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Chirped Envelope Optical Solitons For Non-paraxial Nonlinear Schrödinger Equation

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ABSTRACT: *The purpose of this paper is to search the exact nonlinearly chirped soliton solutions of non-paraxial nonlinear Schrödinger equation. Firstly, we apply a special complex envelope traveling-wave method the non-paraxial nonlinear Schrödinger equation. Complex envelope traveling-wave solution is used to reduce the governing equation to an ordinary differential equation. Secondly, we introduce a new chirping ansatz given as an expansion in powers of intensity of the light. It is shown that the phase associated to the obtained pulses has a nontrivial form and possesses an intensity dependent chirping terms. By using this ansatz the ordinary differential equation has been reduced to an elliptic differential equation with a third-degree nonlinear term describing the dynamics of field amplitude in the nonlinear media. Lastly, using an auxiliary equation to construct the analytical chirped solutions of the non-paraxial nonlinear Schrödinger equation. The resulting amplitude equation is then solved to get exact analytical chirped rational, exponential, triangular periodic, solitary wave and Weierstrass doubly periodic solutions for the model. As a result, we derive families of chirped soliton solutions under certain parametric conditions.*

KEYWORDS: *The non-paraxial nonlinear Schrödinger equation, Chirped soliton, Envelope solution.*

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I. INTRODUCTION

The potential applications of nonlinear optical devices in the field of information technology have motivated the increase in research in recent years. Mathematical modeling of dense optical pulse propagation physics via nonlinear Schrödinger (NLS) equations finds applications in numerous fields. Optical parametric amplifiers and oscillators, second harmonic generation and third harmonic generation, Raman scattering, optical duality and solitons are just some of the physical phenomena that have wide practical application [1-2].

Optical beams: NLS-type equations provide an adequate explanation if (i) they are much wider than the carrier wavelength, (ii) low enough intensity, and (iii) propagate along the reference axis. These criteria define the paraxial approach. If all three conditions are not met at the same time, the beam is called "non-paraxial". Non-paraxial beams have received much attention in the literature over the past three decades. While soliton beams are usually solutions of paraxial wave equations, such as the NLS equation, many important information technology applications involve the breakdown of paraxial approximation. When linear effects such as dispersion, diffraction, or propagation are fully balanced with nonlinearity, self-phase modulation, self-focusing or reaction kinetic properties, respectively, solitons can arise [3].

This study is concerned with one-dimensional non-paraxial nonlinear Schrödinger (NNLS) equation of the form

$$\alpha q_{tt} + i q_t + \beta q_{xx} + |q|^2 q = 0, \tag{1}$$

here $q = q(x,t)$ is a complex-valued function of $x \in \Re$ and $t \ge 0$. The parameters α and β are positive. In Eq.(1), non-paraxial effects are mathematically represented by including the quadratic time derivative term and its associated parameter α. Non-paraxiality can occur in miniaturization of devices and in other configurations such as those containing multiplexed beams [4-6].

II. MATHEMATICAL ANALYSIS

It is of aim to find exact chirped soliton solutions of NNLS equation. Firstly, we want to find traveling-wave solutions of Eq.(1) in the form [7-12]

$$q(x,t) = \rho(\xi) e^{i(\chi(\xi) - \omega t)}, \tag{2}$$

where $\xi = kx - vt$, the amplitude and the phase functions denoted by $\rho = \rho(\xi)$ and $\chi = \chi(\xi)$, respectively. In addition, v and ω indicate the wave velocity and the frequency of the wave oscillation, respectively. The chirp is defined by

$$\delta\omega(x,t) = -\frac{\partial}{\partial t}[\chi(\xi) - \omega t] = -\chi'(\xi). \tag{3}$$

By Eq.(2) placed in Eq.(1), the real and imaginary parts provide a pair of relations in two dependent varibles ρ and χ . Real part is equal to

$$\rho^3 + \rho\omega - \alpha\rho\omega^2 + v\rho\chi' - 2v\alpha\rho\omega\chi' - v^2\alpha\rho(\chi')^2 - k^2\beta\rho(\chi')^2 + k^2\beta\rho'' + \alpha v^2\rho'' = 0, \tag{4}$$

while imaginary part deducible as

$$-v\rho' + 2v\alpha\omega\rho' + 2v^2\alpha\rho'\chi' + 2k^2\beta\rho'\chi' + k^2\beta\rho\chi'' + \alpha v^2\rho\chi'' = 0, \tag{5}$$

where primes express the differentiations with respect to ξ . Mutiplying Eq.(5) by ρ and integrating once, the followingequation is obtained:

$$\chi' = \frac{A}{(v^2\alpha + k^2\beta)\rho^2} + \frac{v(1-2\alpha\omega)}{2(v^2\alpha + k^2\beta)}, \tag{6}$$

where *A* is integration constant. Therefore, the resulting chirp is obtained as:

$$\delta\omega = -\frac{A}{(v^2\alpha + k^2\beta)\rho^2} + \frac{v(2\alpha\omega - 1)}{2(v^2\alpha + k^2\beta)}, \tag{7}$$

which indicates the chirping has an intensity dependent chirping term. When Eq.(6) substituing in Eq.(4) gets

$$-\frac{4A^2}{(v^2\alpha + k^2\beta)\rho^3} + \frac{\rho\left(v^2 - 4k^2\beta\omega(-1+\alpha\omega)\right)}{v^2\alpha + k^2\beta} + 4\rho^3 + 4(v^2\alpha + k^2\beta)\rho'' = 0. \tag{8}$$

Multiplying Eq.(8) by ρ' and integrating with respect to ξ, we have

$$(\rho')^2 = \frac{B}{2(v^2\alpha + k^2\beta)} - \frac{A^2}{4(v^2\alpha + k^2\beta)^2\rho^2} + \left(\frac{-v^2 - 4k^2\beta\omega + 4k^2\alpha\beta\omega^2}{16(v^2\alpha + k^2\beta)^2}\right)\rho^2 - \frac{1}{2(v^2\alpha + k^2\beta)}\rho^4. \tag{9}$$

where *B* is second integration constant. Eq.(9) describes the evolution of wave amplitude in nonlinear environment that governed by Eq.(1).

III. CHIRPED SOLITON SOLUTIONS

We offer distinct chirped soliton solutions for the Eq.(1) under different parametric conditions. Before finding exact solutions to Eq.(9), by changing variables of amplitude function of the form

$$\rho^2(\xi) = U(\xi), \tag{10}$$

Thus, Eq.(9) converts the following auxiliary elliptic equation [13-18]:

$$(U')^2 = a_0 + a_1 U + a_2 U^2 + a_3 U^3, \tag{11}$$

where

$$a_0 = -\frac{4A^2}{(v^2\alpha + k^2\beta)^2}, \quad a_1 = \frac{2B}{v^2\alpha + k^2\beta}, \quad a_2 = \frac{-v^2 - 4k^2\beta\omega + 4k^2\alpha\beta\omega^2}{4(v^2\alpha + k^2\beta)^2}, \quad a_3 = -\frac{2}{v^2\alpha + k^2\beta}. \tag{12}$$

1st case: Eq. (11) has a polynomial type solutions:

i) For $a_1 = a_2 = a_3 = 0,\ a_0 > 0$,

$$U(\xi) = \sqrt{a_0}\,\xi. \tag{13}$$

Based on above finding, we get the first chirped soliton solution for Eq.(1) of the form

$$q(x,t) = \left[\sqrt{a_0}\,\xi\right]^{\frac{1}{2}} e^{i[\chi(\xi) - \omega t]}. \tag{14}$$

The chirping takes the form

$$\delta\omega = \frac{v}{2k^2} - \frac{A}{k^2\sqrt{a_0}\,\xi}. \tag{15}$$

ii) For $a_1 \neq 0,\ a_2 = a_3 = 0$,

$$U(\xi) = -\frac{a_0}{a_1} + \frac{1}{4}a_1\xi^2. \tag{16}$$

We get the chirped soliton solution for Eq.(1) of the form

$$q(x,t)=\left[-\frac{a_0}{a_1}+\frac{1}{4}a_1\xi^2\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}. \tag{17}$$

Then, the chirping will be

$$\delta\omega=\frac{v}{2k^2}-\frac{A}{k^2-\frac{a_0}{a_1}+\frac{1}{4}a_1\xi^2}. \tag{18}$$

iii) For $a_0=a_1=a_2=0$,

$$U(\xi)=\frac{1}{a_3\xi^2}. \tag{19}$$

The chirped soliton solution for Eq.(1) of the form

$$q(x,t)=\frac{1}{\sqrt{a_3}\xi}e^{i[\chi(\xi)-\omega t]}. \tag{20}$$

And, the corresponding chirping is

$$\delta\omega=\frac{v}{2k^2}-\frac{Aa_3\xi^2}{k^2}. \tag{21}$$

2nd case: For $a_0=\frac{a_1^2}{4a_2},\ a_2>0,\ a_3=0$, Eq.(11) has an exponential type solution:

$$U(\xi)=-\frac{a_1}{2a_2}+\exp\left(\sqrt{a_2}\xi\right). \tag{22}$$

The chirped soliton solution

$$q(x,t)=\left[-\frac{a_1}{2a_2}+\exp\left(\sqrt{a_2}\xi\right)\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}. \tag{23}$$

The corresponding chirping is

$$\delta\omega=\frac{v}{2k^2}-\frac{A}{k^2\left(-\frac{a_1}{2a_2}+\exp\left(\sqrt{a_2}\xi\right)\right)}. \tag{24}$$

3rd case: Eq.(11) has a triangular type solutions:

i) For $a_0=0,\ a_2<0,\ a_3=0$,

$$U(\xi)=-\frac{a_1}{2a_2}+\frac{a_1}{2a_2}\sin\left(\sqrt{-a_2}\xi\right). \tag{25}$$

The chirped soliton solution

$$q(x,t)=\left[-\frac{a_1}{2a_2}+\frac{a_1}{2a_2}\sin\left(\sqrt{-a_2}\xi\right)\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}. \tag{26}$$

The corresponding chirping is

$$\delta\omega=\frac{v}{2k^2}-\frac{2a_2A}{a_1k^2\left(\sin\left(\sqrt{-a_2}\xi\right)-1\right)}. \tag{27}$$

ii) For $a_0=a_1=0,\ a_2<0$,

$$U(\xi)=-\frac{a_2}{a_3}\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right). \tag{28}$$

The chirped soliton solution

$$q(x,t)=\left[-\frac{a_2}{a_3}\sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right)\right]^{\frac{1}{2}}e^{i[\chi(\xi)-\omega t]}. \tag{29}$$

The corresponding chirping is

$$\delta\omega = \frac{v}{2k^2} + \frac{a_3 A}{a_2 k^2 \sec^2\left(\frac{\sqrt{-a_2}}{2}\xi\right)}. \quad (30)$$

4th case: Eq.(11) has a hyperbolic type solutions:

i) For $a_0 = 0,\ a_2 > 0,\ a_3 = 0$,

$$U(\xi) = -\frac{a_1}{2a_2} + \frac{a_1}{2a_2}\sinh\left(\sqrt{a_2}\xi\right). \quad (31)$$

The chirped soliton solution

$$q(x,t) = \left[-\frac{a_1}{2a_2} + \frac{a_1}{2a_2}\sinh\left(\sqrt{a_2}\xi\right)\right]^{\frac{1}{2}} e^{i[\chi(\xi)-\omega t]}. \quad (32)$$

The chirping is given as

$$\delta\omega = \frac{v}{2k^2} - \frac{2a_2 A}{a_1 k^2\left(\sinh\left(\sqrt{a_2}\xi\right)-1\right)}. \quad (33)$$

ii) For $a_0 = a_1 = 0,\ a_2 > 0$,

$$U(\xi) = -\frac{a_2}{a_3}\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right). \quad (34)$$

The chirped soliton solution

$$q(x,t) = \left[-\frac{a_2}{a_3}\operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right)\right]^{\frac{1}{2}} e^{i[\chi(\xi)-\omega t]}. \quad (35)$$

And the chirping will be

$$\delta\omega = \frac{v}{2k^2} + \frac{a_3 A}{a_2 k^2 \operatorname{sech}^2\left(\frac{\sqrt{a_2}}{2}\xi\right)}. \quad (36)$$

5th case: For the last case $a_2 = 0,\ a_3 > 0$ **,** Eq.(11) has a Weierstrass elliptic doubly periodic type solution:

$$U(\xi) = \wp\left(\frac{\sqrt{a_3}}{2}\xi, g_2, g_3\right), \quad (37)$$

where $g_2 = -\frac{4a_1}{a_3}$ and $g_3 = -\frac{4a_0}{a_3}$ are called invariants of Weierstrass elliptic function.

The chirped soliton solution

$$q(x,t) = \left[\wp\left(\frac{\sqrt{a_3}}{2}\xi, g_2, g_3\right)\right]^{\frac{1}{2}} e^{i[\chi(\xi)-\omega t]}. \quad (38)$$

The corresponding chirping takes the form

$$\delta\omega = \frac{v}{2k^2} - \frac{A}{k^2\left(\wp\left(\frac{\sqrt{a_3}}{2}\xi, g_2, g_3\right)\right)}. \quad (39)$$

IV. CONCLUSION

In this study, we have obtained chirped rational, periodic, the hyperbolic and Weierstrass elliptic function solutions for the NNLS equation. Introducing a new ansatz which including a new form of chirping, the solutions searched for a general third order elliptic equation containing many parameters. It has been shown that the wave amplitude satisfies a nonlinear differential equation containing two integration constants such that ca be easily obtained by initial parameters of the wave. We solved the resulting amplitude equation analytically and obtained findings for rational, periodic, the hyperbolic and Weierstrass elliptic function solutions. We have identified the nonlinear chirp associated with each of these soliton solutions.

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