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On Product Connectivity Banhatti Index of Some Graphs

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ABSTRACT: *For a connected graph G with a vertex set $V(G)$ and an edge set $E(G)$, the product connectivity Banhatti index of a graph G is defined as* $PB(G)=\sum_{ue}\frac{1}{\sqrt{d_G(u)d_G(e)}}$, *where ue means that a vertex u and an edge e are incident in G. In this paper, we determine the product connectivity Banhatti index of some cycle related and product related graphs.*

KEYWORDS: *Molecular descriptor, product connectivity Banhatti index, product related graph, cycle related graph.*
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I. INTRODUCTION

In this paper, we are interested with nontrivial, simple, connected, finite, undirected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$ with $|V(G)|=n$ and $|E(G)|=m$. The degree $d_G(v)$ of a vertex v is the number of edges incident to v in G. The degree of an edge $e=uv$ in G is defined by $d_G(e)=d_G(u)+d_G(v)-2$. For undefined graph theoretic terminologies and notations, refer [6] or [7].

In chemical graph theory and in mathematical chemistry, a molecular graph or chemical graph is a representation of the structural formula of a chemical compound in terms of graph theory. A molecular graph is a graph whose vertices correspond to the atoms of the chemical compound and edges to the chemical bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecule is called a topological index of that graph. There are numerous molecular descriptors, which are also referred to as topological indices, see [4], that have found some applications in theoretical chemistry, especially in QSPR/QSAR research. One of the best known and widely used topological index is the product connectivity index (or Randić index, connectivity index) by Randić [13], who has shown this index to reflect molecular branching.

Motivated by Randić definition of the product connectivity index, the sum connectivity index was initiated by Zhou and Trinajstić [14] and [15], which is defined by $\chi(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}$. For more details on these type of connectivity indices, refer [1,3,12].

The first and second K Banhatti indices of a graph G are defined as $B_1(G)=\sum_{ue}[d_G(u)+d_G(e)]$ and $B_2(G)=\sum_{ue}[d_G(u)d_G(e)]$, where ue means that a vertex u and an edge e are incident in G. The K Banhatti indices were introduced by Kulli in [8]. The K Banhatti indices are closely related to Zagreb-type indices. For more details on these two type of indices, refer [5]. Recently, many other indices were also studied, for example [9] and [10]. Kulli et al. initiated the study of one more new topological index of a graph G called as product connectivity Banhatti index [11] defined as $PB(G)=\sum_{ue}\frac{1}{\sqrt{d_G(u)d_G(e)}}$, where ue means that a vertex u and an edge e are incident in G. The authors in [11] have computed expression for product connectivity Banhatti index of some standard class of graphs and have given some bounds for it.

The authors in paper [11] have considered only connected graphs. It is to be noted that this index gives an infinite value for $G = K_2$.

The definition of product connectivity can be re-written as $PB(G) = \sum_{ue} \frac{1}{\sqrt{d_G(u)d_G(e)}}$, where $G \neq K_2$.

The present article also gives an expression for product connectivity Banhatti index of some cycle related and product related graphs.

II. PRODUCT CONNECTIVITY BANHATTI INDEX OF CYCLE RELATED GRAPHS

For $n \geq 4$, the graph $W_n = C_{n-1} + K_1$ is called a *wheel graph*.

Theorem 2.1. *If W_n is a wheel of order n, then*

$$PB(W_n) = (n-1)\left[\frac{1}{\sqrt{n(n-1)}} + \frac{1}{\sqrt{3n}} + \frac{1}{\sqrt{3}}\right].$$

Proof. The wheel W_n has n vertices and $2(n-1)$ edges. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{33} = \{uv \in \ E(W_n) \mid d_{W_n}(u) = 3, d_{W_n}(v) = 3\}$, $d_{W_n}(uv) = 4$, $|E_{33}| = n-1$,

$E_{3(n-1)} = \{uv \in \ E(W_n) \mid d_{W_n}(u) = 3, d_{W_n}(v) = n-1\}$, $d_{W_n}(uv) = n$, $|E_{3(n-1)}| = n-1$.

Now, by the definition of product connectivity Banhatti index of a graph we get

$$\begin{aligned}
PB(W_n) &= \sum_{ue} \frac{1}{\sqrt{d_{W_n}(u)d_{W_n}(e)}} \\
&= \sum_{e=uv \in E(W_n)} \left[\frac{1}{\sqrt{d_{W_n}(u)d_{W_n}(e)}} + \frac{1}{\sqrt{d_{W_n}(v)d_{W_n}(e)}}\right] \\
&= (n-1)\left[\frac{1}{\sqrt{3 \cdot 4}} + \frac{1}{\sqrt{3 \cdot 4}}\right] + (n-1)\left[\frac{1}{\sqrt{n \cdot (n-1)}} + \frac{1}{\sqrt{n \cdot 3}}\right] \\
&= (n-1)\left[\frac{1}{\sqrt{n(n-1)}} + \frac{1}{\sqrt{3n}} + \frac{1}{\sqrt{3}}\right].
\end{aligned}$$

The details of the following special graphs can be found in [2].

Let $C_n^{(t)}$ denote the one-point union of $t \geq 2$ cycles of length n. The graph $C_3^{(t)}$, is called a *Friendship graph.*

Theorem 2.2. *If $C_3^{(t)}$ is a friendship graph of order $2t+1$, then*

$$PB(C_3^{(t)}) = t + \sqrt{t} + 1.$$

Proof. The graph $C_3^{(t)}$ of order $2t+1$ has two types of verices namely $2t$ vertices of degree 2 and 1 vertex of degree $2t$. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in E(C_3^{(t)}) \mid d_{C_3^{(t)}}(u) = 2, d_{C_3^{(t)}}(v) = 2\}$, $d_{C_3^{(t)}}(uv) = 2$, $|E_{22}| = t$,

$E_{2(2t)} = \{uv \in EC_3^{(t)}) \mid d_{C_3^{(t)}}(u) = 2, d_{C_3^{(t)}}(v) = 2t\}$, $d_{C_3^{(t)}}(uv) = 2t$, $|E_{2n}| = 2t$.

From the definition of product connectivity Banhatti index of a graph and above data we get,

$$
\begin{aligned}
PB(C_3^{(t)}) &= \sum_{ue} \frac{1}{\sqrt{d_{C_3^{(t)}}(u) d_{C_3^{(t)}}(e)}} \\
&= \sum_{e=uv \in E(C_3^{(t)})} \left[\frac{1}{\sqrt{d_{C_3^{(t)}}(u) d_{C_3^{(t)}}(e)}} + \frac{1}{\sqrt{d_{C_3^{(t)}}(v) d_{C_3^{(t)}}(e)}} \right] \\
&= t\left[\frac{1}{\sqrt{2\cdot 2}} + \frac{1}{\sqrt{2\cdot 2}} \right] + 2t\left[\frac{1}{\sqrt{2t\cdot 2}} + \frac{1}{\sqrt{2t\cdot 2t}} \right] \\
&= t + \sqrt{t} + 1.
\end{aligned}
$$

The *double cone* $DC_n = C_n + 2K_1$ is a graph with $n+2$ vertices and $3n$ edges.

Theorem 2.3. *If DC_n is a double cone of order $n+2$ and size $3n$, then*

$$PB(DC_n) = 2n\left[\frac{1}{\sqrt{n(n+2)}} + \frac{1}{2\sqrt{n+2}} + \frac{1}{2\sqrt{6}} \right].$$

Proof. The double cone is a graph of order $n+2$ and size $3n$. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{4n} = \{uv \in E(DC_n) \mid d_{DC_n}(u) = 4, d_{DC_n}(v) = n\}$, $d_{DC_n}(uv) = n+2$, $|E_{4n}| = 2n$,

$E_{44} = \{uv \in E(DC_n) \mid d_{DC_n}(u) = 4, d_{DC_n}(v) = 4\}$, $d_{DC_n}(uv) = 6$, $|E_{44}| = n$.

The definition of product connectivity Banhatti index of a graph along with the above data gives,

$$
\begin{aligned}
PB(DC_n) &= \sum_{ue} \frac{1}{\sqrt{d_{DC_n}(u) d_{DC_n}(e)}} \\
&= \sum_{e=uv \in E(DC_n)} \left[\frac{1}{\sqrt{d_{DC_n}(u) d_{DC_n}(e)}} + \frac{1}{\sqrt{d_{DC_n}(v) d_{DC_n}(e)}} \right] \\
&= 2n\left[\frac{1}{\sqrt{n(n+2)}} + \frac{1}{\sqrt{4(n+2)}} \right] + n\left[\frac{1}{\sqrt{4\cdot 6}} + \frac{1}{\sqrt{4\cdot 6}} \right] \\
&= 2n\left[\frac{1}{\sqrt{n(n+2)}} + \frac{1}{2\sqrt{n+2}} + \frac{1}{2\sqrt{6}} \right].
\end{aligned}
$$

The *helm* H_n is a graph obtained from a wheel W_n by attaching a pendant edge at each vertex of a cycle C_n.

Theorem 2.4. *If the graph H_n is a helm of order $2n-1$, then*

$$PB(H_n)=(n-1)\left[\frac{1}{\sqrt{6}}+\frac{1}{2\sqrt{n+1}}+\frac{1}{\sqrt{(n+1)(n-1)}}+\frac{1}{\sqrt{3}}+\frac{1}{2\sqrt{3}}\right].$$

Proof. The helm H_n is a graph of order $2n-1$ and size $3(n-1)$. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{44}=\{uv\in E(H_n)\mid d_{H_n}(u)=4, d_{H_n}(v)=4\}$, $d_{H_n}(uv)=6$, $|E_{44}|=n-1$,

$E_{4(n-1)}=\{uv\in E(H_n)\mid d_{H_n}(u)=4, d_{H_n}(v)=n-1\}$, $d_{H_n}(uv)=n+1$, $|E_{4(n-1)}|=n-1$,

$E_{14}=\{uv\in E(H_n)\mid d_{H_n}(u)=1, d_{H_n}(v)=4\}$, $d_{H_n}(uv)=3$, $|E_{14}|=n-1$.

The definition of product connectivity Banhatti index of a graph shows,

$$\begin{aligned}
PB(H_n) &= \sum_{ue}\frac{1}{\sqrt{d_{H_n}(u)d_{H_n}(e)}}\\
&= \sum_{e=uv\in E(H_n)}\left[\frac{1}{\sqrt{d_{H_n}(u)d_{H_n}(e)}}+\frac{1}{\sqrt{d_{H_n}(v)d_{H_n}(e)}}\right]\\
&= (n-1)\left[\frac{1}{\sqrt{4\cdot 6}}+\frac{1}{\sqrt{4\cdot 6}}\right]+(n-1)\left[\frac{1}{\sqrt{4(n+1)}}+\frac{1}{\sqrt{(n+1)(n-1)}}\right]\\
&\quad+(n-1)\left[\frac{1}{\sqrt{1\cdot 3}}+\frac{1}{\sqrt{4\cdot 3}}\right]\\
&= (n-1)\left[\frac{1}{\sqrt{6}}+\frac{1}{2\sqrt{n+1}}+\frac{1}{\sqrt{(n+1)(n-1)}}+\frac{1}{\sqrt{3}}+\frac{1}{2\sqrt{3}}\right].
\end{aligned}$$

The *closed helm* H_n' is a graph obtained from a helm H_n by joining each pendant vertex to form a cycle.

Theorem 2.5. *If the graph* H_n' *is a closed helm of order* $2n-1$, *then*

$$PB(H_n')=(n-1)\left[\frac{1}{2\sqrt{n+1}}+\frac{1}{\sqrt{(n+1)(n-1)}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{15}}+\frac{1}{2\sqrt{5}}\right].$$

Proof. The closed helm H_n' is a graph of order $2n-1$ and size $4(n-1)$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{44}=\{uv\in E(H_n')\mid d_{H_n'}(u)=4, d_{H_n'}(v)=4\}$, $d_{H_n'}(uv)=6$, $|E_{44}|=n-1$,

$E_{4(n-1)}=\{uv\in E(H_n')\mid d_{H_n'}(u)=4, d_{H_n'}(v)=n-1\}$, $d_{H_n'}(uv)=n+1$, $|E_{4(n-1)}|=n-1$,

$E_{43}=\{uv\in E(H_n')\mid d_{H_n'}(u)=4, d_{H_n'}(v)=3\}$, $d_{H_n'}(uv)=5$, $|E_{43}|=n-1$,

$E_{33}=\{uv\in E(H_n')\mid d_{H_n'}(u)=3, d_{H_n'}(v)=3\}$, $d_{H_n'}(uv)=4$, $|E_{33}|=n-1$.

The definition of product connectivity Banhatti index of a graph gives,

$$
\begin{aligned}
PB(H_n^{'}) &= \sum_{ue} \frac{1}{\sqrt{d_{H_n^{'}}(u) d_{H_n^{'}}(e)}} \\
&= \sum_{e=uv\in E(H_n)} \left[\frac{1}{\sqrt{d_{H_n^{'}}(u) d_{H_n^{'}}(e)}} + \frac{1}{\sqrt{d_{H_n^{'}}(v) d_{H_n^{'}}(e)}} \right] \\
&= (n-1)\left[\frac{1}{\sqrt{4\cdot 6}} + \frac{1}{\sqrt{4\cdot 6}}\right] + (n-1)\left[\frac{1}{\sqrt{4\cdot 5}} + \frac{1}{\sqrt{3\cdot 5}}\right] \\
&\quad + (n-1)\left[\frac{1}{\sqrt{4\cdot 3}} + \frac{1}{\sqrt{4\cdot 3}}\right] + (n-1)\left[\frac{1}{\sqrt{4(n+1)}} + \frac{1}{\sqrt{(n+1)(n-1)}}\right] \\
&= (n-1)\left[\frac{1}{2\sqrt{n+1}} + \frac{1}{\sqrt{(n+1)(n-1)}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{15}} + \frac{1}{2\sqrt{5}}\right].
\end{aligned}
$$

The *gear graph* G_n is a graph obtained from a wheel W_n by adding a vertex between every pair of adjacent vertices of the cycle C_{n-1}.

Theorem 2.6. *If the graph G_n is a gear graph of order $2n-1$, then*

$$PB(G_n) = (n-1)\left[\frac{2}{3} + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{3(n+2)}} + \frac{1}{\sqrt{(n-1)(n+2)}}\right].$$

Proof. The gear graph G_n is a graph of order $2n-1$ and size $3(n-1)$. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{32} = \{uv \in E(G_n) \mid d_{G_n}(u) = 3, d_{G_n}(v) = 2\}$, $d_{G_n}(uv) = 3$, $|E_{32}| = 2(n-1)$,

$E_{3(n-1)} = \{uv \in E(G_n) \mid d_{G_n}(u) = 3, d_{G_n}(v) = n-1\}$, $d_{G_n}(uv) = n+2$, $|E_{3(n-1)}| = n-1$.

By the definition of product connectivity Banhatti index of a graph we have,

$$
\begin{aligned}
PB(G_n) &= \sum_{ue} \frac{1}{\sqrt{d_{G_n}(u) d_{G_n}(e)}} \\
&= \sum_{e=uv\in E(G_n)} \left[\frac{1}{\sqrt{d_{G_n}(u) d_{G_n}(e)}} + \frac{1}{\sqrt{d_{G_n}(v) d_{G_n}(e)}} \right] \\
&= 2(n-1)\left[\frac{1}{\sqrt{3\cdot 3}} + \frac{1}{\sqrt{2\cdot 3}}\right] + (n-1)\left[\frac{1}{\sqrt{3(n+2)}} + \frac{1}{\sqrt{(n-1)(n+2)}}\right] \\
&= (n-1)\left[\frac{2}{3} + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{3(n+2)}} + \frac{1}{\sqrt{(n-1)(n+2)}}\right].
\end{aligned}
$$

The *flower graph* Fl_n is a graph obtained from a helm by joining each pendant vertex to a central vertex of the helm.

Theorem 2.7. *If the graph Fl_n is a flower of order $2n-1$, then*

$$PB(Fl_n)=(n-1)\left[\frac{1}{\sqrt{6}}+\frac{1}{2\sqrt{2n+2}}+\frac{1}{2\sqrt{(n-1)(n+1)}}+\frac{1}{\sqrt{n-1}}+\frac{1}{\sqrt{4}}+\frac{1}{2\sqrt{2}}\right].$$

Proof. The flower graph Fl_n is a graph of order $2n-1$ and size $4(n-1)$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{44}=\{uv\in E(Fl_n)\mid d_{Fl_n}(u)=4, d_{Fl_n}(v)=4\}$, $d_{Fl_n}(uv)=6$, $|E_{44}|=n-1$,

$E_{4(2n-2)}=\{uv\in E(Fl_n)\mid d_{Fl_n}(u)=4, d_{Fl_n}(v)=2n-2\}$, $d_{Fl_n}(uv)=2n+2$, $|E_{4(2n-2)}|=n-1$,

$E_{42}=\{uv\in E(Fl_n)\mid d_{Fl_n}(u)=4, d_{Fl_n}(v)=2\}$, $d_{Fl_n}(uv)=4$, $|E_{42}|=n-1$,

$E_{2(2n-2)}=\{uv\in E(Fl_n)\mid d_{Fl_n}(u)=2, d_{Fl_n}(v)=2n-2\}$, $d_{Fl_n}(uv)=2n-2$, $|E_{2(2n-2)}|=n-1$.

By the definition of product connectivity Banhatti index of a graph we have,

$$\begin{aligned}
PB(Fl_n) &= \sum_{ue}\frac{1}{\sqrt{d_{Fl_n}(u)d_{Fl_n}(e)}}\\
&= \sum_{e=uv\in E(Fl_n)}\left[\frac{1}{\sqrt{d_{Fl_n}(u)d_{Fl_n}(e)}}+\frac{1}{\sqrt{d_{Fl_n}(v)d_{Fl_n}(e)}}\right]\\
&= (n-1)\left[\frac{1}{\sqrt{4\cdot 6}}+\frac{1}{\sqrt{4\cdot 6}}\right]+(n-1)\left[\frac{1}{\sqrt{4(2n+2)}}+\frac{1}{\sqrt{(2n-2)(2n+2)}}\right]\\
&\quad+(n-1)\left[\frac{1}{\sqrt{4\cdot 4}}+\frac{1}{\sqrt{2\cdot 4}}\right]+(n-1)\left[\frac{1}{\sqrt{2(2n-2)}}+\frac{1}{\sqrt{2(2n-2)}}\right]\\
&= (n-1)\left[\frac{1}{\sqrt{6}}+\frac{1}{2\sqrt{2n+2}}+\frac{1}{2\sqrt{(n-1)(n+1)}}+\frac{1}{\sqrt{n-1}}+\frac{1}{\sqrt{4}}+\frac{1}{2\sqrt{2}}\right]
\end{aligned}$$

The *sunflower graph* SF_n is a graph obtained from a wheel with central vertex c, n-cycle $v_0, v_1, \ldots, v_{n-1}$ and additional n vertices $w_0, w_1, \ldots, w_{n-1}$ where w_i is joined by edges to v_i, v_{i+1} for $i=0,1,\ldots,n-1$ where $i+1$ is taken modulo n.

Theorem 2.8. *If the graph SF_n is a sunflower of order $2n+1$, then*

$$PB(SF_n)=n\left[\frac{1}{\sqrt{n(n+3)}}+\frac{1}{\sqrt{5(n+3)}}+\frac{1}{\sqrt{10}}+\frac{2}{\sqrt{10}}+\frac{2}{5}\right].$$

Proof. The sunflower graph SF_n is a graph of order $2n+1$ and size $4n$. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{5n}=\{uv\in E(SF_n)\mid d_{SF_n}(u)=5, d_{SF_n}(v)=n\}$, $d_{SF_n}(uv)=n+3$, $|E_{5n}|=n$,

$E_{55} = \{uv \in E(SF_n) \mid d_{SF_n}(u) = 5, d_{SF_n}(v) = 5\}$, $d_{SF_n}(uv) = 8$, $|E_{55}| = n$,

$E_{52} = \{uv \in E(SF_n) \mid d_{SF_n}(u) = 5, d_{SF_n}(v) = 2\}$, $d_{SF_n}(uv) = 5$, $|E_{52}| = 2n$.

The definition of product connectivity Banhatti index of a graph and above data together gives,

$$PB(SF_n) = \sum_{ue} \frac{1}{\sqrt{d_{SF_n}(u) d_{SF_n}(e)}}$$

$$= \sum_{e=uv \in E(SF_n)} \left[\frac{1}{\sqrt{d_{SF_n}(u) d_{SF_n}(e)}} + \frac{1}{\sqrt{d_{SF_n}(v) d_{SF_n}(e)}} \right]$$

$$= n\left[\frac{1}{\sqrt{n(n+3)}} + \frac{1}{\sqrt{5(n+3)}} \right] + n\left[\frac{1}{\sqrt{5 \cdot 8}} + \frac{1}{\sqrt{5 \cdot 8}} \right]$$

$$+ 2n\left[\frac{1}{\sqrt{5 \cdot 5}} + \frac{1}{\sqrt{2 \cdot 5}} \right]$$

$$= n\left[\frac{1}{\sqrt{n(n+3)}} + \frac{1}{\sqrt{5(n+3)}} + \frac{1}{\sqrt{10}} + \frac{2}{\sqrt{10}} + \frac{2}{5} \right].$$

The graph $F_n = P_n + K_1$ is called a *fan graph*, where $P_n : u_1 u_2 ... u_n$ is a path.

Theorem 2.9. *If the graph F_n is a fan graph of order $n+1$, then*

$$PB(F_n) = 2\left[\frac{1}{\sqrt{2n}} + \frac{1}{n} + \frac{1}{\sqrt{6}} + \frac{1}{3} + \frac{n-3}{\sqrt{7}} \right] + (n-2)\left[\frac{1}{\sqrt{3(n+1)}} + \frac{1}{\sqrt{n(n+1)}} \right].$$

Proof. The fan graph F_n is a graph of order $n+1$ and size $2n-1$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{2n} = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = n\}$, $d_{F_n}(uv) = n$, $|E_{2n}| = 2$,

$E_{23} = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 3\}$, $d_{F_n}(uv) = 3$, $|E_{23}| = 2$,

$E_{33} = \{uv \in E(F_n) \mid d_{F_n}(u) = 3, d_{F_n}(v) = 3\}$, $d_{F_n}(uv) = 4$, $|E_{33}| = n-3$,

$E_{3n} = \{uv \in E(F_n) \mid d_{F_n}(u) = 3, d_{F_n}(v) = n\}$, $d_{F_n}(uv) = n+1$, $|E_{3n}| = n-2$.

The definition of product connectivity Banhatti index of a graph gives,

$$
\begin{aligned}
PB(F_n) &= \sum_{ue} \frac{1}{\sqrt{d_{F_n}(u)d_{F_n}(e)}} \\
&= \sum_{e=uv\in E(F_n)} \left[\frac{1}{\sqrt{d_{F_n}(u)d_{F_n}(e)}} + \frac{1}{\sqrt{d_{F_n}(v)d_{F_n}(e)}} \right] \\
&= 2\left[\frac{1}{\sqrt{2\cdot n}} + \frac{1}{\sqrt{n\cdot n}} \right] + 2\left[\frac{1}{\sqrt{2\cdot 3}} + \frac{1}{\sqrt{3\cdot 3}} \right] \\
&\quad + (n-3)\left[\frac{1}{\sqrt{4\cdot 3}} + \frac{1}{\sqrt{4\cdot 3}} \right] + (n-2)\left[\frac{1}{\sqrt{3(n+1)}} + \frac{1}{\sqrt{n(n+1)}} \right] \\
&= 2\left[\frac{1}{\sqrt{2n}} + \frac{1}{n} + \frac{1}{\sqrt{6}} + \frac{1}{3} + \frac{n-3}{\sqrt{7}} \right] + (n-2)\left[\frac{1}{\sqrt{3(n+1)}} + \frac{1}{\sqrt{n(n+1)}} \right].
\end{aligned}
$$

The graph $DF_n = P_n + 2k_1$ is called a *double fan*.

Theorem 2.10. *If the graph DF_n is a double fan graph of order $n+2$, then*

$$
PB(DF_n) = 4\left[\frac{1}{\sqrt{3(n+1)}} + \frac{1}{\sqrt{n(n+1)}} \right] + 2(n-2)\left[\frac{1}{2\sqrt{n+2}} + \frac{1}{\sqrt{n(n+2)}} \right] + \frac{(n-3)}{\sqrt{6}} + \frac{2}{\sqrt{15}} + \frac{1}{\sqrt{5}}.
$$

Proof. The double fan graph DF_n is a graph of order $n+2$ and size $3n-1$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{3n} = \{uv \in E(DF_n) \mid d_{DF_n}(u) = 3, d_{DF_n}(v) = n\}$, $d_{DF_n}(uv) = n+1$, $|E_{3n}| = 4$,

$E_{34} = \{uv \in E(DF_n) \mid d_{DF_n}(u) = 3, d_{DF_n}(v) = 4\}$, $d_{DF_n}(uv) = 5$, $|E_{34}| = 2$,

$E_{4n} = \{uv \in E(DF_n) \mid d_{DF_n}(u) = 4, d_{DF_n}(v) = n\}$, $d_{DF_n}(uv) = n+2$, $|E_{4n}| = 2n-4$,

$E_{44} = \{uv \in E(DF_n) \mid d_{DF_n}(u) = 4, d_{DF_n}(v) = 4\}$, $d_{DF_n}(uv) = 6$, $|E_{44}| = n-3$.

From the definition of product connectivity Banhatti index of a graph we arrive at the following.

$$
\begin{aligned}
PB(DF_n) &= \sum_{ue} \frac{1}{\sqrt{d_{DF_n}(u)d_{DF_n}(e)}} \\
&= \sum_{e=uv\in E(DF_n)} \left[\frac{1}{\sqrt{d_{DF_n}(u)d_{DF_n}(e)}} + \frac{1}{\sqrt{d_{DF_n}(v)d_{DF_n}(e)}} \right]
\end{aligned}
$$

$$= 4\left[\frac{1}{\sqrt{3(n+1)}}+\frac{1}{\sqrt{n(n+1)}}\right]+2\left[\frac{1}{\sqrt{3\cdot 5}}+\frac{1}{\sqrt{4\cdot 5}}\right]$$

$$+(2n-4)\left[\frac{1}{\sqrt{4(n+2)}}+\frac{1}{\sqrt{n(n+2)}}\right]+(n-3)\left[\frac{1}{\sqrt{4\cdot 6}}+\frac{1}{\sqrt{4\cdot 6}}\right]$$

$$= 4\left[\frac{1}{\sqrt{3(n+1)}}+\frac{1}{\sqrt{n(n+1)}}\right]+2(n-2)\left[\frac{1}{2\sqrt{n+2}}+\frac{1}{\sqrt{n(n+2)}}\right]$$

$$+\frac{(n-3)}{\sqrt{6}}+\frac{2}{\sqrt{15}}+\frac{1}{\sqrt{5}}.$$

III. PRODUCT CONNECTIVITY BANHATTI INDEX OF PRODUCT RELATED GRAPHS

The graph $S_m \times P_2$ (where S_m is a star with $m+1$ vertices) is called a *book graph* B_m.

Theorem 3.1. *If the graph* B_m *is a book graph, then*

$$PB(B_m) = m\left[\frac{2}{m+1}+\sqrt{\frac{2}{m+1}}+1\right]+\sqrt{\frac{2}{m(m+1)}}.$$

Proof. The book graph B_m is a graph of order $2m+2$ and size $3m+1$. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in E(B_m) \mid d_{B_m}(u)=2, d_{B_m}(v)=2\}$, $d_{B_m}(uv)=2$, $|E_{22}| = m$,

$E_{2(m+1)} = \{uv \in E(B_m) \mid d_{B_m}(u)=2, d_{B_m}(v)=m+1\}$, $d_{B_m}(uv)=m+1$, $|E_{2(m+1)}| = 2m$,

$E_{(m+1)(m+1)} = \{uv \in E(B_m) \mid d_{B_m}(u)=m+1, d_{B_m}(v)=m+1\}$, $d_{B_m}(uv)=2m$,

$|E_{(m+1)(m+1)}| = 1$.

From the definition of product connectivity Banhatti index of a graph we have,

$$PB(B_m) = \sum_{ue}\frac{1}{\sqrt{d_{B_m}(u)d_{B_m}(e)}}$$

$$= \sum_{e=uv\in E(B_m)}\left[\frac{1}{\sqrt{d_{B_m}(u)d_{B_m}(e)}}+\frac{1}{\sqrt{d_{B_m}(v)d_{B_m}(e)}}\right]$$

$$= m\left[\frac{1}{\sqrt{2\cdot 2}}+\frac{1}{\sqrt{2\cdot 2}}\right]+2m\left[\frac{1}{\sqrt{2(m+1)}}+\frac{1}{\sqrt{(m+1)(m+1)}}\right]$$

$$+\left[\frac{1}{\sqrt{(m+1)2m}}+\frac{1}{\sqrt{(m+1)2m}}\right]$$

$$= m\left[\frac{2}{m+1}+\sqrt{\frac{2}{m+1}}+1\right]+\sqrt{\frac{2}{m(m+1)}}.$$

The graph $B_t = P_2 + tK_1$ where $t \geq 1$ is called a *book with triangular pages*.

Theorem 3. 2. *If* B_t *is a book with triangular pages, then*

$$PB(B_t) = \left[\sqrt{\frac{2}{t(t+1)}} + t\sqrt{\frac{2}{t+1}} + \frac{2t}{t+1}\right].$$

Proof. The book B_t with triangular pages has order $t+2$ and size $2t+1$. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{(t+1)(t+1)} = \{uv \in E(B_t) \mid d_{B_t}(u) = t+1, d_{B_t}(v) = t+1\}$, $d_{B_t}(uv) = 2t$, $|E_{(t+1)(t+1)}| = 1$,

$E_{2(t+1)} = \{uv \in E(B_t) \mid d_{B_t}(u) = 2, d_{B_t}(v) = t+1\}$, $d_{B_t}(uv) = t+1$, $|E_{2(t+1)}| = 2t$.

Taking the definition of product connectivity Banhatti index of a graph and above data we have,

$$\begin{aligned} PB(B_t) &= \sum_{ue} \frac{1}{\sqrt{d_{B_t}(u) d_{B_t}(e)}} \\ &= \sum_{e=uv\in E(B_t)} \left[\frac{1}{\sqrt{d_{B_t}(u) d_{B_t}(e)}} + \frac{1}{\sqrt{d_{B_t}(v) d_{B_t}(e)}}\right] \\ &= \left[\frac{1}{\sqrt{(t+1)2t}} + \frac{1}{\sqrt{(t+1)2t}}\right] + 2t\left[\frac{1}{\sqrt{2(t+1)}} + \frac{1}{\sqrt{(t+1)(t+1)}}\right] \\ &= \left[\sqrt{\frac{2}{t(t+1)}} + t\sqrt{\frac{2}{t+1}} + \frac{2t}{t+1}\right]. \end{aligned}$$

The graph $G = P_m \times P_n$ is called a *planar grid.*

Theorem 3.3. *Let* G *be a planar grid, then*

$$PB(G) = \frac{(2mn - 5(m+n) + 12)}{\sqrt{6}} + \left[\frac{m+n-4}{\sqrt{5}}\right]\left[\frac{2}{\sqrt{3}} + 1\right] + \frac{2(m+n-6)}{\sqrt{3}} + 8\left[\frac{1}{3} + \frac{1}{\sqrt{6}}\right].$$

Proof. The planar grid G is a graph of order mn and size $m(n-1) + n(m-1)$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $d_G(uv) = 3$, $|E_{23}| = 8$,

$E_{33} = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 3\}$, $d_G(uv) = 4$, $|E_{33}| = 2(n-3) + 2(m-3)$,

$E_{34} = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}$, $d_G(uv) = 5$, $|E_{34}| = 2(m-2) + 2(n-2)$,

$E_{44} = \{uv \in E(G) \mid d_G(u) = 4, d_G(v) = 4\}$, $d_G(uv) = 6$, $|E_{44}| = 2mn - 5(m+n) + 12$.

By the definition of product connectivity Banhatti index of a graph we have,

$$\begin{aligned} PB(G) &= \sum_{ue} \frac{1}{\sqrt{d_G(u) d_G(e)}} \\ &= \sum_{e=uv\in E(G)} \left[\frac{1}{\sqrt{d_G(u) d_G(e)}} + \frac{1}{\sqrt{d_G(v) d_G(e)}}\right] \end{aligned}$$

$$= 8\left[\frac{1}{\sqrt{2\cdot 3}}+\frac{1}{\sqrt{3\cdot 3}}\right]+\left[2(n-3)+2(m-3)\right]\left[\frac{1}{\sqrt{3\cdot 4}}+\frac{1}{\sqrt{3\cdot 4}}\right]$$
$$+\left[2(n-2)+2(m-2)\right]\left[\frac{1}{\sqrt{3\cdot 5}}+\frac{1}{\sqrt{4\cdot 5}}\right]$$
$$+(2mn-5(m+n)+12)\left[\frac{1}{\sqrt{4\cdot 6}}+\frac{1}{\sqrt{4\cdot 6}}\right]$$
$$= \frac{(2mn-5(m+n)+12)}{\sqrt{6}}+\left[\frac{m+n-4}{\sqrt{5}}\right]\left[\frac{2}{\sqrt{3}}+1\right]+\frac{2(m+n-6)}{\sqrt{3}}+8\left[\frac{1}{3}+\frac{1}{\sqrt{6}}\right].$$

The graph $P_n \times P_2$ is called a *ladder graph* L_n.

Theorem 3.4. *If* L_n *is a ladder graph, then*

$$PB(L_n)=2\left[1+\frac{2}{\sqrt{3}}\right]+\frac{1}{3}\left[(3n-8)+\frac{4}{\sqrt{2}}\right].$$

Proof. The ladder graph L_n is a graph of order $2n$ and size $3n-2$. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22}=\{uv\in E(L_n) \mid d_{L_n}(u)=2, d_{L_n}(v)=2\}$, $d_{L_n}(uv)=2$, $|E_{22}|=2$,

$E_{23}=\{uv\in E(L_n) \mid d_{L_n}(u)=2, d_{L_n}(v)=3\}$, $d_{L_n}(uv)=3$, $|E_{23}|=4$,

$E_{33}=\{uv\in E(L_n) \mid d_{L_n}(u)=3, d_{L_n}(v)=3\}$, $d_{L_n}(uv)=4$, $|E_{33}|=3n-8$.

The definition of product connectivity Banhatti index of a graph and above data gives,

$$PB(L_n) = \sum_{ue}\frac{1}{\sqrt{d_{L_n}(u)d_{L_n}(e)}}$$
$$= \sum_{e=uv\in E(L_n)}\left[\frac{1}{\sqrt{d_{L_n}(u)d_{L_n}(e)}}+\frac{1}{\sqrt{d_{L_n}(v)d_{L_n}(e)}}\right]$$
$$= 2\left[\frac{1}{\sqrt{2\cdot 2}}+\frac{1}{\sqrt{2\cdot 2}}\right]+4\left[\frac{1}{\sqrt{2\cdot 3}}+\frac{1}{\sqrt{3\cdot 3}}\right]$$
$$+(3n-8)\left[\frac{1}{\sqrt{3\cdot 4}}+\frac{1}{\sqrt{3\cdot 4}}\right]$$
$$= 2\left[1+\frac{2}{\sqrt{3}}\right]+\frac{1}{3}\left[(3n-8)+\frac{4}{\sqrt{2}}\right].$$

The graph $H=P_n\circ K_1$ is called a *comb graph*.

Theorem 3.5. *If* H *is a comb graph, then*

$$PB(H)=\frac{n-3}{\sqrt{3}}+\frac{n}{\sqrt{2}}\left[1+\frac{1}{\sqrt{3}}\right]+\frac{8}{3}.$$

Proof. The comb H is a graph of order $2n$ and size $2n-1$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{12} = \{uv \in E(H) \mid d_H(u) = 1, d_H(v) = 2\}$, $d_H(uv) = 1$, $|E_{12}| = 2$,

$E_{13} = \{uv \in E(H) \mid d_H(u) = 1, d_H(v) = 3\}$, $d_H(uv) = 2$, $|E_{13}| = n-2$,

$E_{23} = \{uv \in E(H) \mid d_H(u) = 2, d_H(v) = 3\}$, $d_H(uv) = 3$, $|E_{23}| = 2$,

$E_{33} = \{uv \in E(H) \mid d_H(u) = 3, d_H(v) = 3\}$, $d_H(uv) = 4$, $|E_{33}| = n-3$.

Now, by the definition of product connectivity Banhatti index of a graph we have

$$\begin{aligned}
PB(H) &= \sum_{ue} \frac{1}{\sqrt{d_H(u)d_H(e)}} \\
&= \sum_{e=uv\in E(H)} \left[\frac{1}{\sqrt{d_H(u)d_H(e)}} + \frac{1}{\sqrt{d_H(v)d_H(e)}}\right] \\
&= 2\left[\frac{1}{\sqrt{1\cdot 1}} + \frac{1}{\sqrt{2\cdot 1}}\right] + 2\left[\frac{1}{\sqrt{2\cdot 3}} + \frac{1}{\sqrt{3\cdot 3}}\right] + (n-2)\left[\frac{1}{\sqrt{1\cdot 2}} + \frac{1}{\sqrt{3\cdot 2}}\right] \\
&\quad + (n-3)\left[\frac{1}{\sqrt{3\cdot 4}} + \frac{1}{\sqrt{3\cdot 4}}\right] \\
&= \frac{n-3}{\sqrt{3}} + \frac{n}{\sqrt{2}}\left[1 + \frac{1}{\sqrt{3}}\right] + \frac{8}{3}.
\end{aligned}$$

The graph $X = P_n \circ 2K_1$ is called a *double comb graph.*

Theorem 3.6. *If X is a double comb graph, then*

$$PB(X) = \frac{n-3}{\sqrt{6}} + (n-2)\sqrt{3} + \frac{4}{\sqrt{2}}\left[1 + \frac{1}{\sqrt{3}}\right] + \frac{3}{\sqrt{15}}.$$

Proof. The double comb X is a graph of order $3n$ and size $3n-1$. By algebraic method, there are four types of edges based on the degree of the end vertices of each edge as follows:

$E_{13} = \{uv \in E(X) \mid d_X(u) = 1, d_X(v) = 3\}$, $d_X(uv) = 2$, $|E_{13}| = 4$,

$E_{14} = \{uv \in E(X) \mid d_X(u) = 1, d_X(v) = 4\}$, $d_X(uv) = 3$, $|E_{14}| = 2n-4$,

$E_{34} = \{uv \in E(X) \mid d_X(u) = 3, d_X(v) = 4\}$, $d_X(uv) = 5$, $|E_{34}| = 2$,

$E_{44} = \{uv \in E(X) \mid d_X(u) = 4, d_X(v) = 4\}$, $d_X(uv) = 6$, $|E_{44}| = n-3$.

Now, by the definition of product connectivity Banhatti index of a graph we have

$$\begin{aligned}
PB(X) &= \sum_{ue} \frac{1}{\sqrt{d_X(u)d_X(e)}} \\
&= \sum_{e=uv\in E(X)} \left[\frac{1}{\sqrt{d_X(u)d_X(e)}} + \frac{1}{\sqrt{d_X(v)d_X(e)}}\right]
\end{aligned}$$

$$= \quad 4\left[\frac{1}{\sqrt{1\cdot 2}}+\frac{1}{\sqrt{3\cdot 2}}\right]+(2n-4)\left[\frac{1}{\sqrt{1\cdot 3}}+\frac{1}{\sqrt{4\cdot 3}}\right]+2\left[\frac{1}{\sqrt{3\cdot 5}}+\frac{1}{\sqrt{4\cdot 5}}\right]$$

$$+(n-3)\left[\frac{1}{\sqrt{4\cdot 6}}+\frac{1}{\sqrt{4\cdot 6}}\right]$$

$$= \quad \frac{n-3}{\sqrt{6}}+(n-2)\sqrt{3}+\frac{4}{\sqrt{2}}\left[1+\frac{1}{\sqrt{3}}\right]+\frac{3}{\sqrt{15}}.$$

The graph $C_n \circ K_1$ is called a *crown graph*.

Theorem 3.7. *If A is a crown graph, then*

$$PB(A) = n\left[\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{6}}\right].$$

Proof. The crown graph has $2n$ vertices and $2n$ edges. By algebraic method, there are two types of edges based on the degree of the end vertices of each edge as follows:

$E_{13} = \{uv \in E(A) \mid d_A(u)=1, d_A(v)=3\}$, $d_A(uv)=2$, $|E_{13}| = n$,

$E_{33} = \{uv \in \quad E(A) \mid d_A(u)=3, d_A(v)=3\}$, $d_A(uv)=4$, $|E_{33}| = n$.

The definition of product connectivity Banhatti index of a graph gives,

$$PB(A) \quad = \quad \sum_{ue}\frac{1}{\sqrt{d_A(u)d_A(e)}}$$

$$= \quad \sum_{e=uv\in E(A)}\left[\frac{1}{\sqrt{d_A(u)d_A(e)}}+\frac{1}{\sqrt{d_A(v)d_A(e)}}\right]$$

$$= \quad n\left[\frac{1}{\sqrt{1\cdot 2}}+\frac{1}{\sqrt{3\cdot 2}}\right]+n\left[\frac{1}{\sqrt{3\cdot 4}}+\frac{1}{\sqrt{3\cdot 4}}\right]$$

$$= \quad n\left[\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{6}}\right].$$

Two even cycles of the same order, say C_n, sharing a common vertex with m pendant edges attached at the common vertex is called a *butterfly graph* $By_{m,n}$.

Theorem 3.8. *If $By_{m,n}$ is a butterfly graph, then*

$$PB(By_{m,n}) = 2(n-2)+\frac{4}{\sqrt{m+4}}\left[1+\frac{1}{\sqrt{2}}\right]+\frac{m}{\sqrt{m+3}}\left[1+\frac{1}{\sqrt{m+4}}\right].$$

Proof. The graph $By_{m,n}$ has $2n+m-1$ vertices and $2n+m$ edges. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in E(By_{m,n}) \mid d_{By_{m,n}}(u)=2, d_{By_{m,n}}(v)=2\}$, $d_{By_{m,n}}(uv)=2$, $|E_{22}| = 2(n-2)$,

$E_{2(m+4)} = \{uv \in \quad E(By_{m,n}) \mid d_{By_{m,n}}(u)=2, d_{By_{m,n}}(v)=m+4\}$, $d_{By_{m,n}}(uv)=m+4$,

$|E_{2(m+4)}|=4$,

$E_{(m+4)1}=\{uv\in E(By_{m,n})\,|\,d_{By_{m,n}}(u)=m+4, d_{By_{m,n}}(v)=1\}$, $d_{By_{m,n}}(uv)=m+3$,

$|E_{(m+4)1}|=m$.

Considering the definition of product connectivity Banhatti index of a graph along with above data we get,

$$PB(By_{m,n}) = \sum_{ue}\frac{1}{\sqrt{d_{By_{m,n}}(u)d_{By_{m,n}}(e)}}$$

$$= \sum_{e=uv\in E(By_{m,n})}\left[\frac{1}{\sqrt{d_{By_{m,n}}(u)d_{By_{m,n}}(e)}}+\frac{1}{\sqrt{d_{By_{m,n}}(v)d_{By_{m,n}}(e)}}\right]$$

$$= 2(n-2)\left[\frac{1}{\sqrt{2\cdot 2}}+\frac{1}{\sqrt{2\cdot 2}}\right]+4\left[\frac{1}{\sqrt{2(m+4)}}+\frac{1}{\sqrt{(m+4)(m+4)}}\right]$$

$$+m\left[\frac{1}{\sqrt{(m+4)(m+3)}}+\frac{1}{\sqrt{m+3}}\right]$$

$$= 2(n-2)+\frac{4}{\sqrt{m+4}}\left[1+\frac{1}{\sqrt{2}}\right]+\frac{m}{\sqrt{m+3}}\left[1+\frac{1}{\sqrt{m+4}}\right].$$

The *triangular snake* T_n is obtained from the path P_n by replacing each edge of a path by a triangle C_3.

Theorem 3.9. *If T_n is a triangular snake, then*

$$PB(T_n)=(n-1)\sqrt{2}\left[\frac{1}{\sqrt{3}}+\frac{1}{2}\right]+\frac{2(n-3)}{\sqrt{10}}+2.$$

Proof. The graph T_n has $2n-1$ vertices and $3(n-1)$ edges. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22}=\{uv\in E(T_n)\,|\,d_{T_n}(u)=2, d_{T_n}(v)=2\}$, $d_{T_n}(uv)=2$, $|E_{22}|=2$,

$E_{24}=\{uv\in E(T_n)\,|\,d_{T_n}(u)=2, d_{T_n}(v)=4\}$, $d_{T_n}(uv)=4$, $|E_{24}|=2(n-1)$,

$E_{44}=\{uv\in E(T_n)\,|\,d_{T_n}(u)=4, d_{T_n}(v)=4\}$, $d_{T_n}(uv)=6$, $|E_{44}|=n-3$.

In view of the definition of product connectivity Banhatti index of a graph and above data we can write,

$$
\begin{aligned}
PB(T_n) &= \sum_{ue} \frac{1}{\sqrt{d_{T_n}(u) d_{T_n}(e)}} \\
&= \sum_{e=uv\in E(T_n)} \left[\frac{1}{\sqrt{d_{T_n}(u) d_{T_n}(e)}} + \frac{1}{\sqrt{d_{T_n}(v) d_{T_n}(e)}} \right] \\
&= 2\left[\frac{1}{\sqrt{2\cdot 2}} + \frac{1}{\sqrt{2\cdot 2}} \right] + 2(n-1)\left[\frac{1}{\sqrt{2\cdot 4}} + \frac{1}{\sqrt{4\cdot 4}} \right] \\
&\quad + (n-3)\left[\frac{1}{\sqrt{4\cdot 6}} + \frac{1}{\sqrt{4\cdot 6}} \right] \\
&= (n-1)\left[\frac{1}{\sqrt{2}} + \frac{1}{2} \right] + \frac{(n-3)}{\sqrt{6}} + 2.
\end{aligned}
$$

The *alternate triangular snake* $A(T_n)$ is obtained from the path $P_n : u_1, u_2 ... u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by a triangle C_3.

Theorem 3.10. *If* $A(T_n)$ *is an alternate triangular snake, then*

$$PB(A(T_n)) = 2(n-2) + \frac{4}{\sqrt{6}} + \frac{4}{3} + \frac{1}{\sqrt{3}}.$$

Proof. The graph $A(T_n)$ has $2n$ vertices and $2n+1$ edges. By algebraic method, there are three types of edges based on the degree of the end vertices of each edge as follows:

$E_{22} = \{uv \in \ E(A(T_n)) \mid d_{A(T_n)}(u) = 2, d_{A(T_n)}(v) = 2\}$, $d_{A(T_n)}(uv) = 2$, $|E_{22}| = 2(n-2)$,

$E_{23} = \{uvï \in ï\ E(A(T_n)) \mid d_{A(T_n)}(u) = 2, d_{A(T_n)}(v) = 3\}$, $d_{A(T_n)}(uv) = 3$, $|E_{23}| = 4$,

$E_{33} = \{uv \in \ E(A(T_n)) \mid d_{A(T_n)}(u) = 3, d_{A(T_n)}(v) = 3\}$, $d_{A(T_n)}(uv) = 4$, $|E_{33}| = 1$.

By the definition of product connectivity Banhatti index of a graph we have the following.

$$
\begin{aligned}
PB(A(T_n)) &= \sum_{ue} \frac{1}{\sqrt{d_{A(T_n)}(u) d_{A(T_n)}(e)}} \\
&= \sum_{e=uv\in E(A(T_n))} \left[\frac{1}{\sqrt{d_{A(T_n)}(u) d_{A(T_n)}(e)}} + \frac{1}{\sqrt{d_{A(T_n)}(v) d_{A(T_n)}(e)}} \right] \\
&= 2(n-2)\left[\frac{1}{\sqrt{2\cdot 2}} + \frac{1}{\sqrt{2\cdot 2}} \right] + 4\left[\frac{1}{\sqrt{2\cdot 3}} + \frac{1}{\sqrt{3\cdot 3}} \right] \\
&\quad + \left[\frac{1}{\sqrt{3\cdot 4}} + \frac{1}{\sqrt{3\cdot 4}} \right] \\
&= 2(n-2) + \frac{4}{\sqrt{6}} + \frac{4}{3} + \frac{1}{\sqrt{3}}.
\end{aligned}
$$

IV. CONCLUSION

In this paper, we have made an important observation that the product connectivity Banhatti index cannnot be applied for P_2 (considering the connected graphs). We have also computed the expression for product connectivity Banhatti index of few graphs obtained by some graph operations. Finding the product connectivity Banhatti index of graph operations in general which is a complicated task remains as an open problem.

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