International Journal of Mathematics and Statistics Invention (IJMSI)
E-ISSN: 2321 – 4767 P-ISSN: 2321 - 4759
www.ijmsi.org Volume 7 Issue 1 || January, 2019 || PP-01-06

On some decompositions of nano ∗ - continuity

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ABSTRACT: *In this paper, we introduce the notions of NIrg* closed sets, NG - I - LC*-sets and weakly NG - I - LC* - sets in nano ideal topological spaces and obtained some decompositions of nano∗ - continuity. Further, we introduce the notions of weakly NGLC* - sets, and weakly NGLC*-continuous maps in nano topological spaces and obtained decompositions of nano-continuity.*
KEYWORDS: *NG - I - LC*- sets, weakly NG - I - LC*- sets and NIrg*- sets.*
2010 Mathematics Subject Classifications: 54A05, 54A10, 54B05.

Date of Submission: 03-02-2019 Date Of Acceptance: 19-02-2019

I. INTRODUCTION

The notion of nano topological space was introduced by Lellis Thivagar [5], [6] which was defined in terms of lower approximations, upper approximations and boundary region of a subset X of an universe using an equivalence relation on it. Lellis Thivagar et al. [6] studied a new class of functions called nano continuous functions and their characterizations in nano topological spaces. Stephan Antony Raj et al. [12] studied the expansion of nano-open sets and obtained decompositions of nano continuity in nano topological spaces. The concept of ideals in topological spaces is treated in the classic text by Kuratowski [4] and Vaidyanathaswamy [14]. The notion of a nano ideal topological space was introduced by Lellis Thivagar et al. [7]. The concept of NIg-closed sets in nano ideal topological spaces was introduced by Parimala et al. [9]. In this paper, we obtained some decompositions of nano ∗-continuity in nano ideal topological spaces by introducing NIrg* -closed sets and NG - I -LC* - sets.

II. PRELIMINARIES

Definition 1.1 [6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.
Let $X \subseteq U$. Then,
1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = U\,\{R(a) : R(a) \subseteq X, a \in U\}$, where R(a) denotes the equivalence class determined by $a \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X) = U\,\{R(a):R(a) \cap X \neq \phi, a \in U\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 1.2 [5] If (U,R) is an approximation space and $X, Y \subseteq U$; then
1. $L_R(X) \subseteq X \subseteq L_R(X)$.
2 $L_R(X) = U_R(X)= \phi$ and $L_R(U) = U_R(U) = U$.
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
9. $U_RU_R(X) = L_RU_R(X) = U_R(X)$.

10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.3 [6] Let U be the universe and R be an equivalence relation on U. Then for $X \subseteq U$, $\tau_R(X) = \{ \phi, L_R(X), U_R(X), B_R(X), U \}$ is called the nano topology on U. Then by property 1.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

We call $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called nano - open sets and the complement of a nano - open set is a nano - closed set.

If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano interior of the set A is defined as the union of all nano - open subsets contained in A and is denoted by Nint(A).

(ii) The nano closure of the set A is defined as the intersection of all nano-closed subsets containing A and is denoted by Ncl(A).

Definition 1.4 [4] An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties:

1. $A \in I$ and $B \in A$ imply $B \in I$ (heredity),
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$ (finite additivity).

Definition 1.5 [7] A nano topological space $(U, \tau_R(X))$ with an ideal I on U is called a nano ideal topological space or nano ideal space and denoted by $(U, \tau_R(X), I)$.

Definition 1.6 [7] Let $(U, \tau_R(X), I)$ be a nano ideal topological space. A set operator $A_n^* : P(U) \rightarrow P(U)$, is called the nano local function of I on $\tau_R(X)$ and is defined as $A_n^* = \{ x \in : U \cap A \notin I;$ for every $U \in \tau_R(X) \}$. The nano closure operator is defined as $Ncl^*(A) = A \cup (A_n^*)$.

Definition 1.7 [5] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be nano regular open if $A = Nint(Ncl(A))$.

Definition 1.8 A subset A of a nano topological space $(U, \tau_R(X))$ is said to be

1. Ng-closed [1] if $Ncl(A) \subseteq S$ whenever $A \subseteq S$ and S is nano-open.
2. Nrg-closed [13] if $Ncl(A) \subseteq S$ whenever $A \subseteq S$ and S is nano regular open.
3. Ng^*- closed [10] if $Ncl(A) \subseteq S$ whenever $A \subseteq S$ and S is Ng - open.
4. Nrg^*- closed [8] if $Ncl(A) \subseteq S$ whenever $A \subseteq S$ and S is Nrg - open in U .

The complement of a Ng - closed (resp. Nrg - closed, Ng^* - closed, Nrg^* - closed) set is called Ng - open (resp. Nrg - open, Ng^* - open, Nrg^* - open).

Definition 1.9 [3] A subset A of a nano topological space $(U, \tau_R(X))$ is said to be

1. Nano Locally Closed (briefly NLC) if $G \cap F$ where G is nano - open and F is nano - closed.
2. $NGLC^*$ - set if $G \cap F$ where G is Ng - open and F is nano - closed.

Definition 1.10 [9] A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is nano $*$ - closed (briefly n$*$ - closed) if $A_n^* \subset A$.

Result 1.11 [7] Let $(U, \tau_R(X), I)$ be a nano topological space with an ideal I on X and A is a subset of X. If $A \subset A_n^*$, then $A_n^* = Ncl(A_n^*) = Ncl(A) = cl^*(A)$.

Definition 1.12 Let $(U, \tau_R(X))$ and $(V, \tau_R(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be

1. nano continuous [6] if $f^{-1}(A)$ is nano-closed in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.
2. Ng -continuous [2] if $f^{-1}(A)$ is Ng -closed in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.
3. NLC -continuous [3] if $f^{-1}(A)$ is a NLC -set in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.
4. $NGLC^*$ -continuous [3] if $f^{-1}(A)$ is a $NGLC^*$ -set in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.
5. Nrg^*-continuous [8] if $f^{-1}(A)$ is Nrg^* -closed set in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.

6. Ng^*-continuous [10] if $f^{-1}(A)$ is Ng^* -closed set in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.

Theorem 1.13 [11] Every n $*$- closed set is NIg^* -closed.

Definition 1.14 [11] A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be NIg^*- closed if $A_n^* \subset S$ whenever $A \subset S$ and S is Ng - open in U.

III. NG-I-LC* SETS

Definition 2.1 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be NG -I - LC*-set if $A = G \cap F$, where G is Ng-open and F is n $*$ -closed.

Definition 2.2 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be weakly NG - I - LC*- set if $A = G \cap F$, where G is Nrg - open and F is n $*$ -closed.

Proposition 2.3 For a subset A of a nano ideal topological space $(U, \tau_R (X), I)$, the following hold.
1. If A is Ng - open, then A is a NG - I - LC* - set.
2. If A is n $*$- closed, then A is a NG - I - LC* - set.
3. If A is NG - I - LC*- set, then A is a weakly NG - I - LC* - set.

Proof. It is obvious from Definition 2.1 and Definition 2.2.

The converse of Proposition 2.3 need not be true as seen from the following example.

Example 2.4 Let U = {a, b, c, d} be the universe, X = {a, b} $\subseteq$ U with U\R = {{a}, {c}, {b, d}}, $\tau_R(X)$ = { ϕ, {a}, {b, d}, {a, b, d}, U} and the ideal I = {ϕ, {a}}. Then
1. A = {c} is a NG - I - LC* -set but not Ng - open.
2. A = {b} is a NG - I - LC* - set but not n $*$ -closed.
3. A = {b, c} is a weakly NG - I - LC* - set but not a NG - I - LC*- set.

Theorem 2.5 Let $(U, \tau_R(X), I)$ be a nano ideal topological space and A be a NG - I - LC*- subset of X. Then the following hold.
1. If B is a n $*$ - closed set, then $A \cap B$ is a NG - I - LC*- set.
2. If B is a Ng - open set, then $A \cap B$ is a NG - I - LC*- set.
3. If B is a NG - I - LC*- set, then $A \cap B$ is a NG - I - LC*- set.

Proof. 1. Let B be a n $*$ -closed set. Then $A \cap B = (G \cap F) \cap B = G \cap (F \cap B)$, where $F \cap B$ is n $*$ -closed. Hence $A \cap B$ is a NG - I - LC* - set.
2. Let B be a Ng - open set. Then $A \cap B = (G \cap F) \cap B = (G \cap F) \cap B$, where $G \cap B$ is Ng - open [1]. Hence $A \cap B$ is a NG - I - LC* - set.
3. Let B be a NG - I - LC* - set. Then $A \cap B = (G \cap F) \cap (U \cap V) = (G \cap U) \cap (F \cap V)$, where $G \cap U$ is Ng - open and $F \cap V$ is n $*$ -closed. Hence $A \cap B$ is a NG - I - LC* - set.

Definition 2.6 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be $NIrg^*$ - closed if $A_n^* \subset S$ whenever $A \subset S$ and S is Nrg - open in U.

Theorem 2.7 For a subset A of a nano ideal topological space $(U, \tau_R(X), I)$, the following hold.
1. If A is n $*$ -closed, then A is $NIrg^*$ - closed.
2. If A is n $*$ -closed, then A is a weakly NG - I - LC* - set.
3. If A is $NIrg^*$ - closed, then A is NIg^* - closed.

Proof. Suppose that A is a subset of a nano ideal topological space $(U, \tau_R(X), I)$.
1. Let A be a n $*$ -closed. Then $A_n^* \subset A$. Now $A_n^* \subset U$ whenever $A \subset U$, where U is Nrg - open. Hence A is a $NIrg^*$ - closed set.
2. Follows from Proposition 2.3.
3. Let A be a $NIrg^*$ - closed set. That is, $A_n^* \subset G$ whenever $A \subset G$, where G is Nrg - open in U. Let $G \subseteq U$. Since U is Ng - open, G is Ng - open. Therefore, $A_n^* \subset G$ whenever $A \subset G$, where G is Ng - open in U. Hence A is NIg^* - closed.

The Converse of Theorem 2.7 need not be true as seen from the following example.

Example 2.8 Let U = {a, b, c, d, e} be the universe, X = {a, b, c} $\subseteq$ U with U\R = {{a, b},{c, d},{e}},

$\tau_R(X) = \{ \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, U\}$ and the ideal $I = \{ \phi, \{a\}\}$. Then

1. $A = \{a, b, c, e\}$ is a $NIrg^*$ - closed set but not a $n*$ - closed set.
2. $A = \{b\}$ is a weakly NG - I - LC^* - set but not a $n*$ - closed set.
3. $A = \{b, e\}$ is a NIg^*- closed set but not a $NIrg^*$ - closed set.

Theorem 2.9 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$, is $n*$-closed if and only if it is a weakly NG - I -LC^*- set and a $NIrg^*$- closed set.
Proof. Necessity is trivial. We shall prove only sufficiency. Let A be a weakly NG - I - LC^* - set and a $NIrg^*$ - closed set. Since A is a weakly NG - I - LC^* - set, $A = G \cap F$ where G is Nrg - open and F is $n*$-closed. So we have $A = G \cap F \subset G$. Since A is $NIrg^*$ - closed, $A_n^* \subset G$. Also $A = G \cap F \subset F$ and F is $n*$- closed implies $A_n^* \subset F$. Consequently, we have $A_n^* \subset G \cap F = A$ and hence A is $n*$-closed.

Theorem 2.10 For a subset A of a nano ideal topological space $(U, \tau_R(X), I)$, the following are equivalent.

1. A is $n*$-closed.
2. A is a NG - I - LC^* - set and a $NIrg^*$ - closed set.
3. A is a NG - I - LC^*- set and a NIg^*- closed set.

Proof. $(1) \Rightarrow (2)$: This is obvious.
$(2) \Rightarrow (3)$: Follows from the fact that every $NIrg^*$ - closed set is NIg^* - closed set [Theorem 2.7 (3)].
$(3) \Rightarrow (1)$: Let A be a NG - I - LC^*- set and a NIg^* - closed set. Since A is a NG - I - LC^* - set, $A = G \cap F$ where G is Ng - open and F is $n*$-closed. Now $A \subset G$ and A is NIg^*- closed implies $A_n^* \subset G$. Also $A \subset F$ and F is $n*$-closed implies that $A_n^* \subset F$. Therefore, $A_n^* \subset G \cap F = A$. Hence A is $n*$ -closed.

Remark 2.11

1. The notions of NG - I - LC^*- sets and NIg^* - closed sets are independent.
2. The notions of NG - I - LC^* -sets and $NIrg^*$ - closed sets are independent.
3. The notions of weakly NG - I - LC^* - sets and $NIrg^*$- closed sets are independent.

Example 2.12 Let $U = \{p, q, r, s, t\}$ be the universe, $X = \{p, s\} \subseteq U$ with $U\backslash R = \{\{p, q\},\{r, t\}, \{s\}\}$, $\tau_R(X) = \{ \phi, \{p, q\}, \{s\}, \{p, q, s\}, U\}$ and the ideal $I = \{ \phi , \{p\}\}$. Then,

1. $A = \{r, t\}$ is a NIg^* - closed set but not a NG - I - LC^* - set.
2. $A = \{p, q, s\}$ is a NG - I - LC^*- set but not a NIg^*- closed set.
3. $A = \{p, q, r, s\}$ is a NG - I - LC^* - set but not a $NIrg^*$- closed set.
4. $A = \{q, r, t\}$ is a $NIrg^*$ - closed set but not a NG - I - LC^* - set.
5. $A = \{p, q, s, t\}$ is a weakly NG - I - LC^* - set but not a $NIrg^*$- closed set.
6. $A = \{r, s, t\}$ is a $NIrg^*$- closed set but not a weakly NG - I - LC^* - set.

IV. A NEW SUBSET OF A NANO TOPOLOGICAL SPACE

Definition 3.1 Let A be a subset of a nano topological space $(U, \tau_R(X))$. Then the nano g - kernel of the set A, denoted by Ng - ker(A) is the intersection of all Ng - open supersets of A.

Definition 3.2 A subset A of a nano topological space $(U, \tau_R(X))$ is called nano Λ_g - set if A = Ng - ker(A).

Definition 3.3 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is called $N\lambda g$ -I-closed if $A = G \cap F$ where G is a nano Λ_g - set and F is $n*$ -closed.

Proposition 3.4 In a nano ideal topological space $(U, \tau_R(X), I)$, every $n*$ - closed set is $N\lambda g$ -I-closed.
Proof. It is obvious from Definition 3.3

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5 Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subseteq U$ with $U\backslash R = \{\{a\}, \{c\}, \{b, d\}\}$, $\tau_R(X) = \{ \phi , \{a\}, \{b, d\}, \{a, b, d, U\}$ and the ideal $I = \{ \phi , \{a\}\}$. Then the set $A = \{a\}$ is $N\lambda g$ -I-closed but not $n*$-closed.

Lemma 3.6 For a subset A of a nano ideal topological space $(U, \tau_R(X), I)$, the following are equivalent.

1. A is $N\lambda g$ - I - closed.
2. $A = P \cap Ncl^*(A)$ where P is a nano Λ_g - set.
3. $A = Ng\text{-}ker(A) \cap Ncl^*(A)$.

Proof. (1) $\Rightarrow$ (2) : Let A be a $N\lambda g$ - I - closed set. Then $A = P \cap Q$, where P is a $N\lambda g$ - I - set and Q is $n*$-closed. Clearly, $A \subset P \cap Ncl^*(A)$. Since Q is $n*$-closed, $Ncl^*(A) \subset Ncl^*(Q) = Q$ and so $P \cap Ncl^*(A) \subset P \cap Q = A$. Therefore, $A = P \cap Ncl^*(A)$.
(2) $\Rightarrow$ (3) : Let $A = P \cap Ncl^*(A)$, where P is a nano Λ_g - set. Since P is a nano Λ_g - set, we have $A = Ng\text{-}ker(A) \cap Ncl^*(A)$.
(3) $\Rightarrow$ (1) : Let $A = Ng\text{-}ker(A) \cap Ncl^*(A)$. By Definition 3.2 and the notion of $n*$-closed set, we get A is $N\lambda g$ - I - closed.

Lemma 3.7 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is NIg^*- closed if and only if $Ncl^*(A) \subset Ng\text{-}ker(A)$.
Proof. Assume that $A \subseteq U$ is NIg^* - closed set. Let $x \in Ncl^*(A)$. Suppose $x \notin Ng\text{-}ker(A)$. Then there exists a Ng - open set U containing A such that $x \notin U$. Since A is a NIg^* - closed set, $A \subseteq G$ and G is Ng - open implies that $Ncl^*(A) \subseteq G$ and so $x \notin Ncl^*(A)$, which is a contradiction. Therefore, $Ncl^*(A) \subset Ng\text{-}ker(A)$.
Conversely, let $Ncl^*(A) \subseteq Ng\text{-}ker(A)$. If P is any Ng -open set containing A, then $Ncl^*(A) \subseteq Ng\text{-}ker(A) \subseteq P$. Therefore A is NIg^* -closed.

Remark 3.8 The notions of NIg^*- closed sets and $N\lambda g$ - I - closed sets are independent.

Example 3.9 Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, b\} \subseteq U$ with $U \backslash R = \{\{b, d\}, \{a\}, \{c\}\}$, $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$ and the ideal $I = \{\phi, \{a\}\}$. Then
1. The set $A = \{b\}$ is $N\lambda g$ - I -closed but not NIg^*-closed.
2. The set $A = \{b, c\}$ is NIg^*-closed but not $N\lambda g$ - I -closed.

Theorem 3.10 A subset of a nano ideal topological space $(U, \tau_R(X), I)$ is $n*$-closed if and only if it is both NIg^* -closed and $N\lambda g$ - I -closed.
Proof. Necessity is obvious from Theorem 1.13 and Proposition 3.4. We shall prove Sufficiency.
Let A be a NIg^* -closed set and a $N\lambda g$ - I -closed set. As A is a $N\lambda g$ - I -closed set, $A = P \cap Q$, where P is a nano Λ_g -set and Q is $n*$-closed. Now, $A \subset P$ and A is NIg^* -closed set implies $A_n^* \subset P$. Also, $A \subset Q$ and Q is $n*$-closed set implies $A_n^* \subset Q$. Thus $A_n^* \subset P \cap Q = A$. Hence A is $n*$-closed.

V. DECOMPOSITIONS OF NANO $*$-CONTINUITY

Definition 4.1 A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is said to be nano $*$-continuous (briefly $n*$ - continuous) if $f^{-1}(A)$ is a $n*$-closed in $(U, \tau_R(X), I)$ for every nano-closed set A of $(V, \tau_R(Y))$.

Definition 4.2 A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is said to be NIg^* - continuous (resp. $NIrg^*$ - continuous, NG - I - LC^* - continuous, weakly NG - I - LC^* - continuous) if $f^{-1}(A)$ is NIg^* - closed (resp. $NIrg^*$ - closed, NG - I – LC^* - set, weakly NG - I – LC^* - set) in $(U, \tau_R(X), I)$ for every nano-closed set A of $(V, \tau_R(Y))$.

Definition 4.3 A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is said to be $N\lambda g$ - I - continuous if $f^{-1}(A)$ is a $N\lambda g$ - I -closed set in $(U, \tau_R(X), I)$ for every nano-closed set A in $(V, \tau_R(Y))$.

Theorem 4.4 A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is $n*$-continuous if and only if it is weakly NG - I - LC^*-continuous and $NIrg^*$-continuous.
Proof: This is an immediate consequence of Theorem 2.9.

Definition 4.5 A subset A of a nano topological space $(U, \tau_R(X))$ is weakly $NGLC^*$ -set if $A = G \cap F$, where G is Nrg -open and F is nano-closed.

Definition 4.6 A function $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be weakly $NGLC^*$ -continuous if $f^{-1}(A)$ is weakly $NGLC^*$ -set in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \tau_R(Y))$.

Corollary 4.7 Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $I = \{\phi\}$, then a function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is continuous if and only if it is weakly $NGLC^*$ -continuous and Nrg^* - continuous.

Theorem 4.8 For a function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$, the following are equivalent.
1. f is n $*$ -continuous;
2. f is NG - I - LC^* - continuous and $NIrg^*$ - continuous;
3. f is NG - I - LC^* - continuous and NIg^* - continuous.

Proof. This is an immediate consequence of Theorem 2.10.

Corollary 4.9 Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $I = \{ \phi \}$, then for a function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$, the following are equivalent.
1. f is nano continuous;
2. f is $NGLC^*$ - continuous and Nrg^* - continuous;
3. f is $NGLC^*$ - continuous and Ng^* - continuous.

Theorem 4.10 A function $f : (U, \tau_R(X), I) \rightarrow (V, \tau_R(Y))$ is n $*$ -continuous if and only if it is both NIg^*- continuous and Nλg -I- continuous.

Proof. This is an immediate consequence of Theorem 3.10.

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J. Jayasudha" On some decompositions of nano $*$ - continuity" International Journal of Mathematics and Statistics Invention (IJMSI), vol. 07, no. 01, 2019, pp. 01-06