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Bölcsföldi-Birkás-Ferenczi prime numbers (Full prime numbers)

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ABSTRACT: *After defining full prime numbers, full prime numbers will be presented from 23 to 2232323. How many full prime numbers are there in the interval (10^{p-1},10^p) (where p is a prime number)? On the one hand, it has been counted by computer among the prime numbers with up to 13-digits. On the other hand, the function (1) gives the approximate number of full prime numbers in the interval (10^{p-1},10^p). The notion of decomposable full prime numbers will be defined. Based on the examination of the set of decomposable full prime numbers a near-proof reasoning will be provided regarding the number of full prime numbers. Another near-proof reasoning has emerged from the conformity of Mill's prime numbers with full prime numbers.*
The set of full prime numbers is probably infinite.

KEYWORDS: *number, frequency, frequency-function, full prime numbers, decomposable full prime numbers*

I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-prime numbers (the sum of the digits is a prime number too) [8], factorial prime numbers (with the form of n!-1 or n!+1), twin prime-pairs (p and p+2 are prime-pairs), Mersenne-prime numbers (with the form of 2^n-1), Chen-prime numbers (p is a prime number, and p+2 is a prime number or semi-prime, i.e. the multiplication of two prime numbers), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of full prime numbers [3].

II. FULL PRIME NUMBERS

Definition: a positive integer number is a full prime number if a/ the positive integer number is prime, b/ all digits are prime, c/ the number of digits is prime. Prime numbers that meet the conditions a/ and b/ have been known for a long time; these are prime numbers containing prime digits [1]. The set of prime numbers meeting the conditions a/ and c/ is also well-known; it is the set of prime-long prime numbers. Prime numbers meeting all the three conditions (a/, b/, c/) at the same time are full prime numbers.
In German: "Vollprimzahlen" [3].
Full prime number p_v has the following sum form:

2.1 $$p_v=\sum_{j=0}^{k(p_v)} e_j(p_v).10^j \quad \text{where } e_j(p_v) \text{ € } \{2,3,5,7\}=A \text{ and } k(p_v)+1 \text{ is a prime number.}$$

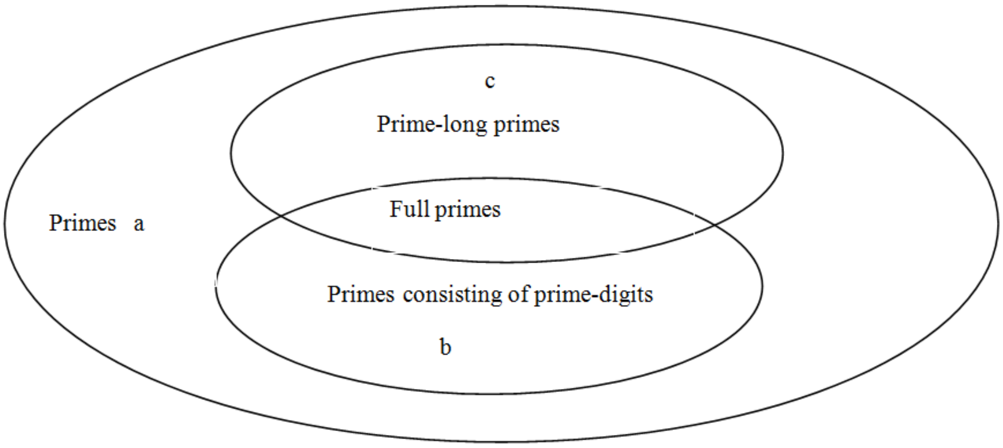
Full prime numbers are as follows (the last digit can only be 3 or 7):
23, 37, 53, 73.
223, 227, 233, 257, 277, 337, 353, 373, 523, 557, 577, 727, 733, 757, 773,
22273, 22277, 22573, 22727, 22777, 23227, 23327, 23333, 23357, 23537,
23557, 23753, 23773, 25237, 25253, 25357, 25373, 25523, 25537, 25577,
25733, 27253, 27277, 27337, 27527, 27733, 27737, 27773, 32233, 32237,
32257, 32323, 32327, 32353, 32377, 32533, 32537, 32573, 33223, 33353,
33377, 33533, 33577, 33757, 33773, 35227, 35257, 35323, 35327, 35353,
35527, 35533, 35537, 35573, 35753, 37223, 37253, 37273, 37277, 37337,
37357, 37537, 37573, 52223, 52237, 52253, 52553, 52727, 52733, 52757,
53233, 53323, 53327, 53353, 53377, 53527, 53773, 53777, 55333, 55337,
55373, 55733, 57223, 57373, 57527, 57557, 57727, 57737, 57773, 72223,
72227, 72253, 72277, 72337, 72353, 72533, 72577, 72727, 72733, 73237,
73277, 73327, 73523, 73553, 73727, 73757, 75223, 75227, 75253, 75277,
75323, 75337, 75353, 75377, 75527, 75533, 75553, 75557, 75577, 75773,
77237, 77323, 77377, 77527, 77557, 77573, 77723, 77773,
2222273,2222327,2222333,2222377,2222527,2222533,2222537,2222573,
2222723,2223233,2223253,2223757,2223773,2225233,2225323,2225533,2225557,2225753,
2225777,2227223, 2227273,2227327,2227333,2227723,2227727,2227777,2232257,2232323, etc.

The factual number of full primes and the number of full primes calculated according to function (1) are as follows:

Number of digits p	The factual number of full primes among the p-digit primes H(p)	The number of full primes among the p-digit primes according to function (1)	H/G
2	4	4	1
3	15	10	1,5
5	128	100	1,28
7	1325	1189	1,11
11	214904	202599	1,06
13	2884201	2810500	1,03
17		558971271	
19		8053515082	
23		$1,717373252 \times 10^{12}$	

The relation between the three sets (a/, b/, c/) is the following:

Fig.1



H(p) is the frequency of full prime numbers in the interval $(10^{p-1}, 10^{p})$.
Consequently, H(2)=4, H(3)=15, H(5)=128, H(7)=1325, H(11)=214432, H(13)=2884201,…
If p is the highest element of a twin prime-pair (p-2, p), the number of full prime numbers with p-digits is approximately
10 times higher than the number of full prime numbers with (p-2)-digits. (H(5)~10.H(3), H(7)~10.H(5), H(13)~10.H(11)).
On the other hand, the number of full prime numbers with 13-digits

ending in	23	is 360661,
„	27	„ 360596,
„	33	„ 360671,
„	37	„ 360409,
„	53	„ 360891,
„	57	„ 361010,
„	73	„ 359844,
„	77	„ 360119.

Thus, it can be stated that full prime numbers are evenly distributed in the interval $(10^{p-1}, 10^{p})$.

G(p) is the number of full prime numbers in the interval $(10^{p-1}, 10^{p})$. We think that $G(p)= c.\,(10^{k.p}/p).(1+O_p(1))$, where $c>0$ and $0<k<1$ are appropriate constants, p is any arbitrary prime number, and $O_p(1)$ is the compensation-function. For instance, for c=0,4 and k=0,6 the compensation-function is $O_p(1)=p^{1/7}-1$. Therefore, the estimation is $G(p)=0,4\,(10^{0,6p}/p)(1+p^{1/7}-1)$,

or more precisely: $G(p)= 0,4\,(10^{0,6p}/p^{6/7})$. (1)

III. DECOMPOSABLE FULL PRIME NUMBERS [3].

Definition: Full prime number p_v is decomposable if $p_v=a+10^r.b$, where a and b are full prime numbers, and r is a positive integer number. In other words: If a full prime number can be decomposed so that both elements are full prime numbers, the original full prime number is a decomposable full prime number. Exactly one of the two numbers a and b must have two digits. Both parts can be full prime numbers if the original full prime number has at least 5 digits. We shall examine the full prime numbers that contain only the digits 3 and 7.

We examine the full prime numbers

$$p_w= \sum_{j=0}^{k(p_w)} e_j(p_w).10^j \text{ where } e_j(p_w) \in \{3,7\}=A_w \text{ and } k(p_w)+1 \text{ is a prime number.}$$

In this set the decomposable full prime numbers with no more than 7-digits are the following:

37337 parts: (373 and 37) or (37 and 337), 33773 parts 337 and 73, 7777337 parts: 77773 and 37.

Further examples:

3333377773373 parts	:	33333777733 and 73,
3373337737373 „	:	33733377373 and 73,
3377373337337 „	:	33773733373 and 37,
3377377777373 „	:	33773777773 and 73,
3733773333737 „	:	(37337733337 and 37) or (37 and 33773333737).

The number of full prime numbers consisting of only the digits 3 and 7 is:

5-digits	5,	2 of them are decomposable,		
7-digits	16, „	1	„	,
13-digits	591, „	20	„	,
17-digits	27243, „	454	„	, etc.

We have found a decomposable full prime number with 109-digits:

…………. …77337373337733737. The dots mean digit 3.

If the full prime number 37 will be deleted at the end of this full prime number, a full prime number with 107-digits remains. [6]

Supposition/2: The number of elements of full prime numbers consisting of only the digits 3 and 7 is infinite.

IV. NUMBER OF THE ELEMENTS OF THE SET OF FULL PRIME NUMBERS [3].

4.1 If the number of digits in a full prime number is the highest element of a twin prime-pair, the full prime number can be decomposed. Otherwise the full prime number is not decomposable. The connection between the numbers of digits is the following:

5=3+2, 7=5+2, 13=11+2, 19=17+2, 31=29+2, 43=41+2, etc. In general terms:

p=(p-2)+2, where (p-2) and p are the elements of a twin prime-pair. If the number of twin prime-pairs is infinite, the necessary condition of decomposability of full prime numbers containing 3 and 7 (p=(p-2)+2) takes place infinite times, and the sufficient condition (both parts are full prime numbers) can take place infinite times. If the number of twin prime-pairs is finite, the number of elements of the set of decomposable full prime numbers is finite. Thus, it can be said that the number of elements of the set of full prime numbers is probably infinite.

4.2 Let's take the set of Mills' prime numbers!

Definition: The number $m=[\,M \text{ ad } 3^n\,]$ is a prime number,

where M=1,3063778838630806904686144926 02… is the Mills' constant, and n=1,2,3, … is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection $n \rightarrow m$ is the following: 1→2, 2→11, 3→1361, 4→2521008887,…

The Mills' prime number $m=[\,M \text{ ad } 3^n\,]$ corresponds with the interval $(10^{m-1}, 10^m)$ and vice versa. For instance, $2\rightarrow(10, 10^2)$, $11\rightarrow(10^{10}, 10^{11})$, $1361\rightarrow(10^{1360}, 10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. In the interval $(10^{m-1}, 10^m)$ the number of full prime numbers is according to estimates (1) $G(m)=0{,}4(10^{0{,}6m}/m^{6/7})$. Thus, it can be stated that the number of the elements of the set of full prime numbers is probably infinite.

V. CONCLUSION

Our respected mathematical ancestors have become acquainted with 64 different sets of special prime numbers within the set of prime numbers in the last ca. 2,500 years. We have found the 65th set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be the task of researchers of the future.

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