanku, m.), and clay to make the bricks (Sanskrit, iṣṭakā, f.). Concision is achieved in the sūtra, by not explicitly mentioning what the adjective "transverse" qualifies; however, from the feminine form of the (Sanskrit) adjective used, it is easily inferred to qualify "cord." Similarly, in the second stanza, "bricks" are not explicitly mentioned, but inferred again by the feminine plural form of "North-pointing." Finally, the first stanza, never explicitly says that the first layer of bricks are oriented in the East-West direction, but that too is implied by the explicit mention of "North-pointing" in the second stanza; for, if the orientation was meant to be the same in the two layers, it would either not be mentioned at all or be only mentioned in the first stanza. All these inferences are made by the officiant as he recalls the formula from his memory

With the increasing complexity of mathematics and other exact sciences, both writing and computation were required. Consequently, many mathematical works began to be written down in manuscripts that were then copied and re-copied from generation to generation.

India today is estimated to have about thirty million manuscripts, the largest body of handwritten reading material anywhere in the world. The literate culture of Indian science goes back to at least the fifth century B.C. ... as is shown by the elements of Mesopotamian omen literature and astronomy that entered India at that time and (were) definitely not ... preserved orally.

The earliest mathematical prose commentary was that on the work, Āryabhaṭīya (written 499 CE), a work on astronomy and mathematics. The mathematical portion of theĀryabhaṭīya was composed of 33 sūtras (in verse form) consisting of mathematical statements or rules, but without any proofs. However, according to (Hayashi 2003, p. 123), "this does not necessarily mean that their authors did not prove them. It was probably a matter of style of exposition." From the time of Bhaskara I (600 CE onwards), prose commentaries increasingly began to include some derivations (upapatti). Bhaskara I's commentary on the Āryabhaṭīya, had the following structure.

Rule ('sūtra') in verse by Āryabhaṭa

Commentary by Bhāskara I, consisting of:

Elucidation of rule (derivations were still rare then, but became more common later)

Example (uddeśaka) usually in verse.

Setting (nyāsa/sthāpanā) of the numerical data.

Working (karana) of the solution.

Verification (pratyayakaraṇa, literally "to make conviction") of the answer. These became rare by the 13th century, derivations or proofs being favoured by then.

Typically, for any mathematical topic, students in ancient India first memorised the sūtras, which, as explained earlier, were "deliberately inadequate" in explanatory details (in order to pithily convey the bare-bone mathematical rules). The students then worked through the topics of the prose commentary by writing (and drawing diagrams) on chalk- and dust-boards (i.e. boards covered with dust). The latter activity, a staple of mathematical work, was to later prompt mathematician-astronomer, Brahmagupta (fl. 7th century CE), to characterise astronomical computations as "dust work" (Sanskrit: dhulikarman).

It is well known that the decimal place-value system in use today was first recorded in India, then transmitted to the Islamic world, and eventually to Europe. The Syrian bishop Severus Sebokht wrote in the mid-7th century CE about the "nine signs" of the Indians for expressing numbers. However, how, when, and where the first decimal place value system was invented is not so clear.

The earliest extant script used in India was the Kharoṣṭhī script used in the Gandhara culture of the north-west. It is thought to be of Aramaic origin and it was in use from the 4th century BCE to the 4th century CE. Almost contemporaneously, another script, the Brāhmī script, appeared on much of the sub-continent, and would later become the foundation of many scripts of South Asia and South-east Asia. Both scripts had numeral symbols and numeral systems, which were initially not based on a place-value system.

The earliest surviving evidence of decimal place value numerals in India and southeast Asia is from the middle of the first millennium CE.A copper plate from Gujarat, India mentions the date 595 CE, written in a decimal place value notation, although there is some doubt as to the authenticity of the plate. Decimal numerals recording the years 683 CE have also been found in stone inscriptions in Indonesia and Cambodia, where Indian cultural influence was substantial.

There are older textual sources, although the extant manuscript copies of these texts are from much later dates. Probably the earliest such source is the work of the Buddhist philosopher Vasumitra dated likely to the 1st century CE. Discussing the counting pits of merchants, Vasumitra remarks, "When [the same] clay counting-piece is in the place of units, it is denoted as one, when in hundreds, one hundred."Although such references seem to imply that his readers had knowledge of a decimal place value representation, the "brevity of their allusions and the ambiguity of their dates, however, do not solidly establish the chronology of the development of this concept."

A third decimal representation was employed in a verse composition technique, later labelled Bhuta-sankhya (literally, "object numbers") used by early Sanskrit authors of technical books.Since many early technical works were composed in verse, numbers were often represented by objects in the natural or religious world that correspondence to them; this allowed a many-to-one correspondence for each number and made verse composition easier. According to Plofker 2009, the number 4, for example, could be represented by the word "Veda" (since there were four of these religious texts), the number 32 by the word "teeth" (since a full set consists of 32), and the number 1 by "moon" (since there is only one moon). So, Veda/teeth/moon would correspond to the decimal numeral 1324, as the convention for numbers was to enumerate their digits from right to left. The earliest reference employing object numbers is a ca. 269 CE Sanskrit text, Yavanajātaka (literally "Greek horoscopy") of Sphujidhvaja, a versification of an earlier (ca. 150 CE) Indian prose adaptation of a lost work of Hellenistic astrology. Such use seems to make the case that by the mid-3rd century CE, the decimal place value system was familiar, at least to readers of astronomical and astrological texts in India.

It has been hypothesized that the Indian decimal place value system was based on the symbols used on Chinese counting boards from as early as the middle of the first millennium BCE. According to Plofker 2009,

These counting boards, like the Indian counting pits, ..., had a decimal place value structure . Indians may well have learned of these decimal place value "rod numerals" from Chinese Buddhist pilgrims or other travelers, or they may have developed the concept independently from their earlier non-place-value system; no documentary evidence survives to confirm either conclusion."

Vedic and Puranic texts describe units of Kala measurements, from Paramaṇu (about 17 microseconds) to Maha-Manvantara (311.04 trillion years). According to these texts, the creation and destruction of the universe is a cyclic process, which repeats itself forever. Each cycle starts with the birth and expansion (lifetime) of the Universe equaling 311.04 trillion years, followed by its complete annihilation (which also prevails for the same duration). This is currently 51st year of Brahma, and this is the "year" when the solar system was created according to Hindu astrolog, and is the first mahayuga for humanity.

Various units of time are used across the Vedas, Puranas, Mahabharata, Suryasidhanta etc. Especially, Nimesha's multiple, it varies to 3, 10, 15, 18, 20, 27, 30, 45, 48, 60. At the lower end, these are pretty consistent. The Complete Hindu metrics of time (KālaVyavahāra) can be summarised as below.

A Tithi or lunar day is defined as the time it takes for the longitudinal angle between the moon and the Sun to increase by 12°. Tithis begin at varying times of day and vary in duration from approximately 19 to approximately 26 hours.

A **Paksa** (also Pakṣa) or lunar fortnight consists of 15 tithis.

A **M□sa** or lunar month (approximately 29.5 days) is divided into 2 Pakṣas: the one between new moon and full moon (waxing) is called gaura or (bright) or ŚuklaPakṣa; the one between full moon and new moon (waning) Kṛiṣhṇa (dark) paksha

A **□itu** (or season) is 2 Māsa

An **Ayana** is 3 Ṛitus

A **year** is two **Ayanas**

TROPICAL METRICS

A **Yama** = 1/4 of a day (light) or night [= 7½ **Ghatis** (घटि = (3¾ **Muhurtas** = 3 **Hora**s (होरा( ]

Four **Y□mas** make half of the day (either day or night)

Eight **Y□mas** make an **Ahoratras** (day + night)

An Ahorātra is a tropical day (Note: A day is considered to begin and end at sunrise, not midnight.)

Reckoning of time among other entities

AMONG THE PITRAS

1 human fortnight (15 days) = 1 day (light) or night of the Pitṛas.

1 human month (30 days) = 1 day (light) and night of the Pitṛas.

30 days of the Pitṛas = 1 month of the Pitṛas = (30 × 30 = 900 human days).

12 months of the **Pit□as** = 1 year of the Pitṛas = (12 months of Pitṛas × 900 human days = 10800 human days).

The lifespan of the Pitṛas is 100 years of the Pitṛas (= 36,000 Pitṛa days = 1,080,000 human days = 3000 human years)

1 day of the **Devas** = 1 human year

1 month of the **Devas** = 30 days of the Devas(30 human years)

1 year of the **Devas** (1 divine year) = 12 months of the Devas(360 years of humans)

AMONG THE DEVAAS

The life span of any Hindu deva spans nearly (or more than) 4.5 million years. Statistically, we can also look it as:

12000 **Deva** Years = Life Span of Devas = 1 Mahā-Yuga.

The ViṣṇuPurāṇa Time measurement section of the ViṣṇuPurāṇa Book I Chapter III explains the above as follows:

2 **Ayanas** (6-month periods, see above) = 1 human year or 1 day of the devas

4,000 + 400 + 400 = 4,800 divine years (= 1,728,000 human years) = 1 Satya Yuga

3,000 + 300 + 300 = 3,600 divine years (= 1,296,000 human years) = 1 Tretā Yuga

2,000 + 200 + 200 = 2,400 divine years (= 864,000 human years) = 1 Dvāpara Yuga

1,000 + 100 + 100 = 1,200 divine years (= 432,000 human years) = 1 Kali Yuga

12,000 divine year = 4 Yugas (= 4,320,000 human years) = 1 Mahā-Yuga (also is equaled to 12000 Daiva (divine) Yuga)

[2*12,000 = 24,000 divine year = 12000 revolutions of sun around its dual]

FOR BRAHMA

1000 **Mah□-Yugas** = 1 **Kalpa** = 1 day (day only) of Brahma

(2 **Kalpas** constitute a day and night of Brahma, 8.64 billion human years)

30 days of **Brahma** = 1 month of **Brahma** (259.2 billion human years)

12 months of **Brahma** = 1 year of **Brahma** (3.1104 trillion human years)

50 years of **Brahma** = 1 **Par□rdha**

2 **parardhas** = 100 years of Brahma = 1 Para = 1 **Mah□-Kalpa** (the lifespan of Brahma)(311.04 trillion human years)
One day of Brahma is divided into 1000 parts called charaṇas. The charaṇas are divided as follows:
The cycle repeats itself, so altogether there are 1,000 cycles of Mahā-Yuga in one day of Brahma.
One cycle of the above four Yugas is one **Mah□-Yuga** (4.32 million solar years)
as is confirmed by the GītāŚloka 8.17 (statement) "sahasra-yuga-paryantamaharyad brahmaṇo viduḥrātriṁyuga-sahasrāntāṁte 'ho-rātra-vidojanāḥ", meaning, a day of brahma is of 1000 Mahā-Yuga. Thus a day of Brahma, **Kalpa**, is of duration: 4.32 billion solar years. Two Kalpas constitute a day and night (AdhiSandhi) of Brahma.
A **Manvantara** consists of 71 Mahā-Yuga (306,720,000 solar years). Each Manvantara is ruled by a Manu.
After each **Manvantara** follows one Saṃdhi Kāla of the same duration as a Kṛta Yuga (1,728,000 = 4 Charaṇas). (It is said that during a Saṃdhi Kāla, the entire earth is submerged in water.)
A **Kalpa** consists of a period of 4.32 Billion solar years followed by 14 **Manvataras** and **Sa□ dhiKalas**.
A day of Brahma equals
(14 times 71 **Mah□-Yuga**) + (15 × 4 **Chara□as**)
= 994 Mahā-Yuga + (15 * 4800)
= 994 Mahā-Yuga + (72,000 years)[deva years] / 6 = 12,000[deva years] viz. one mahayuga.
= 994 Mahā-Yuga + 6 Mahā-Yuga
= 1,000 **Mah□-Yuga**

THE SURYA SIDDHANTA DEFINITION OF TIMESCALES

The Surya Siddhanta [Chapter 14 Mānādhyāyah (*मानाध्यायः*)], documents a comprehensive model of nine divisions of time called māna (*मान*) which span from very small time units (Prāṇa [*प्राण*] - 4 seconds) to very large time scales (Para [*पर*] - 300000.04 Trillion solar years).
Currently, 50 years of Brahma have elapsed. The last Kalpa at the end of 50th year is called Padma Kalpa. We are currently in the first 'day' of the 51st year. This Brahma's day, Kalpa, is named as Shveta-VarahaKalpa. Within this Day, six Manvantaras have already elapsed and this is the seventh Manvantara, named as – VaivasvathaManvantara (or SraddhadevaManvantara). Within the VaivasvathaManvantara, 27 Mahayugas (4 Yugas together is a Mahayuga), and the Krita, Treta and Dwapara Yugas of the 28th Mahayuga have elapsed. This Kaliyuga is in the 28th Mahayuga. This Kaliyuga began in the year 3102 BCE in the proleptic Julian Calendar. Since 50 years of Brahma have already elapsed, this is the second Parardha, also called as DvithiyaParardha.
The time elapsed since the current Brahma has taken over the task of creation can be calculated as
432000 × 10 × 1000 × 2 = 8.64 billion years (2 Kalpa (day and night))
8.64 × 10^9 × 30 × 12 = 3.1104 Trillion Years (1 year of Brahma)
3.1104 × 10^{12} × 50 = **155.52 Trillion Years** (50 years of Brahma)
(6 × 71 × 4320000) + 7 × 1.728 × 10^6 = 1852416000 years elapsed in first six Manvataras, and SandhiKalas in the current Kalpa
27 × 4320000 = 116640000 years elapsed in first 27 Mahayugas of the current Manvantara
1.728 × 10^6 + 1.296 × 10^6 + 864000 = 3888000 years elapsed in current Mahayuga
3102 + 2016 = 5118 years elapsed in current Kaliyuga.
So the total time elapsed since current Brahma is
155520000000000 + 1852416000 + 116640000 + 3888000 + 5115 = 155,521,972,949,117 years (one hundred fifty-five trillion, five hundred twenty-one billion, nine hundred seventy-two million, nine hundred forty-nine thousand, one hundred seventeen years) as of 2016 AD
The current Kali Yuga began at midnight 17 February / 18 February in 3102 BCE in the proleptic Julian calendar. As per the information above about Yuga periods, only 5,118 years are passed out of 432,000 years of current Kali Yuga, and hence another 426,882 years are left to complete this 28th Kali Yuga of VaivaswathaManvantara.

IX. SUTRA OF VEDIC MATHEMATICS

There are plethoras of ancient techniques which provide quick and accurate solutions for various types of problems. Some of the basic methods are listed in the Table.

SUTRAS	MEANING
EKADHIKINA PURVENA	BY ONE MORE THAN THE PREVIOUS ONE
NIKHILAM NAVATASHCARAMAM DASHATAH	ALL FROM 9 AND THE LAST FROM 10
URDHVA-TIRYAGBYHAM	VERTICALLY AND CROSSWISE
PARAAVARTYA YOJAYET	TRANSPOSE AND ADJUST
SHUNYAM SAAMYASAMUCCAYE	WHEN THE SUM IS THE SAME THAT SUM IS ZERO.
(ANURUPYE) SHUNYAMANYAT	IF ONE IS IN RATIO, THE OTHER IS ZERO
SANKALANA-VYAVAKALANABHYAM	BY ADDITION AND BY SUBTRACTION
PURANAPURANABYHAM	BY THE COMPLETION OR NON-COMPLETION
CHALANA-KALANABYHAM	DIFFERENCES AND SIMILARITIES
YAAVADUNAM	WHATEVER THE EXTENT OF ITS DEFICIENCY
VYASHTISAMANSTIH	PART AND WHOLE
SHESANYANKENA CHARAMENA	THE REMAINDERS BY THE LAST DIGIT
SOPAANTYADVAYAMANTYAM	THE ULTIMATE AND TWICE THE PENULTIMATE

EKANYUNENA PURVENA	BY ONE LESS THAN THE PREVIOUS ONE
GUNITASAMUCHYAH	THE PRODUCT OF THE SUM IS EQUAL TO THE SUM OF THE PRODUCT
GUNAKASAMUCHYAH	THE FACTORS OF THE SUM IS EQUAL TO THE SUM OF THE FACTORS

Table 1

X. LIST OF LATIN ABBREVIATIONS WHICH WE OFTEN MISSPELT

Abbreviations deriving from Latin terms and phrases can be troublesome for us non-Latin speakers. Here's the long and short of the most common short forms adopted into English from the classical language:

10.1 e.g.-This abbreviation of *exempli gratia* ("for example") is not only often left bereft of its periods (or styled *eg.*), it's also frequently confused for a similar abbreviation you'll find below. Use *e.g.* (followed by a comma) to signal sample examples.

10.2 etc.-This sloppily formed abbreviation of et cetera ("and so forth") is often misspelled ect., perhaps because we're accustomed to words in which c precedes t, but not vice versa. (Curiously, Merriam-Webster spells out etcetera as such as a noun, but at the end of an incomplete list, retain the two-word form, or translate it.) A comma should precede it.Refrain from using etc. in an e.g. list; the abbreviations are essentially redundant, and note that etc. is also redundant in a phrase that includes including.

10.3 et al.-This abbreviation of et alia ("and others"), used almost exclusively to substitute for the names of all but the primary author in a reference to a multiauthor publication or article but occasionally applied in other contexts, should have no period after et, because that word in particular is not an abbreviation.Also, unlike as in the case of etc., refrain from preceding it with a comma, presumably because only one name precedes it. Fun fact: We use a form of the second word in this term — alias — to mean "otherwise known as" (adverb) or "an assumed name" (noun).

10.4 i.e.-This abbreviation of id est ("that is") is, like e.g., is frequently erroneously styled without periods (or as ie.). It, followed by a comma, precedes a clarification, as opposed to examples, which e.g. serves to introduce.

10.5 fl.-This abbreviation of flourit ("flourished") is used in association with a reference to a person's heyday, often in lieu of a range of years denoting the person's life span.

10.6 N.B.-This abbreviation for nota bene ("note well"), easily replaced by the imperative note, is usually styled with uppercase letters and followed by a colon.

10.7 per cent.-This British English abbreviation of per centum ("for each one hundred") is now often (and in the United States always) spelled percent, as one word and without the period.

10.8 re-This abbreviation, short for in re ("in the matter of") and often followed by a colon, is often assumed to be an abbreviation for reply, especially in email message headers.

10.9 viz.-This abbreviation of videlicet ("namely"), unlike e.g., precedes an appositive list — one preceded by a reference to a class that the list completely constitutes: "Each symbol represents one of the four elements, viz. earth, air, fire, and water." Note the absence of a following comma.

10.10 vs.-This abbreviation of versus ("against") is further abbreviated to v. in legal usage. Otherwise, the word is usually spelled out except in informal writing or in a jocular play on names of boxing or wrestling matches or titles of schlocky science fiction movies. ("In this title bout of Greed vs. Honesty, the underdog never stood a chance.")

XI. CONCLUSIONS

We need to be little bit more frugal to deal with symbols, notations and primary concepts. Mathematics is not only a subject but a language of modern computing. Fallacies are present everywhere like we find exceptions in Chemistry and bugs in Computer Science. Confluence of Vedic Mathematics and modern computer programming will embellish and uplift techniques of modern computing.

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