International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 4 Issue 8 || October. 2016 || PP-27-30

Note on Sidharth's Unification of Electromagnetism and Gravitation

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ABSTRACT : *In this note, it is shown that the Sidharth's Unification of Electromagnetism and Gravitation is close to the Einstein Tetrad Field and far of Einstein–Cartan–Evans theory. Mathematically in the sector Gravitation-Electromagnetism, Sidharth approach through non commutativity, where there is a minimum space time length, allows to link the vector magnetic potential with quantum mechanics and unify electromagnetism and gravitation under self dual configuration of electromagnetic fields.*

KEYWORDS: *Tetrad, Einstein, Evans theory, gravitation, electromagnetism, Sidharth, vector potential*

I. INTRODUCTION

It is shown in our paper, [1], that the electromagnetic field defined as the torsion form of differential geometry proposed by Myron W. Evans, in his **Einstein–Cartan–Evans theory,** (ECE) [2-5] , is in contradictions with the Einstein Tetrad Field. Mathematically in the sector unification of Gravitation-Electromagnetism, having severe refutations when ECE theory is compared with the Einstein tetrad Field. [6, 7]. Several of the published contributions in this theory have been shown to be mathematically incorrect, [8-12]. On the other hand, a reconciliation of Electromagnetism and Gravitation was made by Sidharth, [13-14]. Despite nearly a century of work, it has not been possible to achieve a real unification of gravitation and electromagnetism. The tools used, be it Quantum Theory or General Relativity are deeply entrenched in differentiable space time manifolds (and point particles) - the former with Minkowski space time and the latter with curved space time. The challenge has been, as Wheeler noted [15], the introduction of Quantum Mechanical spin half into General Relativity on the one hand and the introduction of curvature into Quantum Mechanics on the other. It was only through the artifice of renormalization that 't Hooft could finally circumvent this vexing problem, in the 1970s [16]. Sidharth argues below is that once the underlying non commutative nature of the geometry is recognized then it is possible to reconcile electromagnetism and gravitation. In this letter we review this hypothesis based on article [1], showing that the Sidharth' s approach is close to Tetrad Einstein Theory and far from the ECE theory.

II. ECE THEORY - EINSTEIN TETRAD FIELD

Following [1], in ECE theory the electromagnetic field for example is defined by the ansatz:

$$A_{\mu}^{a} = A^{(0)} q_{\mu}^{a} \tag{1}$$

$$F_{\mu}^{a} = A^{(0)} T_{\mu}^{a} \tag{2}$$

Where $cA^{(0)}$ is the primordial voltage, c being the speed of Light in vacuo and $A^{(0)}$ the potential magnitude of the electromagnetic field. The gravitational field is also defined by the tetrad, the symmetric metric being:

$$g_{\mu\nu} = q_{\mu}^{a} q_{\nu}^{b} \eta_{ab} \tag{3}$$

Where η_{ab} is the metric in the tangent space-time of Cartan geometric at point P in the base manifold. However Evans define this Riemann connection in antisymmetrical form. According Evans and collaborators [17], . "*It has been shown (www.aias.us Paper 122) that the Riemannian connection is antisymmetric, and that in consequence, the Riemannian torsion is identically non-zero. The Einstein field equation is therefore fundamentally incorrect because of its arbitrary neglect of spacetime torsion. All metrics and inferences based on the Einstein field equation are also incorrect, notably the Einsteinian gravitational theory, the theory of Big Bang and black holes, and derivative dogma such as dark matter.*
so",

$$\Gamma^{\alpha}{}_{\mu\nu} = -\Gamma^{\alpha}{}_{\nu\mu} \tag{4}$$

and the torsion tensor is

$$T^{\alpha}{}_{\mu\nu} = 2\Gamma^{\alpha}{}_{\mu\nu} \tag{5}$$

$$T^{\alpha}{}_{\mu\nu} = e^{\alpha} F_{\mu\nu} = 2\Gamma^{\alpha}{}_{\mu\nu} \tag{6}$$

from here, Evans obtains

$$T^{\alpha}{}_{\mu\nu} = e^{\alpha} F_{\mu\nu} \tag{7}$$

$$R^{\alpha}{}_{\mu\beta\nu}(\Gamma) = 2(\partial_{\beta}\Gamma^{\alpha}{}_{\nu\mu} + \Gamma^{\alpha}{}_{\beta\rho}\Gamma^{\rho}{}_{\mu\nu}) \tag{8}$$

This equation corresponds to Eq. (10.50) of reference [17]. which is not zero unless $\Gamma^{\alpha}{}_{\mu\nu} = 0$ so Evans et al, obtain in vacuum $G_{\mu\nu} \neq 0$.

On the other hand, following [1], In the gravitational sector, according the Einstein Theory 1928, the symmetrical Riemann connection is in modern notation

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_{\mu} e_{\nu} e_{\beta} + \partial_{\mu} e_{\beta} e_{\nu} + \partial_{\nu} e_{\mu} e_{\beta}) + \frac{1}{2} g^{\alpha\beta}(\partial_{\nu} e_{\beta} e_{\mu} - \partial_{\beta} e_{\mu} e_{\nu} - \partial_{\beta} e_{\nu} e_{\mu}) \tag{9}$$

where $\Gamma^{\alpha}{}_{\mu\nu}$ is symmetrical and we make $e_{\nu} = q_{\nu}$ of Evans and $e_{\nu} = h_{\nu}$ of Einstein notation,

$$\Gamma^{\alpha}{}_{\mu\nu} = \Gamma^{\alpha}{}_{\nu\mu} \tag{10}$$

The tetrad field allows define the Einstein connection as

$$^{E}\Gamma^{\alpha}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\mu\nu} = K^{\alpha}{}_{\mu\nu} \tag{11}$$

and the torsion is

$$^{E}\Gamma^{\alpha}{}_{\mu\nu} - {}^{E}\Gamma^{\alpha}{}_{\nu\mu} = T^{\alpha}{}_{\mu\nu} \tag{12}$$

where $^{E}\Gamma^{\alpha}{}_{\mu\nu}$ is the connection with absolute paralellism, $K^{\alpha}{}_{\mu\nu}$, is the contortion tensor [6, 7]

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}(-e^{\alpha} f_{\mu\nu} + f^{\alpha}{}_{\mu} e_{\nu} + f^{\alpha}{}_{\nu} e_{\mu}) \tag{13}$$

This tensor is equivalent to

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}(-T^{\alpha}{}_{\mu\nu} + T_{\nu}{}^{\alpha}{}_{\mu} + T_{\mu}{}^{\alpha}{}_{\nu})$$

where $T^{\alpha}{}_{\mu\nu} = e^{\alpha} F_{\mu\nu}$ is the torsion tensor [18, 19] and e^{α} is a vector tetrad.

With the Einstein Theory for the curvature tensor we obtain

$$R^{\alpha}{}_{\mu\beta\nu}({}^{E}\Gamma) = 0 \tag{14}$$

So with the Einstein theory without matter-energy tensor we obtain the Einstein tensor $G_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - 1/2(R g_{\mu\nu}) = 0 \tag{15}$$

This equation is consistent with self dual electromagnetic fields when tensor momentum-energy is zero.

III. SIDHARTH APPROACH

According to Sidharth, the following non commutative geometry [13,14] is defined as:

$$[x, y] \approx O(l^2), \quad [p_x, p_y] \approx \frac{\hbar^2}{l^2} O(l^2) \tag{16}$$

where l, τ are the extensions of the space time coordinates and $g_{\mu\nu}$ of Einstein equation (15) is taken into account. Let us now introduce this effect into the usual distance formula in flat space

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{17}$$

Rewriting the product of the two coordinate differentials in (17) in terms of the symmetric and non symmetric combinations, we get

$$g_{\mu\nu} = \eta_{\mu\nu} + k h_{\mu\nu} \tag{18}$$

where the first term on the right side of (18) denotes the usual flat space time and the second term denotes the effect of the non commutativity, k being a suitable constant. It must be noted that if $l, \tau \to 0$ then equations (16) and also (18) reduce to the usual formulation. The idea is to obtain $A_{\mu} = \partial^{\nu} h_{\mu\nu}$ for the magnetic potential.

The effect of the non commutative geometry is therefore to introduce a departure from flat space time, as can be seen from (18). Infact, remembering that the second term of the right side of (18) is small, this can straightaway be seen to lead to a linearized theory of General Relativity [15]. Exactly as in this reference we could now deduce the General Relativistic relation

$$\partial_{\lambda}\partial^{\lambda} h^{\mu\nu} - (\partial_{\lambda}\partial^{\nu} h^{\mu\lambda} + \partial_{\lambda}\partial^{\mu} h^{\nu\lambda}) - \eta^{\mu\nu}\partial_{\lambda}\partial^{\lambda} h + \eta^{\mu\nu}\partial_{\lambda}\partial_{\sigma} h^{\lambda\sigma} = -k\Theta_{\mu\nu} \tag{19}$$

Let us discuss about tensor momentum-energy $\Theta_{\mu\nu}$. A remarkable property of self-dual configurations is that they carry zero energy and momentum. This property can be verified by applying the self-duality condition to the expressions for the energy density $\varepsilon = (1/8\pi)(\mathbf{E}^2 + \mathbf{H}^2)$ and the Poynting vector $\mathbf{S} = (c/4\pi)(\mathbf{E} \times \mathbf{H})$ Note that given an antisymmetric field $F_{\mu\nu}$ in Minkowski space, the self-duality condition can be expressed as

$$*F_{\mu\nu} = \pm iF_{\mu\nu} \tag{20}$$

where the Hodge dual field $*F_{\mu\nu}$ is defined by $*F_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. Equation (20) is identical to Eq. $\mathbf{E} = i\mathbf{H}$ because $\mathbf{E}$ and $\mathbf{H}$ are expressed in terms of $F_{\mu\nu}$ as $E_i = F_{0i}$ and $H_i = (1/2)\varepsilon_{ijk}F_{jk}$ so equation $\mathbf{E} = i\mathbf{H}$ is $F_{0i} = i(1/2)\varepsilon_{ijk}F_{jk}$

If the Bianchi identity

$$\partial_\mu * F^{\mu\nu} = 0 \tag{21}$$

is compared with the equations of motion for a free electromagnetic field,

$$\partial_\mu F^{\mu\nu} = 0 \tag{22}$$

it becomes apparent that if $F_{\mu\nu}$ obeys Eqs. (20) and (21) then $F_{\mu\nu}$ automatically obeys Eq. (22).

Self-dual configurations possess trivial energy-momentum contents. The stress-energy tensor

$$\Theta_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\lambda F_{\lambda\mu} + \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{23}$$

can be brought into the form

$$\Theta_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\lambda F_{\lambda\mu} + *F_\mu{}^\lambda * F_{\lambda\mu}\right) \tag{24}$$

The proof is simple; see, for example, Ref. [20], and thus

$$\Theta_{\mu\nu} = \frac{1}{4\pi}\left(F_\mu{}^\lambda + i * F_\mu{}^\lambda)(+F_{\lambda\nu} + i * F_{\lambda\nu}\right) \tag{25}$$

If $*F_{\mu\nu} = \pm iF_{\mu\nu}$ then

$$\Theta_{\mu\nu} = 0 \tag{26}$$

Now returning to equation (19) using (26) we have

$$\partial_\lambda\partial^\lambda h^{\mu\nu} - (\partial_\lambda\partial^\nu h^{\mu\lambda} + \partial_\lambda\partial^\mu h^{\nu\lambda}) - \eta^{\mu\nu}\partial_\lambda\partial^\lambda h + \eta^{\mu\nu}\partial_\lambda\partial_\sigma h^{\lambda\sigma} = 0 \tag{27}$$

so under this condition, $\Theta_{\mu\nu} = 0$, the Sidharth's approach is self-consistent and cannot be consistent with the Evans theory supported by [5, 17] , the Academic Council of EUIEE and the editor of Ingeniare.

Let us now consider the non commutative relation (1) for the momentum components. Then, it can be shown using (16) and (18) that [16],

$$\frac{\partial}{\partial x^\lambda}\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\lambda} \text{ goes over to } \frac{\partial}{\partial x^\lambda}\Gamma^\nu_{\mu\nu} - \frac{\partial}{\partial x^\mu}\Gamma^\nu_{\lambda\nu} \tag{28}$$

Normally in conventional theory the right side of (20) would vanish. Let us designate this nonvanishing part on the right by

$$\frac{e}{c\hbar}F^{\mu\nu} \tag{29}$$

From (29) we have

$$A_\mu = \hbar\Gamma^\rho_{\mu\rho} \tag{30}$$

where B is the magnetic field, if we are to identify $F^{\mu\nu}$ with the electromagnetic tensor[16]. It will be recognized that (21) gives the celebrated expression for the magnetic monopole, and indeed it has also being shown that a non commutative space time at the extreme scale throws up the monopole [17, 18]. We have shown here that the non commutativity in momentum components leads to an effect that can be identified with electromagnetism and infact from expression (29) we have

$$A_\mu = \hbar\Gamma^\rho_{\mu\rho} \tag{31}$$

where A_μ is the electromagnetic four potential. Thus non commutativity as expressed in equations (16) generates both gravitation and electromagnetism. thus according to Sidhardth, electromagnetism is deduced in by considering an imaginary shift, $x^\mu \to x^\mu + ia^\mu$, (a^μ is the Compton scale) in a Quantum Mechanical context.

$$i\hbar \frac{\partial}{\partial x^\mu} \to i\hbar \frac{\partial}{\partial x^\mu} + \frac{\hbar}{a^\mu} \qquad (32)$$

and the second term on the right side of (24) was shown to be the electromagnetic vector potential A_μ so

$$A_\mu = \hbar \Gamma^\rho_{\mu\rho} = \hbar / a_\mu \qquad (33)$$

With the results of [5, 17], it is not possible arrive to Sidharth' s unification of electromagnetism and gravitation, because equation (6) does not support an expansion like equation (18).

CONCLUSION

In this note, we have showed that the Sidharth' s Unification of Electromagnetism and Gravitation is close to the Einstein Tetrad Field and far of Einstein–Cartan–Evans theory. Mathematically in sector Gravitation-Electromagnetism, Sidharth approach through non commutativity, where there is a minimum space time length, allows to link the vector magnetic potential with quantum mechanics and unify electromagnetism and gravitation under self dual configuration of electromagnetic fields .

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