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Algebraic method of the robust stability of discrete linear interval dynamic systems

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ABSTRACT : *The algebraic method of researches of robust stability is considered discrete interval dynamic systems. The results specifying and supplementing earlier known are received. The algorithm of definition of robust stability discrete interval dynamic systems are given.*
Keywords: *robust stability, discrete interval dynamical systems, angular polynoms of Kharitonov, consecutive separate angular coefficients.*

I. Introduction

V. L. Kharitonov’s [1] work has caused huge interest in a problem of researches of a robustness of interval dynamic systems [2-12]. In the modern theory of interval dynamic systems there are two alternative approaches [10-15]: is algebraic or Kharitonov’s approach; is frequency or Tsypkin's- Polyak approach.

In the algebraic or Kharitonov’s direction of researches of a problem of robust stability works of many authors are widely known [2-6, 10-12, 16-19].

In works [2-6] reviews and statements of problems of robust stability which have been caused by the known of V.L.Kharitonov’s work [1] are submitted. In B.T.Polyak's work, P. S. Scherbakova [11] the concept of uperstability of linear control systems is offered. At the same time superstability systems have the properties of camber allowing simple solutions of many classical tasks of the theory of control, in particular, of a problem of robust stabilization at matrix uncertainty. But essential restriction of such systems is the practical narrowness of their class, determined by conditions of existence of the dominating diagonal elements of a matrix of system with negative coefficients [20]. In V. M. Kuntsevich’s [12] work interesting results on robust stability for linear discrete systems are received. At the same time the matrix of system is set in a class of attendants a characteristic polynom of system, i.e. in the Frobenius form [20] that also narrows a class of the considered real systems. In B.R.Barmish works, etc. [16, 17] are offered counterexamples to Bialas [7] theorem which are cancelled in work [8].

In M. Mansour works, etc. [18, 19] are received discrete analogs of weak and strong theorems of Kharitonov’s [1], which have the restrictions imposed on interval areas of coefficients or [2, 9, 13] difficult procedure of design of roots of polynoms on a piece is applied [-1, 1].

In the real work the algebraic method of research of robust stability of discrete interval dynamic systems, in case of interval matrixes of systems of a general view and without certain restrictions which foundation was initiated in works [8,9] is considered.

1.Problem definition. Discrete linear dynamic systems of an order of n are considered

x (m+1) = Ax(m), m = 1,2,3,…, (1)

where, x (m) is a state vector, $A \in R^{nxn}$ is an interval matrix with elements $a_{ij}, i, j = \overline{1,n}$, the representing interval sizes $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ with angular values $\underline{a}_{ij}, \overline{a}_{ij}, \underline{a}_{ij} \le \overline{a}_{ij}$.

It is required to define conditions of robuststability of systems (1).

2. Main results. In work [8], fundamental for the considered method, results in the form of strictly proved Theorem 1 and Lemma to it about robust stability of continuous interval systems

x`= Ax, x(t_0) = x_0 , (2)under the terms that four angular polynoms of Kharitonov’s are Hurwitz ones, made on consecutive separate coefficients $b_i, (\underline{b}_i, \overline{b}_i, i = \overline{1,n})$ of characteristic polynoms of system (2) are received:

$f(\lambda) = \lambda^n + b_1\lambda^{n-1} + ... + b_n = 0$. (3)

We will provide these the Theorem 1 and a Lemma.

Theorem 1.*In order that position of balance x=0 of system (2) asymptotically was stability at all* $A \in D$ *, or that the interval matrix A was stability, is necessary also sufficiently that all four angular Kharitonov’spolynoms* *are Hurwitz ones, made on consecutive separate coefficients* $b_i, (\underline{b}_i, \overline{b}_i, i = \overline{1, n})$ *of characteristic polynoms (3) systems (2).*

This theorem is provedon the basis of the following Lemma.

Lemma. *Separate slopes* $b_i, (\underline{b}_i, \overline{b}_i, i = \overline{1, n})$ *are formed as the corresponding coefficients of polynoms (3), or at angular values of elements* $a_{ij}, i, j = \overline{1, n}$, *of a matrix A, or at zero values of some elements (if the interval of accessory includes zero).*

As it is easy to see from a Lemma, for finding of coefficients, application of optimizing methods of nonlinear programming generally is necessary [21].

To the Theorem 1 which proof is given in the appendix of work [8] it is necessary to make the following specifying remark.

Remark. *Follows from the main argument of the proof of the Theorem 1 connected with existence of four angular Kharitonov'spolynoms, that in the absence of a full set (set) of four angular polynoms of a condition of the Theorem 1 are necessary, but can be insufficient for stability of system (2).*

The case appropriate to the provided Remark can arise when separate coefficients of polynoms (3) are interconnected and as a result narrow a set of angular polynoms to quantity less than four, including also multiple, coinciding polynoms.

Justice of the proved Theorem 1 is confirmed by cancellation of the known counterexamples to Bialas's theorem [7].

So, the Theorem 1 is approved on various counterexamples of the theorem of Bialas, in particular from work [16] where the matrix is considered

$$A = \Omega_r = \begin{bmatrix} -0.5 - r & -12.06 & -0.06 \\ -0.25 & 0 & 1 \\ 0.25 & -4 & -1 \end{bmatrix},$$

where $r \in [0,1]$, for which justice of the Theorem 1 is confirmed.

But in case of a matrix from [16] it is possible to consider visually justice of the Remark given above to the Theorem 1.

Really, in this case consecutive separate coefficients form an incomplete set of coefficients

$$b_1 = -\sum_{i=1}^{3} a_{ij} = 1.5 + r = b_2 = \sum_{i,j=1}^{3} a_{ii} a_{jj} - \sum_{i,j=1}^{3} a_{ij} a_{ji},$$

$$b_3 = \sum_{i,j,k=1}^{3} a_{ij} a_{jk} a_{kj} - \sum_{i,j,k=1}^{3} a_{ij} a_{jk} a_{ki} - a_{11} a_{22} a_{33} = 4r + 2.06,$$

asfrom there are separate coefficients:

$$\underline{b}_1 = 1.5; \overline{b}_1 = 2.5; \underline{b}_2 = 1.5, \overline{b}_2 = 2{,}5; \underline{b}_3 = 2.06, \overline{b}_3 = 6.06.$$

Respectively, angular polynoms of Kharitonov in this case will be only two

$$\begin{aligned} f_1(\lambda) &= \lambda^3 + 1.5\lambda^2 + 1.5\lambda + 2.06 = f_2(\lambda), \\ f_3(\lambda) &= \lambda^3 + 2.5\lambda^2 + 2.5\lambda + 4.06 = f_4(\lambda), \end{aligned} \qquad (4)$$

i.e. the totality of 4 angular polynoms specified in work [8] won't be.

Therefore, on angular polynoms (4) the system (1.1) everywhere at $r \in [0,1]$ will be stability, though it is known that at $r \in [0.5 - \sqrt{0.06},\ 0.5 + \sqrt{0.06}]$ this system it is unstability.

It is known, that the publication of work [1] has given an impulse for search by many researchers of discrete analogs of theorems of Kharitonov [2-7, 9]. So in work [2] it is specified that "the discrete option of a Kharitonov's condition of four polynomials is absent". Buthere it is noted that now [19,20] discrete analogs weak are receivedand strong theorems of Kharitonov. But these analogs of theorems of Kharitonov have the certain restrictions imposed on interval areas of coefficients [2]. These restrictions have been removed in works [9,13] where analogs of theorems of Kharitonov with use received of the theorem of Schur. Also in [9,13] the theorems which are discrete analogs of results of work [8] on interval matrixes and polyhedrons of matrixes are formulated.

Further, generalization of the results received in work [9,13] taking into account the conclusions given above for continuous systems is considered.
For discrete systems, using z - transformation, obtained interval characteristic polynom

$$f(z) = \det(zI - A) = \sum_{i=0}^{n} b_i z^{n-i},\ b_i \in [\underline{b}_i, \overline{b}_i],\ \underline{b}_i \le \overline{b}_i. \qquad (5)$$

For definition of stability conditions we will use Schur theorem, i.e.
look conditions $|b_o| > |b_n|$,
for the sequence of the polynoms determined by recurrence relations

$f_i(z) = [b_o f(z) - b_n f(1/z) z^n]/z, \ldots, f_{i+1}(z) = [b_{o,i} f_i(z) - b_{n,i} f_i(1/z) z^{n-1}]/z$,

where $b_{o,i}$, $b_{n,i}$ are respectively the senior and younger coefficients of polynom $f_i(z)$, i = 1, 2, 3,…, n-2.
Definition. We will call *change points* for coefficients b_i, i = 0,1,2,…, n , points on the valid axis in which there are transitions of roots of a polynom (5), through a
whichsingle circle on the plane of roots, and *change intervals* are respectively intervals in roots are or inside, or out of a single circle.
In work [9] the main results on definition of conditions of robust stability of discrete interval systems in the form of the corresponding theorems 1-6 are formulated. At the same time it should be noted that as it is stated above on page 3, for a case of continuous systems [8], justice of the theorem 5 has the restriction caused by the Remark to the theorem of 1 work [8] i.e. the theorem 5 is right at a totality from 4 various polynoms of Kharitonov.
Justice of results [9,13] concerning an analog of the strong theorem of Kharitonov are shown on the known counterexamples from [2], etc.
Thus,the algorithm of definition of robust stability discrete interval dynamic systems will be the following.

1. Using lemma formulas to the theorem 1 [8], optimization on elements $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, $i, j = \overline{1, n}$ of interval matrix A, there are separate coefficients $b_i \in [\underline{b}_i, \overline{b}_i]$, $i = \overline{0, n}$, of interval characteristic polynom (5).

2. Four polynoms of Kharitonov corresponding to an interval polynom (5) are defined

$f_1(z): \{\underline{b}_0, \underline{b}_1, \overline{b}_2, \overline{b}_3, \underline{b}_4 \ldots, \}$; $f_2(z): \{\underline{b}_0, \overline{b}_1, \overline{b}_2, \underline{b}_3, \underline{b}_4 \ldots, \}$;

$f_3(z): \{\overline{b}_0, \underline{b}_1, \underline{b}_2, \overline{b}_3, \underline{b}_4 \ldots, \}$; $f_4(z): \{\overline{b}_0, \overline{b}_1, \underline{b}_2, \underline{b}_3, \overline{b}_4 \ldots, \}$;

3. N of inequalities of a look (Item 2) specified in the Appendix of work [9] are formed.

4. Concerning each coefficient b_i, $i = \overline{0, n}$, including other coefficients fixed, consistently there are*change points* for all four polynoms of Kharitonov and on all n to inequalities (see item 3), since smaller orders.

5. If all *change points* on all coefficients b_i, $i = \overline{0, n}$, not belong reaped to the set intervals, then the initial polynom (system) is stability, otherwise is unstability.

Conclusion. The algebraic method of researches of robust stability of interval dynamic systems considered in this work is further development of the main results of works [8, 9], which allows to solve a problem of robust stability at a general view of an interval matrix of system.
It should be noted, that the Remark to the Theorem 1 essentially specifies results of work [8], namely emphasizes necessary of a totality from four angular polynoms of Kharitonov (taking into account frequency rate of polynoms) for definition of robust stability of interval dynamic systems. Also, conditions of necessary and sufficiently on the Theorem 1 correspond to the angular separate coefficients, defined consistently from 1st to n coefficient of a characteristic polynom of system which can be found with use of methods of nonlinear programming [21].

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