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Fuzzy Diagonal Optimal Algorithm to solve Travelling Salesman Problem

P.Rajarajeswari[1]and D.Maheswari[2]
[1]*Department of Mathematics, Chikkanna Government Arts College, Tirupur, Tamilnadu, India*
[2]*Department of Mathematics,CMS College of Science and Commerce,Coimbatore,Tamilnadu, India.*

***ABSTRACT*:** *In this paper diagonal optimal algorithm is proposed to solve the travelling salesman problem. In this proposed method the Fuzzy optimal solution of a fuzzy TSP is obtained by using the diagonal method where the values are taken in interval numbers. A numerical example is solved and the result is validated using the existing method. This method can be applied to solve all kinds of fuzzy assignment problems such as unbalanced fuzzy AP, restrictions in AP and many more.*

KEYWORDS: *Interval numbers, travelling salesman problem, Diagonal Optimal method.*

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I. INTRODUCTION

Travelling Salesman Problem is a classical problem in Graph Theory and Combinatorial Optimization and it has been studied since 1950. Given a set of cities and the distance between each possible pairs, the traveling salesman problem is to find the best possible way of visiting all the cities and returning to the starting point that minimizes the travel distance. In TSP the traveling cost is given between the finite number of cities and the salesman will have to find the cheapest route passing through all the cities and returning back to the starting point. FTSP is a Travelling salesman problem where the cost of distance is considered as fuzzy numbers instead of crisp numbers. The fuzzy set theory which fruitfully handles the uncertainty was first implemented byZadeh [18]. Chen [2] proposed a fuzzy assignment model, by proving some theorems that consider all individuals to have some skills. Hansen [8] applied a tabu search algorithm to solve fuzzy TSP. Jaskiewicz [9] applied a genetic local search algorithm for solving fuzzy TSP. Yan et al., [17] used the evolutionary algorithm to solve fuzzy TSP. Sepideh Fereidouni [15] obtained the optimal solution by using a multi-objective linear programming technique. Rehmat et al., [13] solved the assignment problem by fuzzy linear programming technique. Later Kumar et. al. [11] applied different ways for solving fuzzy traveling salesman problems with different membership functions. Amit Kumar et al., [1] for solving fuzzy TSP proposed an algorithm with LR-fuzzy parameters. By using the Hungarian algorithm Dhanasekar et al.[4] solve the fuzzy TSP with the element-wise subtraction of a fuzzy number.P.Rajarajeswari et al, [12] solved TSP with interval cost constraints. Since fuzzy TSP is a polynomial-time problem, more number of works are still going on to solve the fuzzy TSP

In this paper, the salesman wants to visit cities allotted to him, where the cost or distance or time of journey between every pair of cities is considered as interval numbers which may fluctuate considerably. The salesman is to find the shortest route to cover all the cities.Here diagonal optimal algorithm is used to solve FTSP. The proposed method is easy to apply and understand compared to other existing methods and it is also a systematic, complete enumeration technique. The proposed algorithm is validated through examples with existing methods.

The paper is organized as follows: In section II the basic definitions are given. Section III discusses the proposed algorithm. An example and comparison is given in Section IV and the conclusion is given in Section V.

II. PRELIMINARIES

2.1 Definition

An interval number A is defined as $A= [a,b] =\{x / a \leq x \leq b , x \in \Re \}$. Here $a,b\in\Re$ are the lower and upper bound of the intervals.

2.2 Definition

Arithmetic operations on Interval Numbers
Let $A = [a,b]$ and $B =[c,d]$ be two interval numbers.
Addition: $A+B = [a,b] + [c,d] =[a+c,b+d]$.
Subtraction: $A+B = [a,b] - [c,d] = [a-d,c-b]$.
Multiplication:$A*B = [x,y]$ where $x = \min\{ac,ad,bc,bd\}$ and $y = \max\{ac,ad,bc,bd\}$.

2.3 Definition

Let A = [a, b] and B = [c, d] are two interval numbers. Let $\alpha = \frac{a+b}{2}$ and $\beta = \frac{c+d}{2}$

If $\alpha \le \beta$ then A ≤ B. If $\alpha \ge \beta$ then A ≥ B.

2.4 Definition

Equivalent Interval number: Two interval numbers A = [a, b] and B = [c, d] are said to be equivalent if their crisp values [R(A)= R(B)] are equal.

2.5 Definition

Yager'sRanking Y ($\overline{A}$) = $\int_0^1 0.5(a_\alpha^L + a_\alpha^U)$where a_α^L=lower αcut,a_α^U =upper α cut. If Y ($\overline{s}$) <Y ($\overline{t}$)then $\overline{s} < \overline{t}$ .Yager's ranking technique satisfies Compensation, Linearity and additive properties.

III. PROPOSED ALGORITHM

The diagonal optimal algorithm is as follows:

- For each row/column, the fuzzy penalty is calculated by subtracting the minimum fuzzy cost and the next minimum fuzzy cost and put it against the corresponding row./column.
- Among all the penalties choose the maximum fuzzy penalty. If it is along the row find the minimum fuzzy cost in that corresponding row and remove the corresponding row and the corresponding column of the fuzzy element. If it is along the column locate the minimum fuzzy cost in that corresponding column and omit the corresponding row and the corresponding column of the fuzzy element.
- Let $\widetilde{a_{ij}}$ be the assigned cost for all the columns. Subtracting $\widetilde{a_{ij}}$ from each entry of $\widetilde{c_{ij}}$ the corresponding column of the assignment matrix
- Choose each unassigned cell, and form a rectangular loop in which one corner contains a negative penalty value and the remaining two corners are assigned cost values in corresponding row and column. Find the sum of diagonal cells of the unassigned element, say $\widetilde{d_{ij}}$. Choose the most negative $\widetilde{d_{ij}}$.and exchange the assigned cell of the diagonals. Repeat the process until all $\widetilde{d_{ij}}$.>$\tilde{0}$. If any$\widetilde{d_{ij}}$.≈$\tilde{0}$,then exchange the cells of diagonals at the end.

3.1 Numerical Example

Consider the TSP discussed by[12] where the cost matrix [C_{ij}] is given by

	A	**B**	**C**	**D**	**E**
A	∞	[0,7]	[8,15]	[4,11]	[1,12]
B	[-2,5]	∞	[2,11]	[-3,6]	[-2,5]
C	[2,9]	[3,13]	∞	[5,15]	[8,15]
D	[4,12]	[1,10]	[0,7]	∞	[-1,6]
E	[5,17]	[0,7]	[9,16]	[5,13]	∞

Applying the proposed algorithm,

	A	**B**	**C**	**D**	**E**	**Row penalty**
A	∞	[0,7]	[8,15]	[4,11]	[1,12]	[-6,12]
B	[-2,5]	∞	[2,11]	**[-3,6]**	[-2,5]	[-13,3]
C	[2,9]	[3,13]	∞	[5,15]	[8,15]	[-6,11]
D	[4,12]	[1,10]	[0,7]	∞	[-1,6]	[-6,8]
E	[5,17]	[0,7]	[9,16]	[5,13]	∞	[-2,13]
Column penalty	[-3,11]	[-6,10]	[-5,11]	[-2,14]	[-6,8]	

	A	**B**	**C**	**E**	**Row penalty**
A	∞	[0,7]	[8,15]	[1,12]	[-6,12]
C	[2,9]	[3,13]	∞	[8,15]	[-6,11]
D	[4,12]	[1,10]	**[0,7]**	[-1,6]	[-6,8]
E	[5,17]	[0,7]	[9,16]	∞	[-2,17]
Column penalty	[-5,10]	[-6,10]	[1,15]]	[-5,13]	

	A	**B**	**E**	**Row penalty**
A	∞	[0,7]	[1,12]	[-6,12]
C	[2,9]	[3,13]	[8,15]	[-6,11]
E	[5,17]	**[0,7]**	∞	[-2,17]
Column penalty	[-4,15]	[-4,13]	[-4,14]	

	A	E	Row penalty
A	∞	**[1,12]**	[1,12]
C	**[2,9]**	[8,15]	[-1,13]
Column penalty	[2,9]	[-4,14]	

The initial solution is

	A	B	C	D	E
A	∞	[0,7]	[8,15]	[4,11]	**[1,12]**
B	[-2,5]	∞	[2,11]	**[-3,6]**	[-2,5]
C	**[2,9]**	[3,13]	∞	[5,15]	[8,15]
D	[4,12]	[1,10]	**[0,7]**	∞	[-1,6]
E	[5,17]	**[0,7]**	[9,16]	[5,13]	∞

The optimum solution is

	A	B	C	D	E
$[C_{ij}, D_{ij}]$	**[2,9]**	**[0,7]**	**[0,7]**	**[-3,6]**	**[1,12]**
A	∞	[0,7]	[8,15]	[4,11]	**[1,12]**
B	[-2,5]	∞	[2,11]	**[-3,6]**	[-2,5]
C	**[2,9]**	[3,13]	∞	[5,15]	[8,15]
D	[4,12]	[1,10]	**[0,7]**	∞	[-1,6]
E	[5,17]	**[0,7]**	[9,16]	[5,13]	∞

	A	B	C	D	E
A	∞	[-7,7]	[1,15]	[-2,14]	**[-11,11]**
B	[-11,3]	∞	[-5,11]	**[-9,9]**	[-14,4]
C	**[-7,7]**	[-4,13]	∞	[-1,18]	[-4,14]
D	[-5,10]	[-6,10]	**[-7,7]**	∞	[-13,5]
E	[-4,15]	**[-7,7]**	[2,16]	[-1,16]	∞

$r_{12} = \begin{pmatrix} [-7,7] & [-11,11] \\ [-7,7] & \infty \end{pmatrix} \Rightarrow [-7,7] \geq 0.$

$r_{14} = \begin{pmatrix} [-2,14] & [-11,11] \\ [-9,9] & [-14,4] \end{pmatrix} \Rightarrow [-16,18] \geq 0.$

r_{21} = [-12,21] ≥ 0 , r_{23} = [-5,11] ≥ 0 , r_{25} = [-16,18] ≥ 0 , r_{32} = [-8,28] ≥ 0 , r_{34} = [-12,21] ≥ 0 , r_{35} = [-4,14] ≥ 0 r_{41} = [-5,10] ≥ 0 , r_{42} = [-4,26] ≥ 0, r_{45} = [-12,20] ≥ 0, r_{51} = [-8,28] ≥ 0 , r_{54} = [-1,16] ≥ 0

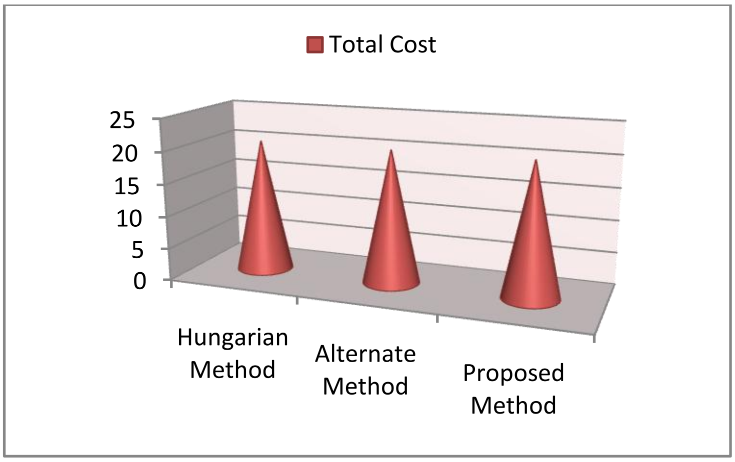
Since all $r_{ij} \geq 0$, the optimum solution is obtained. The final optimum allocation is

	A	B	C	D	E
A	∞	[0,7]	[8,15]	[4,11]	**[1,12]**
B	[-2,5]	∞	[2,11]	**[-3,6]**	[-2,5]
C	**[2,9]**	[3,13]	∞	[5,15]	[8,15]
D	[4,12]	[1,10]	**[0,7]**	∞	[-1,6]
E	[5,17]	**[0,7]**	[9,16]	[5,13]	∞

The optimum assignment is A→E→B→D→C→A. The route condition is satisfied.
The optimum solution is [1,12] + [-3,6] + [2,9] + [0,7] + [0,7] = [0,41].

COMPARISON

Methods	Total Cost
Hungarian Method [12]	**20.5**
Alternate Method [12]	**20.6**
Proposed Method	**20.5**



IV. CONCLUSION

Fuzzy -diagonal optimal algorithm is proposed for solving the travelling salesman problem. This method is very easy and also it is efficient to solve all kinds of fuzzy assignment problems. A numerical example is given to validate the algorithm and the results are verified .Also comparison is done with the existing methods.

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