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Comparison Between Measures of Spatial Autocorrelation

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ABSTRACT: *This work is carried out with the view to analyze and compare Geary's C and Moran's I measures of spatial autocorrelation using the monthly rainfall statistics as the case study for the year 2014 as recorded in CBN Statistical Bulletin, 2014. Both measures were employed to analyze the data and from the empirical results computed, Moran's I coefficient is 0.16 and* r^2 *=0.91 while Geary's C is0.74 and* r^2 *=0.87.These values signify positive autocorrelation or the idea that similar values on the map tend to cluster together.This implies that Geary's C compares favourably with Moran's I as regards to the data used. The scatter plots of Moran's I and Geary's C in figures 2 and 3 show that their relationship is linear and that either of the statistic could be used.*

KEYWORDS: *Spatial autocorrelation, Autocorrelation, Correlation, Weight matrix.*

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I. INTRODUCTION

Spatial autocorrelation is a concept that helps to define spatial analysis.Upton and Fingleton(1985) define spatial autocorrelation as a property that mapped data possess whenever it exhibits an organized pattern..It is central to studies using spatial statistics and spatial econometrics. Climate change seems to be the foremost global challenge facing humans at the moment, even though it seems that not all places on the globe are affected. World leaders, union leaders, pressure groups and others who have shown concern have been meeting to find a lasting solution to the 'acclaimed' dilemma. The assessment of spatial autocorrelation is generally considered to be one of the primary tasks of geographical data analysis (Hubert and Arabie,1991). The scientific community has not been left out as causes and solutions are being proffered and it is expected to linger for a long time. Spatial autocorrelation is more complex than auto-correlation because the correlation is multi-dimensional and bi-directional

LISA statistics (Anselin,1995) was introduced to assess the significance of spatial autocorrelation which indicates the level of spatial autocorrelation at the local scale. Local Moran I and local Geary C are alternative local spatial statistics that have some advantages over $G_i(d)$ statistic.Moran (1950) introduced the first measure of spatial autocorrelation in order to study stochastic phenomenon which are distributed in space in two or more dimensions. Moran's I has been subsequently used in almost all studies employing spatial autocorrelation. To indicate the level of spatial autocorrelation at the local scale,the local Moran statistics is derived for each enumeration area that contains observations of active incidence. The local Moran's *I* reflects how neighbouring values are associated with each other, a high or positive value of local Moran indicates a clustering of similar values and a low or negative value of local Moran indicates a clustering of dissimilar values, between an observation and those in its neighbourhood. Differently from the local Moran, the local Geary statistic is a measure of the weighted sum of square differences between the observed values at the location *i* and those of its surrounding locations.

This work is aimed at evaluating the relationship between rainfall statistics within the states in Nigeria using Geary's C and Moran's I measures of spatial autocorrelation. The study is therefore limited to the monthly rainfall statistics of the year, 2014, with exclusion to Bayelsa, Ekiti, Abia, Jigawa, and Ebonyi states whose statistics was not recorded in CBN Statistical Bulletin, 2014.

1.1 The study Area

Figure 1: Map of Nigeria

II. METHOD

The measures of spatial autocorrelation to be used for this work are Moran's statistic I and Geary's C coefficient. Values of Moran's I range from -1 to +1. Negative values indicate negative spatial auto-correlation and positive values indicate positive spatial auto-correlation, a zero value indicates a random spatial pattern. Moran's *I* is inversely related to Geary's C but it is not identical. Moran's I is a measure of global spatial auto-correlation.

Moran's I is defined as

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2} \qquad (1)$$

where

N=number of spatial units indexed by i and j

x=variable of interest

$\bar{x}$=mean of x

w_{ij}=element of a matrix of spatial weight

The expected value of Moran's *I* under the null hypothesis of no spatial autocorrelation is

Geary's *c*

This is a measure of spatial autocorrelation or an attempt to determine if adjacent observations of the same phenomenon are correlated. Geary's C is more sensitive to local spatial auto-correlation

Geary's C is defined as

$$C = \frac{(N-1)\sum_i \sum_j w_{ij} (x_i - x_j)^2}{2w_{ij} \sum_i (x_i - \bar{x})^2} \qquad (2)$$

where

N=number of spatial units indexed by i and j

x=variable of interest

$\bar{x}$= mean of x

w_{ij}=element of a matrix of spatial weight

W=sum of all w_{ij}

For Geary's C, the cross product uses the actual values while for Moran's I, the cross product is based on deviations from the mean for the two location values.

The weight matrix

$$\begin{pmatrix} & 1 & 2 & . & . & . & . & . & 32 \\ 1 & . & . & . & . & . & . & . & . \\ 2 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 32 & . & . & . & . & . & . & . & . \end{pmatrix}$$

Table 1: The data with the computed d and d^2

S/N	STATES	X	$d=(x-\bar{x})$	$d^2=(x-\bar{x})^2$
1	OGUN	3412.8	1394.2	1943793.6
2	FCT	1700.6	-318	101124
3	ONDO	1407.2	-611.4	373809.96
4	ANAMBRA	1700.1	-318.5	101442.3
5	DELTA	4853.9	2835.3	8038926.1
6	BAUCHI	1410.6	-608	369664
7	EDO	2583.3	564.7	318886.1
8	NIGER	2171.9	153.3	23500.9
9	CROSS-RIVER	7218.9	5200.3	27043120.1
10	ENUGU	1929.6	-89	7921
11	ZAMFARA	705.6	-1313	1723969
12	GOMBE	912.3	-1106.3	1223899.7
13	OYO	3904	1885.4	3554733.2
14	LAGOS	2117.1	98.5	9702.3
15	TARABA	1164.7	-853.9	729145.2
16	KWARA	2466.6	448	200704
17	KANO	1376	-642.6	412934.8
18	KATSINA	481.5	-1537.1	2362676.4
19	KADUNA	2834.6	816	665856
20	NASSARAWA	1332	-686.6	471419.6
21	KOGI	1643.2	-375.4	140925.2
22	PLATEAU	1241.3	-777.3	604195.3
23	BENUE	1253	-765.6	586143.4
24	BORNO	579.1	-1439.5	2072160.3
25	YOBE	1353.8	-664.8	441959
26	OSUN	1649.4	-369.2	136308.6
27	IMO	1958.4	-60.2	3624
28	RIVERS	2575.5	556.9	310137.6
29	SOKOTO	704.2	-1314.4	1727647.4
30	AKWA-IBOM	4077.7	2059.1	4239892.8
31	ADAMAWA	797.3	-1221.3	1491573.7
32	KEBBI	1080.5	-938.1	880031.6
				62311827.16

N/B: From table 1 above**,** $\bar{x} = 2018.6$ **.**

The Moran's I coefficient

$$I=\frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{\sum_i (x_i-\bar{x})^2}$$

where

N=32

$\sum_i \sum_j w_{ij}$=132 (Appendix B)
$\sum_i \sum_j w_{ij} (x_i - \bar{x})(x_j - \bar{x})$=42645911.97 (Appendix D)
$\sum_i (x_i - \bar{x})^2$=62311827.2 (from table 1)
I=$\left(\frac{32}{132}\right)\frac{42645911.97}{62311827.2} = 0.16$

The Geary's c measure

$$C=\frac{(N-1)\sum_i \sum_j w_{ij}(x_i - x_j)^2}{2w_{ij}\sum_i (x_i - \bar{x})^2}$$

N=32
w_{ij}=132 (Appendix B)
$\sum_i (x_i - \bar{x})^2$=62311827.2 (from table 1)
$\sum_i \sum_j w_{ij}(x_i - x_j)^2$=390064749.9
C=$\frac{(32-1)(390064749.9)}{2(132)(62311827.2)} = 0.73$

III. CONCLUSION

.For Moran's I, the cross product is based on the deviations from the mean for the two location values $\{\sum_i \sum_j w_{ij} (x_i - \bar{x})(x_j - \bar{x})\}$,while for Geary's C, the cross products uses the actual values themselves at each location $\{\sum_i \sum_j w_{ij}(x_i - x_j)^2\}$.

1. For Moran's I, our result is 0.16 and r^2=0.91, while for Geary's C, the result is 0.74 and r^2=0.87
2. Moran's I varies on the scale from -1 to1 where

-1 => indicates negative spatial autocorrelation/clustered.
0 => indicates no spatial autocorrelation/random.
1 => indicates positive spatial autocorrelation/dispersed.
While Geary's C varies on the scale from 0 to 2 where,
0 => indicates negative spatial autocorrelation/clustered.
1 => indicates no spatial autocorrelation/random.
2 => indicates positive spatial autocorrelation/dispersed.
Although it can convert to a -1 to 1 scale by calculating C*=1 - C.

3. Moran's I is the most common measure of spatial autocorrelation compared to Geary's C.
4. Moran's I is a measure of global spatial autocorrelation while Geary's C is more sensitive to local spatial autocorrelation.

While their similarities are as follows,

1. They can be used for points and polygons.
2. They can be used for a continuous variable (any value).
3. They involved one variable only, i.e. X.

The measurement of spatial autocorrelation describes the overall pattern across a geographic landscape, supporting spatial prediction and allowing detection of striking deviations, and by graphically portraying the relationship between two quantitative variables measured for the same observation. A scatter-plot relates to the numerical value rendered by a correlation coefficient formula.

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APPEBDIX A : Neighbouring States

	STATE	NEIGHBOURING STATES									
1	OGUN	OYO(13)	OSUN(26)	ONDO(3)	LAGOS(14)						
2	FCT	NIGER(8)	KADUNA(19)	NASSARAWA(20)	KOGI(21)						
3	ONDO	OGUN(1)	OSUN(26)	KOGI(21)	EDO(7)	DELTA(5)					
4	ANAMBRA	DELTA(5)	EDO(7)	KOGI(21)	ENUGU(10)	IMO((27)					
5	DELTA	ONDO(3)	EDO(7)	ANAMBRA(4)	IMO(27)	RIVERS(28)					
6	BAUCHI	PLATEAU(22)	KADUNA(19)	KANO(17)	YOBE(25)	GOMBE(12)	TARABA(15)				
7	EDO	DELTA(5)	ONDO(3)	KOGI(21)	ANAMBRA(4)						
8	NIGER	KEBBI(32)	ZAMFARA11)	KADUNA(19)	FCT(2)	KOGI(21)	KWARA(16)				
9	CROSS-RIVER	BENUE(23)	AKWA IBOM(30)								
10	ENUGU	BENUE(23)	KOGI(21)	ANAMBRA(4)							
11	ZAMFARA	KEBBI(32)	SOKOTO(29)	KATSINA(18)	KADUNA(19)	NIGER(8)					
12	GOMBE	ADAMAWA(31)	BORNO(24)	YOBE(25)	BAUCHI(6)	TARABA(15)					
13	OYO	KWARA(16)	OSUN(26)	OGUN(1)							
14	LAGOS	OGUN (1)									
15	TARABA	BENUE(23)	NASSARAWA(20)	PLATEAU(22)	BAUCHI(6)	GOMBE(12)	ADAMAWA(31)				
16	KWARA	OYO(13)	OSUN(26)	KOGI(21)	NIGER(8)						
17	KANO	KATSINA(18)	BAUCHI(6)	KADUNA(19)							
18	KATSINA	ZAMFARA(11)	KADUNA(19)	KANO(17)							
19	KADUNA	NIGER(8)	ZAMFARA(11)	KATSINA(18)	KANO(7)	BAUCHI(6)	PLATEAU(22)	NASSARAWA(20)	FCT(2)		
20	NASSARAWA	KOGI(21)	FCT(2)	KADUNA(19)	PLATEAU(22)	TARABA(15)	BENUE(23)				
21	KOGI	ONDO(3)	KWARA(16)	NIGER(8)	FCT(2)	NASSARAWA(20)	BENUE(23)	ENUGU(10)	ANAMBRA(4)	EDO(7)	
22	PLATEAU	NASSARAWA(20)	KADUNA(19)	BAUCHI(6)	TARABA(15)						
23	BENUE	KOGI(21)	NASSARAWA(20)	TARABA(15)	CROSS-RIVER(9)	ENUGU(10)					
24	BORNO	YOBE(25)	GOMBE(12)	ADAMAWA(31)							
25	YOBE	BORNO(24)	BAUCHI(6)	GOMBE(12)							
26	OSUN	OGUN(1)	OYO(13)	KWARA(16)	ONDO(3)						
27	IMO	DELTA(5)	ANAMBRA(4)	RIVERS(28)							
28	RIVERS	AKWA IBOM(30)	DELTA(5)	IMO(27)							
29	SOKOTO	KEBBI(32)	ZAMFARA(11)								
30	AKWA-IBOM	RIVERS(28)	CROSS-RIVER(9)								
31	ADAMAWA	TARABA(15)	GOMBE(12)	BORNO(24)							
32	KEBBI	NIGER(8)	ZAMFARA(11)	SOKOTO(29)							

APPEBDIX B : W_{ij} Matrix for Map in Figure 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	TOTAL
1	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	4
2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	4
3	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	5
4	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	5
5	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	5
6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	6
7	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
8	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	6
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	2
10	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	3
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	5
12	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	5
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	3
14	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	1	0	6
16	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	4
17	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3
18	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3
19	0	1	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	8
20	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	6
21	0	1	1	1	0	0	1	1	0	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	9
22	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	4
23	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	5
24	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	3
25	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3
26	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
27	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3
28	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	3
29	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
30	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	2
31	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3
32	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	3
																																	132

APPENDIX C: The Matrix of the States and their locations on the Map

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	0	0	d1d3	0	0	0	0	0	0	0	0	0	d1d13	d1d14	0	0	0	0	0	0	0	0	0	0	0	d1d26	0	0	0	0	0	0
2	0	0	0	0	0	0	0	d2d8	0	0	0	0	0	0	0	0	0	0	d2d19	d2d20	d2d21	0	0	0	0	0	0	0	0	0	0	0
3	d3d1	0	0	0	d3d5	0	d3d7	0	0	0	0	0	0	0	0	0	0	0	0	0	d3d21	0	0	0	0	d3d26	0	0	0	0	0	0
4	0	0	0	0	d4d5	0	d4d7	0	0	d4d10	0	0	0	0	0	0	0	0	0	0	d4d21	0	0	0	0	0	d4d27	0	0	0	0	0
5	0	0	d5d3	d5d4	0	0	d5d7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d5d27	d5d28	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	d7d12	0	0	d6d15	0	d6d17	0	d6d19	0	0	d6d22	0	0	d6d25	0	0	0	0	0	0	0
7	0	0	d7d3	d7d4	d7d5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d7d21	0	0	0	0	0	0	0	0	0	0	0
8	0	d8d2	0	0	0	0	0	0	0	0	d8d11	0	0	0	0	d8d16	0	0	d8d19	0	d8d21	0	0	0	0	0	0	0	0	0	0	d8d32
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d9d23	0	0	0	0	0	0	d9d30	0	0
10	0	0	0	d10d4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d10d21	0	d10d23	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	d11d8	0	0	0	0	0	0	0	0	0	d11d18	d11d19	0	0	0	0	0	0	0	0	0	d11d29	0	0	d11d32
12	0	0	0	0	0	d12d6	0	0	0	0	0	0	0	0	d12d15	0	0	0	0	0	0	0	0	d12d24	d12d25	0	0	0	0	0	d12d31	0
13	d13d1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d13d16	0	0	0	0	0	0	0	0	0	d13d26	0	0	0	0	0	0
14	d14d1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	d15d6	0	0	0	0	0	d15d12	0	0	0	0	0	0	0	d15d20	0	d15d22	d15d23	0	0	0	0	0	0	0	d15d31	0
16	0	0	0	0	0	0	0	d16d8	0	0	0	0	d16d13	0	0	0	0	0	0	0	d16d21	0	0	0	0	d16d26	0	0	0	0	0	0
17	0	0	0	0	0	d17d6	0	0	0	0	0	0	0	0	0	0	0	d17d18	d17d19	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	d18d11	0	0	0	0	0	d18d17	0	d18d19	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	d19d2	0	0	0	d19d6	0	d19d8	0	0	d19d11	0	0	0	0	0	d19d17	d19d18	0	d19d20	0	d19d22	0	0	0	0	0	0	0	0	0	0
20	0	d20d2	0	0	0	0	0	0	0	0	0	0	0	0	d20d15	0	0	0	d20d19	0	d20d21	d20d22	d20d23	0	0	0	0	0	0	0	0	0
21	0	d21d2	d21d3	d21d4	0	0	d21d7	d21d8	0	d21d10	0	0	0	0	0	d21d16	0	0	0	d21d20	0	0	d21d23	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	d22d6	0	0	0	0	0	0	0	0	d22d15	0	0	0	d22d19	d22d20	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	d23d9	d23d10	0	0	0	0	d23d15	0	0	0	0	d23d20	d23d21	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	d24d12	0	0	0	0	0	0	0	0	0	0	0	0	d24d25	0	0	0	0	0	d24d31	0
25	0	0	0	0	0	d25d6	0	0	0	0	0	d25d12	0	0	0	0	0	0	0	0	0	0	0	d25d24	0	0	0	0	0	0	0	0
26	d26d1	0	d26d3	0	0	0	0	0	0	0	0	0	d26d13	0	0	d26d16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	d27d4	d27d5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d27d28	0	0	0	0
28	0	0	0	0	d28d5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d28d27	0	0	d28d30	0	0
29	0	0	0	0	0	0	0	0	0	0	d29d11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d29d32
30	0	0	0	0	0	0	0	0	d30d9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d30d28	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	d31d12	0	0	d31d15	0	0	0	0	0	0	0	0	d31d24	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	d32d8	0	0	d32d11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	d32d28	d32d29	0	0	0

APPENDIX D: Matrix showing the product of the "d"s (neighbouring states)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	-852413.9	0	0	0	0	0	0	0	0	0	262824.7	137329	0	0	0
2	0	0	0	0	0	0	0	-48749.4	0	0	0	0	0	0	0	0	0
3	-852414	0	0	0	-1733502.4	0	-345257.6	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	-903043.1	0	-179856.95	0	0	28346.5	0	0	0	0	0	0	0
5	0	0	-1733502.4	-903043.1	0	0	1601093.9	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	672630.4	0	0	519171.2	0	390700.8
7	0	0	-345257.6	-1733502.4	1601093.9	0	0	0	0	0	0	0	0	0	0	0	0
8	0	-48749.4	0	0	0	0	0	0	0	0	-201282.9	0	0	0	0	68678.4	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	28346.5	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	-201282.9	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	672630.4	0	0	0	0	0	0	0	0	944669.6	0	0
13	2628625	0	0	0	0	0	0	0	0	0	0	0	0	0	0	844659	0
14	137328.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	519171.2	0	0	0	0	0	944669.6	0	0	0	0	0
16	0	0	0	0	0	0	0	68678.4	0	0	0	0	844659.2	0	0	0	0
17	0	0	0	0	0	390700.8	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	2018212	0	0	0	0	0	987740.5
19	0	-259488	0	0	0	-4961128	0	125092.8	0	0	-1071408	0	0	0	0	0	-524361.6
20	0	218338.8	0	0	0	0	0	0	0	0	0	0	0	0	586287.7	0	0
21	0	119377.2	229519.6	119564.9	0	0	-213144.6	-57548.8	0	33410.6	0	0	0	0	0	-168179	0
22	0	0	0	0	0	47259.4	-213144.6	0	0	0	0	0	0	0	663736.5	0	0
23	0	0	0	0	0	0	-213144.6	0	-3981349.7	68138.4	0	0	0	0	653745.8	0	0
24	0	0	0	0	0	0	-213144.6	0	0	0	0	1592518.9	0	0	0	0	0
25	0	0	0	0	0	404198.4	-213144.6	0	0	0	0	735468.2	0	0	0	0	0
26	-514739	0	-225728.9	0	0	0	-213144.6	0	0	0	0	0	-696090	0	0	-165402	0
27	0	0	0	19173.7	-170685.1	0	-213144.6	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	1578978.6	0	-213144.6	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	-213144.6	0	0	0	1725807	0	0	0	0	0	0
30	0	0	0	0	0	0	-213144.6	0	10707937.7	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	-213144.6	0	0	0	0	1351124.2	0	0	1042868	0	0
32	0	0	0	0	0	0	-213144.6	-143810.7	0	0	1231725	0	0	0	0	0	0

APPENDIX D CONTINUES

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	$\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i-x_j)^2$
0	0	0	0	0	0	0	0	3109579.6	0	0	0	0	0	0	9052126.9
0	1285956	135865.96	3294.8	0	0	0	0	0	0	0	0	0	0	0	1647240.46
0	0	0	55696	0	0	0	0	58660.8	0	0	0	0	0	0	17399740.3
0	0	0	3237.6	0	0	0	0	0	66738.9	0	0	0	0	0	10849123.4
0	0	0	0	0	0	0	0	0	8383920.3	5191106.6	0	0	0	0	147656846.6
0	2027776	0	0	28662.5	0	0	3226.2	0	0	0	0	0	0	0	2369631.6
0	0	0	883788	0	0	0	0	0	0	0	0	0	0	0	8202665.8
0	439171.3	0	279523.7	0	0	0	0	0	0	0	0	0	0	1191154	4368856.46
0	0	0	0	0	35591362.8	0	0	0	0	0	0	9867137.4	0	0	45459100.2
0	0	0	82024.96	0	457787.6	0	0	0	0	0	0	0	0	0	593482.86
50220.8	4532641	0	0	0	0	0	0	0	0	0	879093.6	0	0	140550	7752541.3
0	0	0	0	0	0	1110022.2	194912.3	0	0	0	0	0	13225	0	631178.2
0	0	0	0	0	0	0	0	5083221.2	0	0	0	0	0	0	7390617.4
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1678838.5
0	0	27589.3	0	5867.6	7796.9	0	0	0	0	0	0	0	134982.8	0	300809.2
0	0	0	677987.6	0	0	0	0	5896640.9	0	0	0	0	0	0	8727595.4
800130.3	2127513.96	0	0	0	0	0	0	0	0	0	0	0	0	0	2928841.46
0	5537079.6	0	0	0	0	0	0	0	0	0	0	0	0	0	6387430.7
5537079.6	0	2257806.8	0	2538604.9	0	0	0	0	0	0	0	0	0	0	20746549.56
0	2257806.8	0	96845.4	8226.5	6241	0	0	0	0	0	0	0	0	0	2532974.96
0	0	96845.4	0	0	152256	0	0	0	0	0	0	0	0	0	2234654.06
0	2538604.9	8226.5	0	0	0	0	0	0	0	0	0	0	0	0	2581361.5
0	0	6241	152256	0	0	0	0	0	0	0	0	0	0	0	36216044.3
0	0	0	0	0	0	0	600160.1	0	0	0	0	0	251401.96	0	962584.26
0	0	0	0	0	0	600160.1	0	0	0	0	0	0	0	0	798308.6
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8819277.4
0	0	0	0	0	0	0	0	0	0	380812.4	0	0	0	0	6831451.6
0	0	0	0	0	0	0	0	0	380812.4	0	0	2256804.8	0	0	7828523.8
0	0	0	0	0	0	0	0	0	0	0	0	0	0	141601.7	1020685.5
0	0	0	0	0	0	0	0	0	2256804.8	0	0	0	0	0	12123742.2
0	0	0	0	0	0	251401.96	0	0	0	0	0	0	0	0	399609.76
0	0	0	0	0	0	0	0	0	0	0	141601.7	0	0	0	1473305.66
															390064749.9

APPENDIX E : Matrix showing Location of theNeighbouring States

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	x1-x3	0	0	0	0	0	0	0	0	0	x1-x13	x1-x14	0	0	0
2	0	0	0	0	0	0	0	x2-x8	0	0	0	0	0	0	0	0	0
3	x3-x1	0	0	0	x3-x5	0	x3-x7	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	x4-x5	0	x4-x7	0	0	x4-x10	0	0	0	0	0	0	0
5	0	0	x5-x3	x5-x4	0	0	x5-x7	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	x7-x12	0	0	x6-x15	0	x6-x17
7	0	0	x7-x3	x7-x4	x7-x5	0	0	0	0	0	0	0	0	0	0	0	0
8	0	x8-x2	0	0	0	0	0	0	0	0	x8-x11	0	0	0	0	x8-x16	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	x10-x4	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	x11-x8	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	x12-x6	0	0	0	0	0	0	0	0	x12-x15	0	0
13	x13-x1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	x13-x16	0
14	x14-x1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	x15-x6	0	0	0	0	0	x15-x12	0	0	0	0	0
16	0	0	0	0	0	0	0	x16-x8	0	0	0	0	x16-x13	0	0	0	0
17	0	0	0	0	0	x17-x6	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	x18-x11	0	0	0	0	0	x18-x17
19	0	x19-x2	0	0	0	x19-x6	0	x19-x8	0	0	x19-x11	0	0	0	0	0	x19-x17
20	0	x20-x2	0	0	0	0	0	0	0	0	0	0	0	0	x20-x15	0	0
21	0	x21-x2	x21-x3	x21-x4	0	0	x21-x7	x21-x8	0	x21-x10	0	0	0	0	0	x21-x16	0
22	0	0	0	0	0	x22-x6	0	0	0	0	0	0	0	0	x22-x15	0	0
23	0	0	0	0	0	0	0	0	x23-x9	x23-x10	0	0	0	0	x23-x15	0	0
24	0	0	0	0	0	0	0	0	0	0	0	x24-x12	0	0	0	0	0
25	0	0	0	0	0	x25-x6	0	0	0	0	0	x25-x12	0	0	0	0	0
26	x26-x1	0	x26-x3	0	0	0	0	0	0	0	0	0	x26-x13	0	0	x26-x16	0
27	0	0	0	x27-x4	x27-x5	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	x28-x5	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	x29-x11	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	x30-x9	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	x31-x12	0	0	x31-x15	0	0
32	0	0	0	0	0	0	0	x32-x8	0	0	x32-x11	0	0	0	0	0	0

E CONTINUES

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
0	0	0	0	0	0	0	0	x1-x26	0	0	0	0	0	0
0	x2-x19	x2-x20	x2-x21	0	0	0	0	0	0	0	0	0	0	0
0	0	0	x3-x21	0	0	0	0	x3-x26	0	0	0	0	0	0
0	0	0	x4-x21	0	0	0	0	0	x4-x27	0	0	0	0	0
0	0	0	0	0	0	0	0	0	x5-x27	x5-x28	0	0	0	0
0	x6-x19	0	0	x6-x22	0	0	x6-x25	0	0	0	0	0	0	0
0	0	0	x7-x21	0	0	0	0	0	0	0	0	0	0	0
0	x8-x19	0	x8-x21	0	0	0	0	0	0	0	0	0	0	x8-x32
0	0	0	0	0	x9-x23	0	0	0	0	0	0	x9-x30	0	0
0	0	0	x10-x21	0	x10-x23	0	0	0	0	0	0	0	0	0
x11-x18	x11-x19	0	0	0	0	0	0	0	0	0	x11-x29	0	0	x11-x32
0	0	0	0	0	0	x12-x24	x12-x25	0	0	0	0	0	x12-x31	0
0	0	0	0	0	0	0	0	x13-x26	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	x15-x20	0	x15-x22	x15-x23	0	0	0	0	0	0	0	x15-x31	0
0	0	0	x16-x21	0	0	0	0	x16-x26	0	0	0	0	0	0
x17-x18	x17-x19	0	0	0	0	0	0	0	0	0	0	0	0	0
0	x18-x19	0	0	0	0	0	0	0	0	0	0	0	0	0
x19-x18	0	x19-x20	0	x19-x22	0	0	0	0	0	0	0	0	0	0
0	x20-x19	0	x20-x21	x20-x22	x20-x23	0	0	0	0	0	0	0	0	0
0	0	x21-x20	0	0	x21-x23	0	0	0	0	0	0	0	0	0
0	x22-x19	x22-x20	0	0	0	0	0	0	0	0	0	0	0	0
0	0	x23-x20	x23-x21	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	x24-x25	0	0	0	0	0	x24-x31	0
0	0	0	0	0	0	x25-x24	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	x27-x28	0	0	0	0
0	0	0	0	0	0	0	0	0	x28-x27	0	0	x28-x30	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	x29-x32
0	0	0	0	0	0	0	0	0	x30-x28	0	0	0	0	0
0	0	0	0	0	0	x31-x24	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	x32-x29	0	0	0

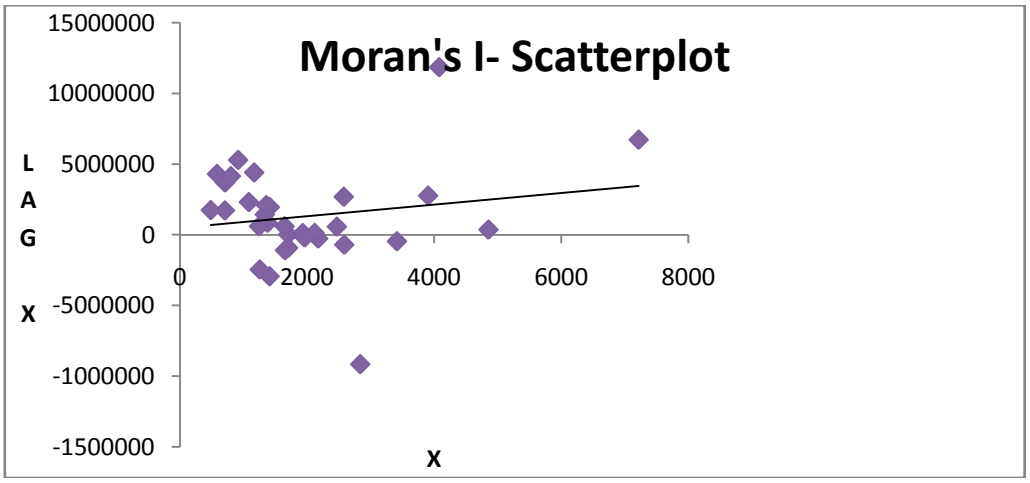

FIG 2: Moran I Scatter Plot

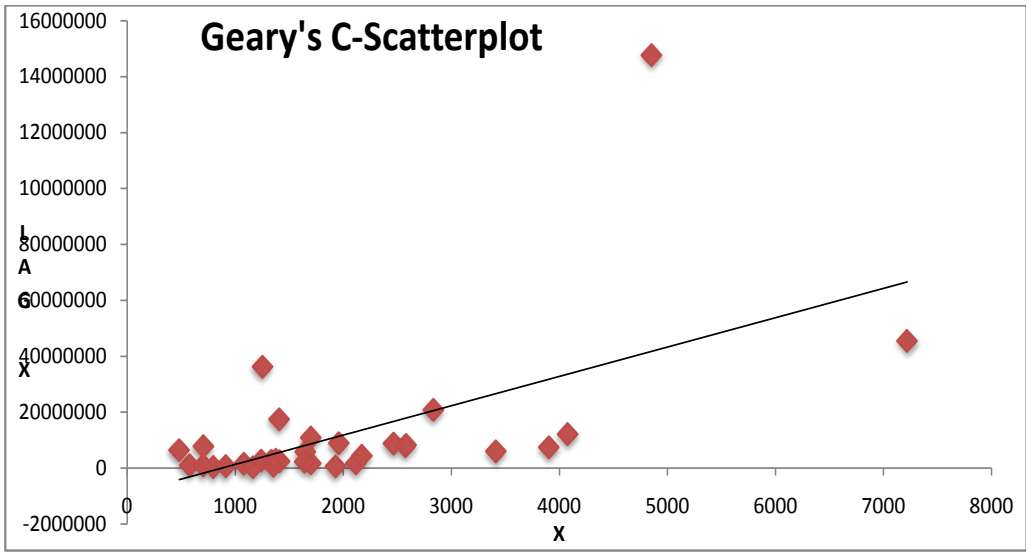

Fig 3: Geary's C- Scatter Plot