

Inclusion Probability Proportional to Size Sampling Plan excluding Adjacent Units For linearly Ordered Samples

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ABSTRACT: The sample selection varies according to different sampling techniques. Probability proportional to size (PPS) is an unequal probability technique of sample selection. When the control is imposed on sample selection, it is called controlled sampling and the samples selected with such procedure is known as preferred samples.

In this paper, an inclusion probability proportional to size sampling plan excluding adjacent units (IPPSEA) separated by at most a distance of $m (\geq 1)$ in linearly ordered units using linear programming problem has been presented.

Keywords: IPPSEA, linear order, linear programming, Microsoft

I. INTRODUCTION

The purpose of sampling theory is to develop methods of sample selection and of estimation that best estimates the population parameter as well as precises our purpose. The most basic sample selection procedure is simple random sampling (SRS), providing an equal chance of selection to all the units in the sample space. When the sample selection of units varies according to size, such a sampling scheme is called the probability proportional to size (PPS) sampling. This scheme is coined by Hartley and Rao (1962)¹. Samford (1967)² describes the IPPS sampling scheme in his literature. The IPPS schemes was available for $n = 2$ but he worked for $n \geq 2$. The plan given by him ensures $\pi_{ij} > 0 \forall i \neq j = 1, 2, \dots, N$ and $\pi_{ij} < \pi_i \pi_j \forall i \neq j = 1, 2, \dots, N$ which is the sufficient condition for non-negativity of variance estimator. IPPS sampling procedures uses Horvitz-Thompson (1952)³ estimator for the estimation of variance. Nigam et al. (1985)⁴ discussed IPPS sampling. Gabler et al. (1987)⁵ gave 'nearest proportional to size sampling designs'. Inclusion probability proportional to size sampling schemes (IPPS) are the sampling schemes in which the first order inclusion probabilities are proportional to size measures. Woong (2005)⁶ suggested an optimal scheme of IPPS. Sahoo et al. (2006)⁷ discussed IPPS sampling scheme. Tiwari et al. (2007)⁸ proposed a one-dimensional optimal controlled IPPS sampling design ensuring zero probability to non-preferred samples. Deshpande and Ajaonkar (2008)⁹ discussed IPPS sampling scheme. Mandal et al. (2008)¹⁰ proposed inclusion probability proportional to size sampling plans excluding adjacent units (IPPSEA plans). IPPSEA plans may be obtained by trial-and-error methods using combinatorial properties of block designs. Mandel et al. (2008)¹⁰ also suggested linear programming approach to obtain IPPSEA plans based on SAS coding for circular as well as linear arrangement of the population units. Sahoo et al. (2010)¹¹ introduced a general class of IPPS sampling schemes. Sahoo et al. (2011)¹² constructed a new IPPS sampling scheme of two units for estimating the total of a finite population. Tiwari and Chilwal (2013)¹³ used a simplified selection scheme for unequal probability sampling without replacement. Ozturk (2020)¹⁴ constructed probability proportional to size ranked set sampling from a stratified population. In this paper, we have proposed the linear programming approach to obtain IPPSEA plans calculated by Microsoft 2019 for linear arrangements of the units. This approach has been discussed in the section 2, some examples described in section 3, followed by conclusion in section 4.

II. LINEAR PROGRAMMING APPROACH TO IPPSEA PLANS

In this section, we discuss a linear programming approach of obtaining an IPPSEA plan. A sample of n units to be drawn from population size N with varying probability without replacement for estimating the population mean $\bar{Y} = N^{-1} \sum_{i=1}^N y_i$. The sampling design is represented by $\{S, p_0(s) | s \in S\}$, where S , the sample space, is the set of all possible ${}^N C_n$ samples s and $p_0(s)$ is the probability of selecting the sample s . $\{p_0(s) | s \in S\}$ is the sampling plan and is also termed as the sampling design. Moreover, $\sum_{s \in S} p_0(s) = 1$.

2.1. Linear IPPS Plans Excluding Adjacent Units

Here, we consider the case of linear arrangement of population units. Let $\Omega_1 = \{(i, j) : \max\{i - j, j - i\} \leq m\}$ for $i \neq j$. Suppose $S_1 \subset S$ denotes the set of non-preferred samples, i.e., the samples which contain adjacent units separated up to a distance of m units, i.e., the pairs in Ω_1 . The similar approach has been used

which is described by Rao and Nigam (1990, 1992)^{15,16}, Then the optimal solution to the IPPS sampling plan excluding adjacent units using linear programming problem follows as:

Minimize the objective function $\phi = \sum_{s \in S_1} p(s)$ with respect to the variables $\{p_0(s) | s \in S\}$ subject to the linear constraints:

- i. $\sum_{s \ni i} p(s) = nP_i \quad \forall i = 1, 2, \dots, N$
- ii. $\sum_{s \ni i, j} p(s) = 0$, if $\max(i - j, j - i) \leq m$, $i \neq j = 1, 2, \dots, N$ equation (1)
- iii. $p(s) \geq 0$ for all $s \in S$
- iv. $\sum_{s \in S} p(s) = 1$

III. EMPIRICAL EXAMPLES

Example 1. Consider a population with $N=7$, $n=2$ and $m=1$ with initial probability of selection units as given below:

Units (i)	1	2	3	4	5	6	7
Initial probability of selection	0.10	0.12	0.14	0.15	0.15	0.16	0.18

The sample space S consists of all possible $\binom{7}{2} = 21$ samples of size 2 and S_1 consists of the samples which contains the pair (i, j) for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 7$.

Sample No.	Samples	Sample No.	Samples
1	1,2	12	3,4
2	1,3	13	3,5
3	1,4	14	3,6
4	1,5	15	3,7
5	1,6	16	4,5
6	1,7	17	4,6
7	2,3	18	4,7
8	2,4	19	5,6
9	2,5	20	5,7
10	2,6	21	6,7
11	2,7		

These 21 possible samples are denoted by s_1, s_2, \dots, s_{21} and their probabilities are $p(s_1), p(s_2), \dots, p(s_{21})$. Let us denote these probabilities by p_1, p_2, \dots, p_{21} respectively.

The preferred samples are the samples without contiguous units which as follows:

Sample No	Probability	Samples
s_2	p_2	1,3
s_3	p_3	1,4
s_4	p_4	1,5
s_5	p_5	1,6
s_6	p_6	1,7
s_8	p_8	2,4
s_9	p_9	2,5
s_{10}	p_{10}	2,6
s_{11}	p_{11}	2,7
s_{13}	p_{13}	3,5
s_{14}	p_{14}	3,6
s_{15}	p_{15}	3,7

Sample No	Probability	Samples
S_{17}	p_{17}	4,6
S_{18}	p_{18}	4,7
S_{20}	p_{20}	5,7

When samples are arranged then it is observed that S_1 consists of sample numbers 1, 7, 12, 16, 19, 21. Hence, objective function is

$$\begin{aligned} \phi &= p(s_1) + p(s_7) + p(s_{12}) + p(s_{16}) + p(s_{19}) + p(s_{21}) \\ &= p(s_1) + 0.p(s_2) + 0.p(s_3) + 0.p(s_4) + 0.p(s_5) + 0.p(s_6) + p(s_7) + 0.p(s_8) + 0.p(s_9) + 0.p(s_{10}) \\ &\quad + 0.p(s_{11}) + p(s_{12}) + 0.p(s_{13}) + 0.p(s_{14}) + 0.p(s_{15}) + p(s_{16}) + 0.p(s_{17}) \\ &\quad + 0.p(s_{18}) + p(s_{19}) + 0.p(s_{20}) + p(s_{21}) \end{aligned}$$

Unit 1 appears in 5 samples. For $i = 1$, the constraint (i) is

$$\pi_1 = p_2 + p_3 + p_4 + p_5 + p_6$$

Similarly, other constraints for $i = 2, 3, \dots, 7$ are set.

As per constraint of equation $p_1 + p_2 + \dots + p_{21} = 1$

Then the first order inclusion probabilities are:

$$\pi_1 = p_2 + p_3 + p_4 + p_5 + p_6$$

$$\pi_2 = p_8 + p_9 + p_{10} + p_{11}$$

$$\pi_3 = p_2 + p_{13} + p_{14} + p_{15}$$

$$\pi_4 = p_3 + p_8 + p_{17} + p_{18}$$

$$\pi_5 = p_4 + p_9 + p_{13} + p_{20}$$

$$\pi_6 = p_5 + p_{10} + p_{14} + p_{17}$$

$$\pi_7 = p_{11} + p_{15} + p_{18} + p_{20}$$

From equations for $N=7, n=2$ and $m=1$, the first order inclusion probabilities are:

$$\pi_i = nP_i \quad i = 1, 2, \dots, 7$$

π_1	0.20
π_2	0.24
π_3	0.28
π_4	0.30
π_5	0.30
π_6	0.32
π_7	0.36

And second order inclusion probabilities are:

$$\pi_{ij} = 0 \quad ; \quad \delta(i, j) \geq m, \quad i \neq j = 1, 2, \dots, N$$

The constraints are arranged according to equation (1) and minimizing the objective function gives the following optimal solution with $\phi = 0$.

Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 1.

Table 1. Linear IPPSEA plan for $N=7, n=2$ and $m=1$.

s_i	$p(s)$	s_i	$p(s)$
s_1	0	s_{12}	0
s_2	0.024473	s_{13}	0.051031
s_3	0.136539	s_{14}	0.086324
s_4	0.038988	s_{15}	0.118172
s_5	0	s_{16}	0
s_6	0	s_{17}	0.154492
s_7	0	s_{18}	0
s_8	0.008968	s_{19}	0

s_9	0.06	s_{20}	0.149981
s_{10}	0.079184	s_{21}	0
s_{11}	0.091848		

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 7$. Therefore, the above sampling plan is IPPSEA plan.

Example 2. Consider a population with $N=9, n=3$ and $m=1$ with initial probability of selection units as given below:

Units (i)	1	2	3	4	5	6	7	8	9
Initial probability of selection	0.137	0.046	0.172	0.08	0.073	0.146	0.082	0.237	0.027

The sample space S consists of all possible $\binom{9}{3} = 84$ samples of size 3 and S_1 consists of the samples which contains the pair (i, j) for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 9$. These 84 possible samples are denoted by s_1, s_2, \dots, s_{84} and their probabilities are $p(s_1), p(s_2), \dots, p(s_{84})$. Let us denote these probabilities by p_1, p_2, \dots, p_{84} respectively.

The preferred samples are the samples without contiguous units which as follows:

Sample No	Probability	Samples	Sample No	Probability	Samples
s_9	p_9	1,3,5	s_{39}	p_{39}	2,4,9
s_{10}	p_{10}	1,3,6	s_{41}	p_{41}	2,5,7
s_{11}	p_{11}	1,3,7	s_{42}	p_{42}	2,5,8
s_{12}	p_{12}	1,3,8	s_{43}	p_{43}	2,5,9
s_{13}	p_{13}	1,3,9	s_{45}	p_{45}	2,6,8
s_{15}	p_{15}	1,4,6	s_{46}	p_{46}	2,6,9
s_{16}	p_{16}	1,4,7	s_{48}	p_{48}	2,7,9
s_{17}	p_{17}	1,4,8	s_{56}	p_{56}	3,5,7
s_{18}	p_{18}	1,4,9	s_{57}	p_{57}	3,5,8
s_{20}	p_{20}	1,5,7	s_{58}	p_{58}	3,5,9
s_{21}	p_{21}	1,5,8	s_{60}	p_{60}	3,6,8
s_{22}	p_{22}	1,5,9	s_{61}	p_{61}	3,6,9
s_{24}	p_{24}	1,6,8	s_{63}	p_{63}	3,7,9
s_{25}	p_{25}	1,6,9	s_{70}	p_{70}	4,6,8
s_{27}	p_{27}	1,7,9	s_{71}	p_{71}	4,6,9
s_{36}	p_{36}	2,4,6	s_{73}	p_{73}	4,7,9
s_{37}	p_{37}	2,4,7	s_{79}	p_{79}	5,7,9
s_{38}	p_{38}	2,4,8			

When samples are arranged then it is observed that S_1 consists of sample numbers 1, 2, 3, 4, 5, 6, 7, 8, 14, 19, 23, 26, 28, 29, 30, 31, 32, 33, 34, 35, 40, 44, 47, 49, 50, 51, 52, 53, 54, 55, 59, 62, 64, 65, 66, 67, 68, 69, 72, 74, 75, 76, 77, 78, 80, 81, 82, 83 and 84. Hence, objective function is $\phi = p(s_1) + p(s_2) + \dots + p(s_8) + p(s_{14}) + p(s_{19}) + p(s_{23}) + \dots + p(s_{84})$
 $= p(s_1) + p(s_2) + \dots + p(s_8) + 0.p(s_9) + 0.p(s_{10}) + 0.p(s_{11}) + 0.p(s_{12}) + 0.p(s_{13})$
 $+ p(s_{14}) + \dots + p(s_{84})$

Now, computing the first order inclusion probabilities just as example 1 and we get:

$$\pi_i = nP_i \forall i = 1, 2, \dots, 9$$

π_1	0.411
π_2	0.138
π_3	0.516
π_4	0.24
π_5	0.219
π_6	0.438
π_7	0.246
π_8	0.711
π_9	0.081

And second order inclusion probabilities are:

$$\pi_{ij} = 0 ; \delta(i, j) \geq m , i \neq j = 1, 2, \dots, N$$

The constraints arranged according to equation (1) and minimizing the objective function gives the following optimal solution with $\phi = 0$.

Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 2.

Table 2. Linear IPPSEA plan for N=9, n=3 and m=1.

s_i	$p(s)$	s_i	$p(s)$	s_i	$p(s)$
s_1	0	s_{29}	0	s_{57}	0.057739
s_2	0	s_{30}	0	s_{58}	0.000435
s_3	0	s_{31}	0	s_{59}	0
s_4	0	s_{32}	0	s_{60}	0.118321
s_5	0	s_{33}	0	s_{61}	0.013188
s_6	0	s_{34}	0	s_{62}	0
s_7	0	s_{35}	0	s_{63}	0.013188
s_8	0	s_{36}	0	s_{64}	0
s_9	0.000435	s_{37}	0.012228	s_{65}	0
s_{10}	0	s_{38}	0.01853	s_{66}	0
s_{11}	0.053216	s_{39}	0	s_{67}	0
s_{12}	0.167169	s_{40}	0	s_{68}	0
s_{13}	0.000434	s_{41}	0.017327	s_{69}	0
s_{14}	0	s_{42}	0.017327	s_{70}	0.115302
s_{15}	0	s_{43}	0	s_{71}	0.014052
s_{16}	0.032919	s_{44}	0	s_{72}	0
s_{17}	0.032919	s_{45}	0	s_{73}	0.014052
s_{18}	0	s_{46}	0	s_{74}	0
s_{19}	0	s_{47}	0	s_{75}	0
s_{20}	0	s_{48}	0	s_{76}	0
s_{21}	0.021012	s_{49}	0	s_{77}	0
s_{22}	0	s_{50}	0	s_{78}	0

s_{23}	0	s_{51}	0	s_{79}	0.012849
s_{24}	0.088655	s_{52}	0	s_{80}	0
s_{25}	0.015176	s_{53}	0	s_{81}	0
s_{26}	0	s_{54}	0	s_{82}	0
s_{27}	0	s_{55}	0	s_{83}	0
s_{28}	0	s_{56}	0.091875	s_{84}	0

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 9$. Therefore, the above sampling plan is IPPSEA plan.

Example 3. Consider a population with $N=10, n=3$ and $m=1$ with initial probability of selection units as given below:

Units (i)	1	2	3	4	5	6	7	8	9	10
Initial probability of selection	0.18	0.14	0.13	0.11	0.10	0.10	0.08	0.07	0.05	0.04

The sample space S consists of all possible $\binom{10}{3} = 120$ samples of size 3 and S_1 consists of the samples which contains the pair (i, j) for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 10$. These 84 possible samples are denoted by s_1, s_2, \dots, s_{120} and their probabilities are $p(s_1), p(s_2), \dots, p(s_{120})$. Let us denote these probabilities by p_1, p_2, \dots, p_{120} respectively. The preferred samples are the samples without contiguous units which as follows:

Sample No	Probability	Samples	Sample No	Probability	Samples
s_{10}	p_{10}	1,3,5	s_1	p_{53}	2,5,9
s_{11}	p_{11}	1,3,6	s_2	p_{54}	2,5,10
s_{12}	p_{12}	1,3,7	s_3	p_{56}	2,6,8
s_{13}	p_{13}	1,3,8	s_4	p_{57}	2,6,9
s_{14}	p_{14}	1,3,9	s_5	p_{58}	2,6,10
s_{15}	p_{15}	1,3,10	s_6	p_{60}	2,7,9
s_{17}	p_{17}	1,4,6	s_7	p_{61}	2,7,10
s_{18}	p_{18}	1,4,7	s_8	p_{63}	2,8,10
s_{19}	p_{19}	1,4,8	s_9	p_{72}	3,5,7
s_{20}	p_{20}	1,4,9	s_{10}	p_{73}	3,5,8
s_{21}	p_{21}	1,4,10	s_{11}	p_{74}	3,5,9
s_{23}	p_{23}	1,5,7	s_{12}	p_{75}	3,5,10
s_{24}	p_{24}	1,5,8	s_{13}	p_{77}	3,6,8
s_{25}	p_{25}	1,5,9	s_{14}	p_{78}	3,6,9
s_{26}	p_{26}	1,5,10	s_{15}	p_{79}	3,6,10
s_{28}	p_{28}	1,6,8	s_{16}	p_{81}	3,7,9
s_{29}	p_{29}	1,6,9	s_{17}	p_{82}	3,7,10
s_{30}	p_{30}	1,6,10	s_{18}	p_{84}	3,8,10
s_{32}	p_{32}	1,7,9	s_{19}	p_{92}	4,6,8
s_{33}	p_{33}	1,7,10	s_{20}	p_{93}	4,6,9
s_{35}	p_{35}	1,8,10	s_{21}	p_{94}	4,6,10
s_{45}	p_{45}	2,4,6	s_{22}	p_{96}	4,7,9

Sample No	Probability	Samples	Sample No	Probability	Samples
s_{46}	p_{46}	2,4,7	s_{23}	p_{97}	4,7,10
s_{47}	p_{47}	2,4,8	s_1	p_{99}	4,8,10
s_{48}	p_{48}	2,4,9	s_2	p_{106}	5,7,9
s_{49}	p_{49}	2,4,10	s_3	p_{107}	5,7,10
s_{51}	p_{51}	2,5,7	s_4	p_{109}	5,8,10
s_{52}	p_{52}	2,5,8	s_5	p_{115}	6,8,10

When samples are arranged then it is observed that S_1 consists of sample numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 16, 22, 27, 31, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 55, 59, 62, 64, 65, 66, 67, 68, 69, 70, 71, 76, 80, 83, 85, 86, 87, 88, 89, 90, 91, 95, 98, 100, 101, 102, 103, 104, 105, 108, 110, 111, 112, 113, 114, 116, 117, 118, 119 and 120. Hence, objective function is

$$\begin{aligned} \phi &= p(s_1) + p(s_2) + \dots + p(s_8) + p(s_9) + p(s_{16}) + p(s_{22}) + \dots + p(s_{120}) \\ &= p(s_1) + p(s_2) + \dots + p(s_8) + p(s_9) + \dots + p(s_{16}) + \dots + 0 \cdot p(s_{21}) + p(s_{22}) + \dots + 0 \cdot p(s_{23}) \\ &\quad + 0 \cdot p(s_{24}) + \dots + p(s_{120}) \end{aligned}$$

Now, computing the first order inclusion probabilities just as example 1 and we get:

$$\pi_i = nP_i \forall i = 1, 2, \dots, 10$$

π_1	0.54
π_2	0.42
π_3	0.39
π_4	0.33
π_5	0.30
π_6	0.30
π_7	0.24
π_8	0.21
π_9	0.15
π_{10}	0.12

And second order inclusion probabilities are:

$$\pi_{ij} = 0 \quad ; \quad \delta(i, j) \geq m \quad , \quad i \neq j = 1, 2, \dots, N$$

The constraints arranged according to equation (1) and minimizing the objective function gives the following optimal solution with $\phi = 0$.

Now, giving the first order inclusion probabilities and solving by Microsoft 2019 Package and the above formulation of linear programming problem gives the following sampling plan given in Table 3.

Table 3. Linear IPPSEA plan for N=10, n=3 and m=1.

s_i	$p(s)$	s_i	$p(s)$	s_i	$p(s)$
s_1	0	s_{41}	0	s_{81}	0.003878
s_2	0	s_{42}	0	s_{82}	0
s_3	0	s_{43}	0	s_{83}	0
s_4	0	s_{44}	0	s_{84}	0.00014
s_5	0	s_{45}	0.053098	s_{85}	0
s_6	0	s_{46}	0.031264	s_{86}	0
s_7	0	s_{47}	0.026684	s_{87}	0
s_8	0	s_{48}	0.018644	s_{88}	0
s_9	0	s_{49}	0.014293	s_{89}	0
s_{10}	0.072543	s_{50}	0	s_{90}	0

s_{11}	0.128821	s_{51}	0.163081	s_{91}	0
s_{12}	0.007477	s_{52}	0.025609	s_{92}	0
s_{13}	0.099127	s_{53}	0.017641	s_{93}	0
s_{14}	0	s_{54}	0.013218	s_{94}	0.005037
s_{15}	0.070259	s_{55}	0	s_{95}	0.000614
s_{16}	0	s_{56}	0.025609	s_{96}	0.005037
s_{17}	0.039147	s_{57}	0.017641	s_{97}	0.000614
s_{18}	0.024617	s_{58}	0.013218	s_{98}	0
s_{19}	0.020036	s_{59}	0	s_{99}	0
s_{20}	0.070333	s_{60}	0	s_{100}	0
s_{21}	0.007645	s_{61}	0	s_{101}	0
s_{22}	0	s_{62}	0	s_{102}	0
s_{23}	0	s_{63}	0	s_{103}	0
s_{24}	0	s_{64}	0	s_{104}	0
s_{25}	0	s_{65}	0	s_{105}	0
s_{26}	0	s_{66}	0	s_{106}	0.004033
s_{27}	0	s_{67}	0	s_{107}	0
s_{28}	0	s_{68}	0	s_{108}	0
s_{29}	0	s_{69}	0	s_{109}	0
s_{30}	0	s_{70}	0	s_{110}	0
s_{31}	0	s_{71}	0	s_{111}	0
s_{32}	0	s_{72}	0	s_{112}	0
s_{33}	0	s_{73}	0	s_{113}	0
s_{34}	0	s_{74}	0.003878	s_{114}	0
s_{35}	0	s_{75}	0	s_{115}	0
s_{36}	0	s_{76}	0	s_{116}	0
s_{37}	0	s_{77}	0	s_{117}	0
s_{38}	0	s_{78}	0.003878	s_{118}	0
s_{39}	0	s_{79}	0	s_{119}	0
s_{40}	0	s_{80}	0	s_{120}	0

It is observed that the inclusion probabilities are proportional to initial probability of selection of the units and $\pi_{ij} = 0$ for $\delta(i, j) = 1, i \neq j = 1, 2, \dots, 10$. Therefore, the above sampling plan is IPPSEA plan.

IV. CONCLUSION

Some previous researchers have given IPPSEA plan based on SAS coding. Comparing my methodology with them I observed that in proposed methodology, no. of preferred samples selected are more. Again, probability of these preferred samples are also high in most of the cases. This methodology of IPPSEA is more efficient on comparing with the SRSWOR. SAS software is an expensive software and one needs to a license to operate it. It is not affordable by everyone as well as its courses are also costly. One must have an interest in complex programming to understand it easily, whereas Microsoft is available with installed windows in computers and it is accessible to all. It is simple to use and ease the calculation to understand. I can say that this methodology is less cumbersome and equally or more efficient than previous work.

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